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CHEMICAL ENGINEERING SERIES

ELEMENTS OF CHEMICAL ENGINEERING

BY

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AND

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WITH AN INTRODUCTION
BY
ARTHUR D. LITTLE

SECOND EDITION
SEVENTEENTH IMPRESSION

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PREFACE TO THE SECOND EDITION

Enough time has elapsed since writing the first edition so that appreciable contributions to many of the subjects treated in this book have appeared in the literature. Our effort in the second edition has been to incorporate as much as possible of this new work without departing from our original idea of an elementary text. Some parts have been rewritten in an attempt to achieve greater clarity or simplicity in the presentation.

The principal changes are in the chapters on Flow of Heat, Evaporation, Drying, Distillation, Gas Absorption, Extraction, and Filtration. Minor changes have been made in other chapters where this seemed desirable. In conformity with recent tendencies, dimensionally consistent units have been used in place of miscellaneous engineering units. The symbols used have been made to correspond, so far as possible, to the recommendations of the Committee on Standard Symbols and Nomenclature of the American Institute of Chemical Engineers.

We wish to acknowledge many helpful suggestions offered by users of the book.

W. L. BADGER.
W. L. McCABE.

ANN ARBOR, MICH.,
August, 1936.

PREFACE TO THE FIRST EDITION

The point of view that chemical engineering is based on the unit operations and their industrial application is relatively recent. Detailed study of the theory of these operations and a rational approach to the design of equipment to carry them out is equally recent. It is, therefore, not surprising that thus far only one important book has appeared to cover this field, namely, "Principles of Chemical Engineering," by Walker, Lewis, and McAdams. This excellent pioneer work, originally published in 1923, was the first to emphasize the fundamental character of the unit operations, and it received hearty approval from chemical engineers.

It is, therefore, with some hesitation that we offer the profession a similar book covering so nearly the same field. Our only excuse for attempting this is that, in spite of the excellence of "Principles of Chemical Engineering," it is rather difficult for beginning students, or for practicing engineers who have not had some training in the underlying theories of the unit operations. Furthermore, the treatment emphasizes the theoretical part of the subject to such an extent that the beginning student, who has had no practical experience and consequently is not familiar with the appearance and performance of equipment on the commercial scale, finds it difficult to realize the concrete application of the theory, even if he can understand it in the abstract.

From a comparison of the titles of this book and the one mentioned above, a general impression of the relation between the two may be obtained. In comparison with "Principles of Chemical Engineering," the present book begins its theoretical discussions from a simpler and more elementary viewpoint. We have tried to develop the theories so begun in the simplest and most systematic manner, even at the expense of being wordy. We have not carried the theory in most cases so far as it has been carried by Walker, Lewis, and McAdams. On the other

hand, important branches of the theory have not been omitted solely because they are difficult. No apology is offered for the use of calculus throughout the text, but graphical methods have been emphasized to render the use of calculus as simple and powerful as possible. We have introduced considerable descriptive matter to provide a background against which the beginning student may orient the theoretical part.

In introducing the descriptive matter, we have made a particular effort to classify the equipment in each field so that a description of the types will have some semblance of organization. Where possible we have tried to distinguish between the fields of usefulness of different types of equipment for carrying out the same process, and in some cases we have tried to correlate features of their design with the theory of the operations they are to perform. We hope that this descriptive material will not be looked on as merely extracts from trade catalogues, but as a serious attempt to correlate the widely diversified types of equipment that the chemical engineer must use.

No pains have been spared in preparing the illustrations. Every one is a line drawing, and all have been prepared especially for this book. In the illustrations of equipment, all unnecessary details have been omitted, but we hope that the figures will emphasize the fact that equipment must be built of definite parts that must be designed, fabricated, and assembled. For this reason the parts that are shown have been drawn as accurately as possible. Because all the illustrations have been drawn especially for this book, and are more or less composite in nature, we have not found it possible to give credit to all the manufacturers from whose catalogues the information was derived. The quality of the illustrations is due entirely to the craftsmanship of Mr. P. A. Badger, who supervised the preparation of every drawing in the book.

It has been impossible to give due credit for all the material in this book. So far as possible, we have tried to refer to the persons who are primarily responsible for the important features of the theories presented. We claim no originality so far as the subject matter is concerned, as practically everything that we have presented has already appeared elsewhere. We do hope that our presentation may have some advantages in clarity and simplicity as compared to the research publications from which

the material has been drawn. We are indebted to Prof. W. P. Wood of the University of Michigan for valuable contributions to the section on Hydraulic Separation.

W. L. BADGER.
W. L. McCABE.

ANN ARBOR, MICH.,
November, 1930.

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INTRODUCTION

Engineering has been defined as the art and science of using the mechanical properties of matter in structures and machines. It first gained recognition as a profession through the activities of those responsible for the construction of engines of war and military works, and it was with such things alone that the engineer was assumed to be concerned. Much later, in fact less than 200 years ago, the growing demand for structures primarily designed for the purposes of peace led to the emergence or segregation of the civil engineers whose business, as defined in 1828 in the charter of the Institution of Civil Engineers (London), was the "art of directing the great sources of power in nature for the use and convenience of man, as the means of production and of traffic in states, both for external and internal trade, as applied in the construction of roads, bridges, aqueducts, canals, river navigation and docks for internal intercourse and exchange, and in the construction of ports, harbors, moles, breakwaters and lighthouses, and in the art of navigation by artificial power for the purposes of commerce, and the construction and adaptation of machinery, and in the drainage of cities and towns."

That was a program to satisfy the most robust ambition, and it is not surprising that it was presently found necessary to divide again the field of engineering. So the mechanical engineer was evolved to take over "the construction and adaptation of machinery," the naval engineer to provide the means for practicing "the art of navigation by artificial power," and the sanitary engineer to assume responsibility for "the drainage of cities and towns" and the sanitation of communities. Once underway, the subdivision of the field of engineering went forward rapidly as the advance of science and the demands of industry required increasing specialization. The expansion of mining brought the mining engineer, and with the flood of petroleum came the petroleum engineer. The new science of electricity created the demand and provided the opportunity for the electrical engineer. Each department of engineering

as thus set off and developed is, despite their diversity of objective, characterized by a common purpose, which is that "of directing the great sources of power in nature for the use and convenience of man."

Chemical industry in any comprehensive sense is a development of the last 100 years. Though it may be said to have begun with the discovery, by LeBlanc, of his process for making soda from common salt, for which he received the prize of the French Academy in 1791, chemical industry was practically non-existent in England and on the Continent in 1800. Its great development in our own country has occurred within the last 35 years.

The building and equipment of chemical plants and the operation of chemical processes have always involved engineering problems which were often of difficult and highly specialized character. Until comparatively recent years, however, the solution of these problems has been left either to chemists who, in the works or elsewhere, had acquired some knowledge of mechanical engineering or to mechanical engineers whose knowledge of chemistry was often casual and superficial. In my own first contacts with industry my recommendations were commonly referred to the stationary engineer of the plant by whom they were usually vetoed with some profanity.

With the increasing number and complexity of the problems arising from the rapid expansion of the chemical industries it came gradually to be recognized that there were need and place for a distinct branch of engineering to which such problems might be assigned. In response to this need we have the development of chemical engineering, not as a composite of chemistry and mechanical and civil engineering, but as a separate branch of engineering, the basis of which is those unit operations referred to in the first chapter of this book, and which, in their proper sequence and coordination, constitute a chemical process as conducted on the industrial scale.

These unit operations are themselves physical, rather than chemical, in character, although directed toward an ultimately chemical result. They are comparatively few in number, but the conditions under which they may be conducted are of the most widely varying character, as determined by the nature of the material in process, the size of the operation, and the temperatures, pressures, and other factors involved in the process itself.

The chemical engineer is thus required to possess accurate knowledge of the chemical and physical properties of the materials under treatment. He must also be familiar with the utility and limitations of materials of construction, the laws of heat flow, the properties of fluid films, and the many other factors upon which the success or failure of a process may depend.

The chemical engineer must have a theoretical understanding and practical knowledge of the many types of equipment that are available for carrying out each specific unit operation in order that he may select those types best adapted to the requirements of the particular case and that plant and process may operate at the highest attainable efficiency and minimum cost. It is obvious, therefore, that chemical engineering is an arduous profession, which makes unusually severe demands on its practitioners. Where the architect finds his highest expression in beautiful and enduring structures like the Church of the Madeleine in Paris, and the civil engineer can point with the satisfaction of great achievement to the Brooklyn Bridge or the Panama Canal, the work of the chemical engineer may lead to triumphs no less notable and useful, as exemplified in great plants for the fixation of atmospheric nitrogen, the refining and cracking of petroleum, or the manufacture of rayon.

New chemical processes are commonly conceived by chemists and have their initial demonstration on the laboratory scale. The many and difficult problems involved in the steps leading from demonstration in the laboratory to operation in the commercial plant fall within the province of chemical engineering.

There is a general agreement among experienced chemical engineers that the proper procedure in process development should involve a series of steps in which the size of the plant and the scale of operation are progressively increased, since it is highly desirable, as Dr. Baekeland puts it, to "make your blunders on the small scale and make your profits on the large scale." In case of an important process which may ultimately require heavy expenditure for plant, we have in our own practice found it advisable to include five or six such steps between laboratory operation and final plant design. Thus the validity of the proposed reactions is confirmed, initial difficulties are disclosed and overcome, yields gradually improved, and any defects in materials of construction or limitations of units of

equipment revealed. Not until the operation of a plant of semiworks size has verified the estimates of costs of production and the product has been sold at a satisfactory price and, as Dr. Whitaker says, "stays sold" can one feel that he has reached the goal of the long road of process development.

The demands upon the chemical engineer are constantly increasing as a result of the rapid influx of new processes, the trend toward operation at high temperatures and pressures, the constant improvement in instruments of control, the frequent offerings of new materials of construction, the development of new methods of heating as by mercury or diphenyl or radiant heat furnaces, and the rapid changes and improvements in equipment, as, for example, in the substitution of pipe stills for batch stills. The chemical engineer must, therefore, be constantly alert to avail himself of these improvements in practice and equipment.

In recent years the education of the chemical engineer has been the subject of widespread discussion. A sound knowledge of mathematics is obviously essential in view of the frequency with which chemical engineering problems require mathematical treatment for their successful attack. Since also the chemical engineer in the course of his work will often be called upon to render reports of progress, upon the adequacy of which the success of the enterprise may depend, it is essential that he should acquire a firm grasp of English as a working tool. The student cannot, moreover, gain any adequate conception of scale or the confidence necessary for handling large equipment by the study of drawings or photographs, and his education must, therefore, include some actual working contact with full-size equipment. After several years of study the Committee on Chemical Engineering Education of the American Institute of Chemical Engineers attempted to evaluate the relative weighting of the several major subjects which, in their opinion, should be included in chemical engineering courses. The following is probably a fair average of their estimate of the percentage of total requirements which might well be allotted to the different subjects in a four-year course:

Cultural	15
Mathematics	12
Physics	8

Mechanics	6
Chemistry	28
Chemical engineering	12
Other engineering	14
Other science	2
Elective	3

In the instruction of chemical engineering students there should always be kept in mind the definite line of demarcation between industrial chemistry, which is concerned with individual processes as entities in themselves, and chemical engineering, which focuses attention upon those unit operations common to many processes and the proper serial grouping of these unit operations for the production of the desired product as efficiently and cheaply as the ruling conditions permit.

Chemical engineering students are fortunate in having available to them in the present volume the experience and conclusions of two such competent and seasoned chemical engineers as the authors of this book.

ARTHUR D. LITTLE.

CAMBRIDGE, MASS.,
November, 1930.

ELEMENTS OF CHEMICAL ENGINEERING

CHAPTER I

GENERAL CONSIDERATIONS

The profession of chemical engineering is of such recent growth that the field it occupies is not always clearly defined. Twenty-five years ago a chemical engineer was a man whose training had been largely that of a chemist, whose background was the chemical laboratory, but who had had enough plant experience to be familiar with the operation of certain types of equipment. He thought of a process in terms of the chemistry involved and analyzed the operations by which the process was conducted in terms of that process only. It was perfectly natural that in this period the field of the chemical engineer was divided according to manufacturing processes such as sulfuric acid, alkalis, rubber, soap, textiles, etc. No common bond was recognized between these different industries and there was little interchange of ideas or equipment among them.

THE UNIT OPERATIONS OF CHEMICAL ENGINEERING

Coincident with the tremendous development of the chemical industries in the United States in the period between 1914 and 1920 (and probably to a large extent because of the intensity of this development and the pioneering nature of much of it), there came a realization that certain engineering operations were common to many processes that had previously been thought to occupy separate water-tight compartments. It was recognized, for instance, that the transportation of liquids and the flow of heat were common to practically all processes. Filtration engineers discovered that a type of filter developed for use in the extraction of gold from gold ores was suited for filtration in most

of the chemical industries. At the same time there also came a realization that many of these operations were little understood and that they all needed examination, development, and elaboration. As soon as this intensive research began, the fact that a given operation involved the same principles, no matter in what industries it might be used, became more apparent. Gradually, as it was recognized that a knowledge of these operations was the real key to an understanding of manufacturing processes, the field of the chemical engineer was recognized as covering unit operations. The first formal statement of this point of view, and the statement responsible for crystallizing this point of view into its present form, was made by Dr. Arthur D. Little in a report to the Corporation of the Massachusetts Institute of Technology in December, 1915.

Although many members of the older branches of engineering are somewhat hazy as to the function of the chemical engineer, there is at present substantial agreement among chemical engineers and chemists as to the significance of the unit operations and the line of demarcation between the chemical engineer and the chemist on the one hand, and between the chemical engineer and the mechanical engineer on the other hand. Within the profession the number, designation, and classification of the unit operations have not yet become standardized, so that the list of these operations as discussed in this book may or may not coincide with a similar list prepared by others. Such differences in nomenclature and classification are, however, of minor significance; and in general it may be taken for granted that practically any chemical manufacturing process can be broken down into a series of the unit operations such as those described in the following pages.

Further, it has appeared that these same unit operations are vital to many industries not primarily chemical. The manufacture of common salt may conceivably be so carried out that not a single chemical reaction occurs, but it consists solely of a sequence of such unit operations as the transportation of liquids, evaporation, crystallization, drying, and size separation. In the same way the unit operations are found underlying many processes really chemical in nature but not ordinarily thought of as chemical industries. As examples may be cited the extraction of metals from ores (especially by wet processes),

the manufacture of many food products, and the working of metals.

FUNDAMENTAL CONCEPTIONS

The replacing of a large list of independent manufacturing processes by a smaller list of general unit operations brings about a considerable simplification in the treatment of chemical engineering and therefore a greater power to those who have mastered this subject. This simplification does not stop here, because the unit operations themselves, although carried out in a wide variety of types of equipment that apparently have nothing in common, are, from the point of view of the theory involved, applications of a very few fundamental laws. In fact, these laws are the fundamental laws of physical sciences that underlie practically all technology. It is the application of these few fundamental laws to the unit operations that is the basis for the theoretical part of this book. These three basic laws are: first, the conservation of matter and energy; second, the relations pertaining to the equilibria of physical and chemical processes; and, third, the laws governing the rate of change in systems not in equilibrium.

Material Balances.—The first fundamental law states that matter cannot be either created or destroyed. Specifically this requires that the materials entering any process must either accumulate or leave the process. There can be no loss or gain during the process. In this book the types of processes considered will not involve accumulation or depletion, and therefore the law of conservation of matter takes the extremely simple form that input must equal output. This law is often used in the form of *material balances.* This implies that the process is debited with everything that enters it, is credited with everything that leaves it, and the sum of the credits must equal the sum of the debits. The importance of this simple but far-reaching statement can hardly be overemphasized. Material balances must hold over the entire process or apparatus, or over any part of it. They must apply to all of the material that enters and leaves the process, to any chemical element that enters and leaves the process, or to any material that passes through the process unchanged.

Heat Balances.—The law of conservation of energy expresses the same fact with regard to the energy input and output of a process or apparatus as does the law of conservation of mass for materials. To be valid, an energy balance must include all types of energy that are involved in the process, whether these energies be heat, mechanical energy, electrical energy, radiant energy, chemical energy, or other forms. It happens that in many cases, however, the only energy involved is heat; and for these cases, heat balances, expressing the equality of heat input to heat output, are valid and extremely useful. Here, again, a heat balance can be struck over the entire apparatus or process or any part of it.

Equilibrium Relationships.—Systems that are undergoing change spontaneously do so in a definite direction. If left to themselves they will eventually reach a state where apparently no further action takes place. Such a state is called an *equilibrium state.* For example, if a piece of hot iron is placed in contact with a piece of cold iron, the hot iron will cool and the cold iron will heat up until an equilibrium point is reached, when the two pieces are at the same temperature. Again, a handful of salt placed in a beaker of water will dissolve until, if there is an excess of salt always present, the concentration of the salt in the solution reaches a definite value if the temperature be kept constant. Here, again, the process apparently stops when the equilibrium point is reached, and the solution is a saturated solution. Such instances are universal, and equilibrium conditions represent end points of naturally occurring processes that cannot be changed without making some change in the conditions governing the system.

Rate of Reaction.—Of even more importance in practice than the end point of the process is the question of how fast the process is tending toward its equilibrium. Indeed, systems that are far from equilibrium but moving toward equilibrium at a very slow rate are, for all practical purposes, at an equilibrium state. On the other hand, where processes are expected to take place, it is in general desired that the rate of the reaction be reasonably large, because time is an all-important factor in practical operations. For instance, consider a large tank of cold, viscous liquid surrounded by a steam jacket. If the liquid is undisturbed it will come to equilibrium with the steam (*i.e.*, reach the tem-

perature of the steam) very slowly and conceivably may require several days. But if the liquid is vigorously stirred, or if it is pumped at high velocity through a pipe surrounded by a steam jacket, it can be made to attain the desired equilibrium in a matter of hours or minutes.

Unfortunately, the laws governing reaction rate are not nearly so exact or precisely known as are the laws of conservation of mass or matter or of equilibrium states. Also, they are more difficult to formulate mathematically and require a more elaborate technique to obtain data for them or to carry out calculations using these data once they are obtained. In general, however, the rate of approach to equilibrium at any particular time can, at least approximately, be expressed as a driving force divided by a resistance. The nature of the driving force will depend upon the type of process considered. The driving force must be such that it is equal to zero at the equilibrium state. The use of the theorem is best illustrated by certain examples. Thus, the driving force that causes heat to flow between a hot body and a cold body is the temperature difference between them. If the temperature difference is zero the bodies are at equilibrium and there is no heat transfer from one to the other. The driving force that causes a fluid to flow through a horizontal pipe line is the pressure difference between the fluid in one section and the fluid in another. If the pressure in the two sections is the same there is no flow and the process is in equilibrium. An electrical flow will take place if there is a difference in electrical potential and will stop when the potentials of the materials in electrical contact are the same. Salt in contact with an unsaturated solution will dissolve, and the driving force for this reaction may be considered to be a difference in concentration between a saturated solution and the concentration of the solution in question.

The above law of rate of reaction must be written in the form of a differential equation which says that the rate is equal to the driving force divided by the resistance. This equation must be a differential equation because, as the operation proceeds, the driving force changes and hence the rate changes. This point will be elaborated later when the basic theory of various operations is considered. Mathematically the equation will take the form:

$$\frac{dQ}{d\theta} \quad \frac{\Delta F}{R} \tag{1}$$

where

Q = the quantity being transferred (which may be heat, matter, or energy in any form).
θ = the time.
ΔF = the driving force.
R = the resistance.

Molecular Units.—In material balance calculations where chemical reactions are involved, molecular units are simpler to use than the ordinary units of weight. A *mol* of any pure substance is defined as that quantity whose weight is numerically equal to its molecular weight. From this definition the meanings of the terms *pound-mol* and *gram-mol* follow. In engineering calculations the molecular unit is usually the pound-mol. The average molecular weight of a mixture of substances is obtained by the use of the following equation:

$$\frac{W_A + W_B + W_C + \cdots}{\frac{W_A}{M_A} + \frac{W_B}{M_B} + \frac{W_C}{M_C} +} = M_a \tag{2}$$

where

W_A, W_B, and W_C = weights of the individual pure components of the mixture.
M_A, M_B, and M_C = the molecular weights of the pure components.
M_{av} = the average molecular weight.

Mol Fraction.—It is often convenient to express compositions not as weight fraction or as weight per cent but rather as mol fraction or mol per cent. The mol fraction is the ratio of the mols of one component to the total number of mols in the mixture. For example, using the same symbols as in Equation (2), the mol fraction of component A is

$$\text{Mol fraction } A = \frac{W_A/M_A}{\frac{W_A}{M_A} + \frac{W_B}{M_B} + \frac{W_C}{M_C} + \cdots} \tag{3}$$

Similarly,

$$\text{Mol fraction } B = \frac{W_B/M_B}{\frac{W_A}{M_A} + \frac{W_B}{M_B} + \frac{W_C}{M_C} + \cdots} \tag{4}$$

It is apparent that the sum of the mol fractions of all the components must equal unity. Mol per cent is, of course, mol fraction multiplied by 100.

Gas Laws.—In addition to the basic and fundamental laws discussed on pages 3 to 6 there are numerous relations of more limited range but of great utility. Some of these, such as Raoult's law, will be discussed in connection with specific problems. For the purposes of this book, however, probably the most generally useful is the ordinary gas law. Although the gas law does not apply precisely to any actual gas, for the great majority of gases and vapors at ordinary temperatures and pressures the law is sufficiently accurate for engineering calculations. This law is usually expressed in the form

$$PV = nRT \tag{5}$$

where

P = the pressure.
V = the volume.
T = the absolute temperature.[1]
R = a constant, equal for all gases.
n = the number of mols of the gas.

The equation is seldom used in this form. It expresses three facts, however: first, that the volume of a gas is directly proportional to the number of mols; second, that the volume is directly proportional to the absolute temperature; and, third, that the volume is inversely proportional to the pressure. These last statements are especially valuable in converting gas volumes from one temperature and pressure to another temperature and pressure. For example, V_1 cu. ft. of a gas at an absolute temperature of T_1 and an absolute pressure of P_1 will occupy a volume of V_2 cu. ft. at another pressure P_2 and another temperature T_2, where the value of V_2 is given by

$$V_2 = V_1\left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right) \tag{6}$$

It is not necessary to memorize Equation (6) in order to determine which term is the numerator in each of the fractions. This may

[1] Absolute temperatures may be expressed as degrees Kelvin (K.), which is temperature in degrees Centigrade + 273; or as degrees Rankine (R.), which is temperature in degrees Fahrenheit + 460.

always be determined qualitatively by remembering that an increase of pressure results in a decrease in volume, and an increase in temperature results in an increase in volume.

The Mol Volume.—Equation (5) shows that a mol of gas under definite conditions of temperature and pressure always occupies a definite volume regardless of the nature of the gas. This volume is called the *mol volume.* A gram-mol of gas at 0°C. and a pressure of 760 mm. of mercury occupies 22.41 l. A pound-mol of any gas occupies 359 cu. ft. under a pressure of 760 mm. and a temperature of 32°F., or 378 cu. ft. under a pressure of 30 in. of mercury and 60°F. A pound-mol volume under any other conditions of temperature and pressure is easily calculated by means of Equation (6).

Another important relationship involving gases is that known as *Dalton's law of partial pressures.* This law states that the total pressure exerted by a mixture of gases may be considered to be the sum of the pressures that would be exerted by each of the gases if it alone were present and occupied the total volume. A mixture of gases may be considered to be either a combination of the partial volumes of the individual gases, each partial volume being taken at the total pressure; or as a combination of gases, each one of which occupies the entire volume at its own partial pressure.

The concept of the mol volume can be applied to a mixture of gases just as it is to a pure gas. In general, a pound-mol is that weight of a gas mixture that will occupy 359 cu. ft. at a pressure of 760 mm. of mercury and 32°F. This quantity is also referred to as the average molecular weight of the gas.

The most useful form in which the laws of a mixture of perfect gases can be put is as follows:

$$\text{Volume } \% = \text{pressure } \% = \text{mol } \% \qquad (7)$$

For example, air contains 79 per cent nitrogen and 21 per cent oxygen by volume. One cubic foot of air under a pressure of P atm. may be considered, therefore, to be a mixture of 0.21 cu. ft. of oxygen at P atm. and 0.79 cu. ft. of nitrogen at P atm. It can also be considered to be a mixture of 1 cu. ft. of oxygen at $0.21P$ atm. and 1 cu. ft. of nitrogen at $0.79P$ atm., so that 21 per cent of the total pressure is due to the oxygen and 79 per cent of the total is due to the nitrogen. Finally, one mol of air

contains 0.21 mol of oxygen and 0.79 mol of nitrogen at all temperatures and pressures.

Example 1.—A solvent recovery system delivers a gas, saturated with benzene vapor (C_6H_6), that analyzes on a benzene-free basis 15 per cent CO_2, 4 per cent O_2, and 81 per cent N_2. This gas is at 70°F. and 750 mm. pressure. It is compressed to 5 atm. and cooled to 70°F. after compression. How many pounds of benzene are condensed by this process, per 1,000 cu. ft. of the original mixture? The vapor pressure of benzene at 70°F. is 75 mm.

Solution.—Since volume per cent equals pressure per cent, the volume of inert gas is $1{,}000 \times \frac{750 - 75}{750} = 900$ cu. ft. This volume may be converted to mols as follows:

$$\frac{900}{359} \times \frac{460 + 32}{460 + 70} \times \frac{750}{760} = 2.30 \text{ mols inert.}$$

The ratio (mols benzene vapor): (mols inert) before and after compression is
Before compression,

$$\frac{75}{750 - 75} = 0.1111$$

After compression,

$$\frac{75}{(760 \times 5) - 75} = 0.0211$$

Hence, 0.1111 − 0.0211, or 0.090 mol benzene is condensed per mol of inert gas; and the weight of benzene condensed is

$$0.090 \times 2.30 \times 78 = 16.1 \text{ lb.}$$

UNITS

In spite of protracted discussions regarding the relative advantages of the English and the metric systems of units, the fact remains that practically all engineering work in the United States is done in terms of the ordinary English units. These units will therefore be used throughout this book. It is true that there are certain chemical processes in which the Centigrade temperature scale is used in plant operation, and there are a very few cases where metric units of weight and measure are used in plant calculations and design. These latter, however, are so rare that they do not need to be considered. The number of plants in which the Centigrade temperature scale is used is rapidly increasing, and it is of importance for the chemical engineer to be able to think freely in either Centigrade or Fahrenheit and not to depend entirely on one or the other system. Consequently, although Fahrenheit temperatures have been very

largely used throughout this book, Centigrade temperatures have also been used without giving the corresponding equivalent in Fahrenheit.

On the other hand, many of the data that the chemical engineer must use must be taken from laboratory work in which metric units are universal. It is, therefore, necessary to be able to transform units from one system to the other and to do this rapidly and precisely. Even though the calculations be kept entirely within the English system, there often may occur cases in which a transformation from one set of units to another set of units becomes necessary. These transformations are simple, involving only a series of ratios; but they seem to be unusually troublesome, not only to students but also to practicing engineers. The most commonly needed factors for such conversions are as follows:

1 inch	= 2.540 cm.
1 foot	= 0.3048 m.
1 pound	= 0.4536 kg.
1°C.	= 1.8°F.
1 kilowatt	= 1.341 hp.
1 kilogram calorie	= 3.968 B.t.u.
1 calorie per gram	= 1.8 B.t.u. per lb.
1 atmosphere	= 760 mm. of mercury.
	= 29.92 in. of mercury.
	= 1,033.3 g. per sq. cm.[1]
	= 14.696 lb. per sq. in.
1 British thermal unit	= 778.6 ft.-lb.
1 gallon	= 231 cu. in.
Gas constant (R)	= 1,543 ft.-lb. per lb.-mol per deg. R.

Example 2.—Suppose a value has been found for a thermal conductivity expressed as (kilogram calories) (meters) per square meter per degree Centigrade per second, and it is desired to convert this to the usual English unit, (B.t.u.) (feet) per square foot per degree Fahrenheit per hour. All that is necessary is to formulate the original units and then substitute the necessary factors, thus:

$$1\left(\frac{\text{kg. cal.} \times \text{m.}}{\text{m.}^2 \times {}^\circ\text{C.} \times \text{sec.}}\right) = \left(\frac{3.968 \times \frac{1}{0.3048}}{\left(\frac{1}{0.3048}\right)^2 \times 1.8 \times \frac{1}{3,600}}\right)\left(\frac{\text{B.t.u.} \times \text{ft.}}{\text{ft.}^2 \times {}^\circ\text{F.} \times \text{hr.}}\right)$$

1 metric unit = 2,420 English units.

[1] In European engineering literature 1 atm. is usually taken as 1,000 g per sq. cm. It is not always easy to tell whether this atmosphere or the one corresponding to 760 mm. of mercury is intended.

USEFUL MATHEMATICAL METHODS

The mathematical technique of the calculations required for the theory of unit operations as presented in this work involves the use of only the most elementary parts of the calculus. It is assumed that the reader is familiar with these elements. There are two mathematical methods, however, which, although they are simple, are so useful in the treatment of this subject that they will be discussed briefly. The first is the use of graphic methods of integration, and the second is the graphical treatment of exponential functions.

Graphical Integration.—It should be remembered from the first principles of integral calculus that the value of a definite integral

$$\int_{x_a}^{x_b} f'(x)dx$$

is the area bounded by the curve of $f'(x)$ *vs.* x, the ordinates $x = x_a$ and $x = x_b$, and the X-axis. Any definite integral can therefore be evaluated numerically without the use of tables by plotting $f'(x)$ against x, drawing the two vertical lines corresponding to the limits, and determining the area enclosed between the curve, the limits, and the X-axis. Thus, in Fig. 1, if the curve *abcde* represents the plot of $f'(x)$ *vs.* x, and if the lines *af* and *eg* correspond to the values of x_a and x_b, respectively, then the entire area *abcdegf* is the desired integral.

The area may be determined by splitting it into a series of rectangles such as the one shown cross-hatched in Fig. 1. The height of this rectangle is so chosen that the area of the little triangle omitted between the rectangle and the curve is approximately equal to the area of the little cross-hatched triangle included in the rectangle above the curve. If the curvature is not too great, and if Δx be chosen sufficiently small, the height of the rectangle can be chosen by eye and yet may fulfill the above condition very accurately. The area desired is, then, the sum of all such rectangles as the one shown in Fig. 1.

The general rule for an integration of any function multiplied by a derivative is to plot the variable of the derivative along the X-axis, and the function along the Y-axis, no matter how complicated the latter part of the expression may be, and then determine the area between the curve and the X-axis between the desired limits.

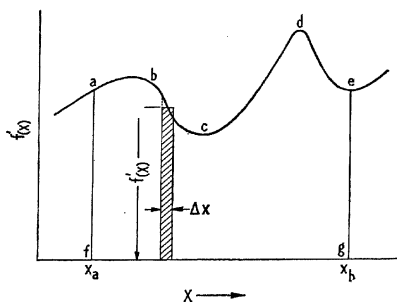



Fig. 1.—Principle of graphic integration.

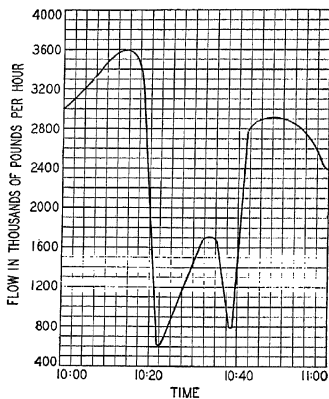



Fig. 2.—Data for Example 3.

This method of integration is especially valuable because in many cases there is no direct mathematical expression for the quantity $f'(x)$. In many cases there may be available an experimental plot of y *vs.* x, which follows no simple mathematical form. It would be extremely difficult to evaluate from such data the integral $\int_{x_a}^{x_b} y dx$ by formal methods, as it would require the fitting of an empirical equation to the data.

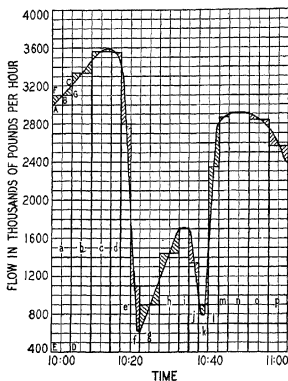



FIG. 3.—Solution of Example 3.

Example 3.—Figure 2 shows the rate of flow of steam in a pipe line, expressed as thousands of pounds per hour. (*a*) What was the total weight of steam flowing between 10:00 and 11:00? (*b*) Plot a curve showing the total steam flow *vs.* time.

Solution.—If R is the rate at any time θ, then during the very small length of time $d\theta$ there will flow dW lb., where

$$dW = R d\theta$$

For the entire time the total flow W_T will be the integral of the above equation, or

$$\int_0^{W_T} dW = \int_{10:00}^{11:00} R d\theta = W_T \qquad (8)$$

In order to evaluate the integral $\int_{10:00}^{11:00} R d\theta$ by formal means it would be necessary to express R in terms of θ. An equation fitting the curve of Fig. 2 would obviously be very difficult to develop, and complicated if obtained. The integral is easily evaluated, however, by graphical methods, since the desired integral is equal to the area bounded by the curve, the time axis, and the two vertical lines corresponding to the values $\theta = 10:00$ and $\theta = 11:00$. This area may be divided into a number of strips, such as

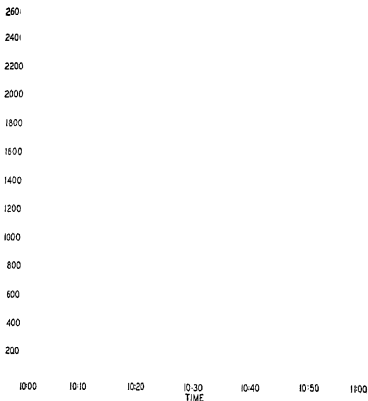



FIG. 4.—Integral curve of Example 3.

a, b, c, etc. (Fig. 3). The true area of strip a is $= ABCDEA$. If the width of the strips be taken sufficiently small, a line FG may be drawn such that the area of the triangle AFB is sensibly equal to the area of the triangle BCG. The area $FBGDEF$ is easily calculated since it is rectangular in shape. Proceeding in this manner the entire area in question can be replaced by a series of rectangles. The heights of the rectangles a, b, c, . . . p are so chosen that the shaded areas above the original curve are equal to those below the curve. In order that the areas measure the flow in pounds, the heights of the rectangles must be expressed in pounds per hour and the bases of the rectangles in hours, so that

$$\frac{\text{lb.}}{\text{hr.}} \times \text{hr.} \quad \text{lb.}$$

The following table shows how the calculation can be carried out. It will be seen that the total area represents 2,412 lb., and this weight is the total weight of steam passing through the pipe from 10:00 to 11:00.

Such an integration not only gives the total flow over the entire time in question but also gives data showing the total flow that has taken place from 10:00 to any other time between 10:00 and 11:00. Thus, referring to item 6 of the table, it is seen that at 10:20 (at the end of rectangle *e*) 1,098 lb. of steam have passed through the pipe, at 10:35, 1,410 lb. have passed, and so on. From the data of items 1 and 6, Fig. 4 can be drawn, showing the total flow up to any time. Such a curve is known as the *integral curve* of Equation (8).

Rectangle	Time	Height of rectangle	Width of rectangle		Area of rectangle, pounds (item 4 × item 2)	Total area = total flow, pounds (sum of items 5)
			Minutes	Hours (item 3 ÷ 60)		
	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6
	10:00	.....	...	......	...	0
a..............	10:05	3,100	5.0	0.0833	258	258
b..............	10:10	3,340	5.0	0.0833	278	536
c..............	10:15	3,560	5.0	0.0833	297	833
d..............	10:17.5	3,550	2.5	0.0417	148	981
e..............	10:20	2,800	2.5	0.0417	117	1,098
f..............	10:22.5	1,100	2.5	0.0417	46	1,144
g..............	10:27.5	900	5.0	0.0833	75	1,219
h..............	10:32.5	1,440	5.0	0.0833	120	1,339
i..............	10:35	1,700	2.5	0.0417	71	1,410
j..............	10:37.5	1,350	2.5	0.0417	56	1,466
k..............	10:40	900	2.5	0.0417	37	1,503
l..............	10:42.5	2,350	2.5	0.0417	98	1,601
m..............	10:45	2,850	2.5	0.0417	119	1,720
n..............	10:50	2,900	5.0	0.0833	242	1,962
o..............	10:55	2,830	5.0	0.0833	236	2,198
p..............	11:00	2,570	5.0	0.0833	214	2,412

Exponential Equations and Log-log Plots.—In many cases, experimental data involving the variables x and y fit an equation of the form

$$y = ax^n \tag{9}$$

where a and n are constants. It is also possible to duplicate many curves by an equation of the form

$$y = a + bx + cx^2 + dx^3 + \qquad (10)$$

While it is true that Equation (10) can be fitted more accurately to many curves than Equation (9), nevertheless Equation (9) fits many experimental data closely enough to warrant its use; and, as will be explained in the following paragraph, the constants of Equation (9) can be very quickly determined. On the other hand, fitting such an equation as (10) to experimental data is a tedious process and often requires the use of a large number of terms.

Equation (9) may be rewritten in the form

$$\log y = \log a + n \log x \qquad (11)$$[1]

It will be seen at once that, if $\log x$ is plotted *vs.* $\log y$, Equation (11) is the equation of a straight line whose slope is n and whose intercept is $\log a$ when x is equal to unity. It is possible to plot a set of data by plotting the logarithms of the numerical values of the two variables, but a much more convenient method is the use of *log-log paper*.

Log-log paper is coordinate paper on which the scales, instead of being uniform, are logarithmic; in other words, the intervals marked 1, 2, 3, etc., are not in proportion to 1, 2, 3 units but to the logarithms of the numbers 1, 2, 3, etc. The graduations on this paper correspond exactly to the graduations on an ordinary slide rule. Consequently if the scales are properly adjusted to the data at hand and a set of values of two variables are plotted on such paper, the result is the same as plotting their logarithms on ordinary rectangular coordinate paper. If the points so plotted fall on a straight line, the equation of this line is Equation (9); in which n is the slope and a is the value of y when $x = 1$, for when $x = 1$, the second term on the right-hand side of Equation (11) is zero.

The only disadvantage of the logarithmic plot is that the scales are rather fine, but in most cases points can be plotted with an accuracy comparable to the accuracy of ordinary engineering

[1] In this book "log" is used to represent logarithms to the base 10, and "ln" to represent logarithms to the base e.

data. An advantage of log-log plots is that deviations from a curve of a given distance represent deviations of a constant per cent of the total value of the variable at that point, irrespective of the part of the plot in which they may lie. In contrast to this, deviations of a given amount from a plot on ordinary coordinate paper represent a constant numerical deviation, which may be a large per cent when the value of the variable is small and a small per cent when the value of the variable is large. Thus a logarithmic plot is of much assistance in representing certain experimental data, since it presents the data with the same percentage of accuracy no matter what the magnitudes of the quantities may be.

Example 4.—An orifice calibration gave the following readings:

Average velocity of water in pipe, feet per second	Orifice manometer reading, millimeters of mercury
3.42	30.3
4.25	58.0
5.25	75.5
5.88	93.5
7.02	137.5
7.30	148.0
10.05	261.0

If the flow through an orifice is known to follow an equation of the type

$$u = kR^n \qquad (12)$$

where

u = the velocity,
R = the reading of the manometer,
determine the values of k and n for this particular orifice.

Solution.—The data are plotted on log-log coordinates in Fig. 5. The data fix the straight line shown with considerable precision. The slope of the line may be determined by reading on a protractor the angle between the line and the X-axis or by measuring the distances a and b. The slope is then the tangent of the angle read by the first method, or b/a in the second method. In either case, the slope is found to be 0.501, hence n may be taken as 0.50 within the limits of accuracy of the data.

The data cannot be extrapolated to determine the intercept where $R = 1$ without going far beyond the scale of the plot. The intercept may be read where $R = 100$ and divided by $100^{0.50}$. This gives a value of 0.605 for k. The desired equation is therefore

$$u = 0.605R^{0.5} = 0.605\sqrt{R}$$

It will be shown later that according to theory the exponent of R must be 0.5.

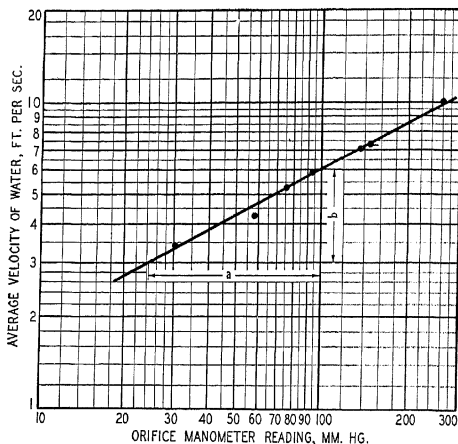



FIG. 5.—Solution of Example 4.

Nomenclature of Chapter I

ΔF = driving force
M = molecular weight
n = number of mols
P = pressure
Q = quantity
R = resistance, gas constant, rate, reading
T = absolute temperature
u = velocity
V = volume
W = weight
sub A, B, C—refer to components A, B, C—of a mixture.
sub av = mean value
θ = time

Problems

1. A flue gas has the following analysis:

	Per cent
O_2	6.0
CO_2	13.4
CO	None
SO_2	0.2
N_2	80.4
	100.0

What is its density in pounds per cubic foot at 100°F. and a pressure of 29.2 in. Hg?

2. A certain process yields 10,000 cu. ft. per day of a mixture of air and dry hydrogen chloride. The hydrogen chloride is 6.2 per cent by volume. The gas is measured at 550°F. and 29.9 in. Hg. How many pounds of limestone, 92 per cent calcium carbonate, will it take to neutralize the acid? How many cubic feet of carbon dioxide, measured at 70°F. and 29.92 in. Hg, would be liberated?

3. The reaction between hydrogen chloride and limestone in the above problem is so carried out that the CO_2 formed, together with all the initial air, issues from the apparatus saturated with water. At 70°F. the vapor pressure of water is 18.8 mm. Hg. What is the volume of the exit gas?

4. What is the equivalent of the metric atmosphere (1 kg. per sq. cm.) in inches of mercury?

5. The constant R in Equation (5) has the value

$$\frac{(82.07)(\text{atm.})(\text{cu. cm.})}{(\text{g.-mols})(^\circ\text{K.})}.$$

What is its numerical value if pressure is expressed in pounds per square inch, volume in cubic feet, weight in pound mols, and temperature in degrees Rankine?

CHAPTER II

FLOW OF FLUIDS

The science of mechanics of fluids is divided into two branches, namely, hydrostatics, which deals with fluids at rest, and hydrodynamics, which deals with fluids in motion. The subject of

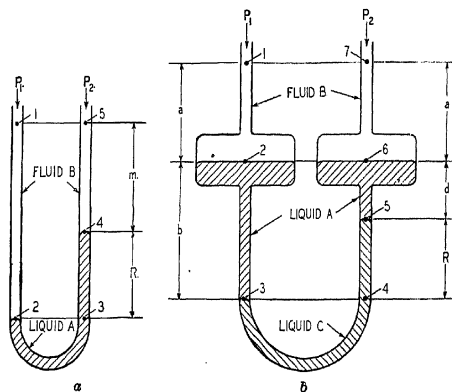



FIG. 6.—Manometers. *a*. Simple manometer. *b*. Differential manometer.

hydrostatics is treated in all elementary books on physics and need not be elaborated here. One important application of hydrostatics will be discussed, however.

Manometers.—An important device for measuring pressure is known as a manometer. Two examples are shown in Fig. 6. Figure 6*a* represents the simplest form of this instrument. Assume that the shaded portion of the U-tube is filled with a

liquid A of density of ρ_A lb. per cu. ft. The arms of the U-tube above liquid A are filled with fluid B, which is immiscible with liquid A and has a density of ρ_B lb. per cu. ft. A pressure of P_1 lb. per sq. ft. is exerted in one arm of the U-tube, and a pressure P_2 on the other. As a result of the difference in pressure $P_1 - P_2$, the meniscus in one branch of the U-tube will be higher than in the other. The vertical distance between these two surfaces is R ft. It is the purpose of the manometer to measure the difference in pressure $P_1 - P_2$ by means of the reading R. In order to derive a relationship between $P_1 - P_2$ and R, start at the point 1 where the pressure is P_1. Then the pressure at the point 2 will be $P_1 + (m + R)\rho_B$. According to the principles of hydrostatics this must also equal the pressure at the point 3. The pressure at point 4 is less than the pressure at point 3 by the amount $R\rho_A$; and the pressure at point 5 will be still less by the amount $m\rho_B$. These statements can be summarized in the equation

$$P_1 + (m + R)\rho_B - R\rho_A - m\rho_B = P_2 \tag{13}$$

Upon simplification of this equation it is found that

$$\Delta P = P_1 - P_2 = R(\rho_A - \rho_B) \tag{14}$$

It will be noted that this relationship is independent of the distance m and also of the dimensions of the U-tube, provided that P_1 and P_2 are measured in the same horizontal plane.

For the measurement of smaller pressure differences, the type of manometer shown in Fig. 6*b*, called the *differential manometer*, is often used. This manometer contains two liquids A and C, which must be immiscible, and whose densities are as nearly equal as possible. Enlarged chambers are inserted in the manometer, so that the position of the meniscus at points 2 and 6 does not change appreciably with changes in the reading R. Consequently the distance between points 1 and 2 may be considered equal to the distance between points 6 and 7. The same principles may be employed in developing the relationship between the reading of this type of manometer and the pressure difference $P_1 - P_2$, as in the previous case. The changes in pressure in passing through the series of points 1 to 7 are shown in the following table:

Point	Total pressure
1	P_1
2	$P_1 + a\rho_B$
3	$P_1 + a\rho_B + b\rho_A$
4	$P_1 + a\rho_B + b\rho_A$
5	$P_1 + a\rho_B + b\rho_A - R\rho_C$
6	$P_1 + a\rho_B + b\rho_A - R\rho_C - d\rho_A$
7	$P_1 + a\rho_B + b\rho_A - R\rho_C - d\rho_A - a\rho_B = P_2$

The last equation may be simplified to

$$P_1 - P_2 = -(b - d)\rho_A + R\rho_C \tag{15}$$

but since $b - d = R$

$$\Delta P = R(\rho_C - \rho_A) \tag{16}$$

From this it follows that the smaller the difference $(\rho_C - \rho_A)$ the larger will be the reading R on the manometer for a given value of ΔP.

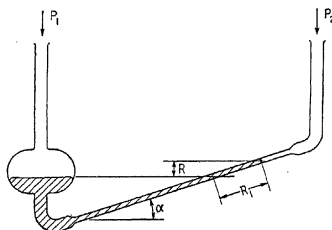



FIG. 7.—Inclined manometer.

For measuring small differences in pressure, the type of manometer shown in Fig. 6*a* may be modified to the form shown in Fig. 7. In this type, it will be noted that the leg containing one meniscus is inclined in such a manner that for a small value of R the meniscus must move a considerable distance along the tube. This distance is equal to R divided by the sine of the angle of inclination α. By making α small, the value of R is multiplied

into a much longer distance R_1. In this type of gage it is necessary to provide an enlargement in the vertical leg so that the movement of the meniscus in this enlargement is negligible within the range of the gage.

BERNOULLI'S THEOREM

It has been stated in Chap. I that one of the most powerful theoretical tools that is available for the attack of any quantitative problem is the principle of the conservation of energy. When this principle is applied to the flow of fluids, the resulting equation is called *Bernoulli's theorem.* It must be distinctly understood, however, that this theorem is only a special case of the more general law of the conservation of energy. Since it

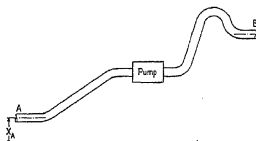



FIG. 8.—Development of Bernoulli's theorem

is theoretically possible for every kind of energy to be involved in a system in which fluids are flowing, Bernoulli's theorem can be written in a general and complicated form. For most cases, however, it degenerates into a comparatively simple equation.

Consider the system represented in Fig. 8 and assume that the temperature is uniform throughout the system. This figure represents a channel conveying a liquid from point A to point B. The pump supplies the energy necessary to cause the flow. Consider 1 lb. of liquid entering at A. Let the pressure at A be P_A lb. per sq. ft., let the velocity of the liquid be u_A ft. per sec., and let the specific volume of the liquid be V_A cu. ft. per lb. Point A is X_A ft. above an arbitrary horizontal datum plane represented by the line MN. The pound of liquid at A has a potential energy, measured above the plane MN, equal to X_A ft.-lb. Since the liquid is in motion at a velocity of u_A ft. per sec., the pound of liquid will have a kinetic energy equal to $u_A^2/2g$ ft.-lb. Furthermore, as the pound of liquid enters the pipe it enters

against a pressure of P_A lb. per sq. ft., and therefore work equal to $P_A V_A$ ft.-lb. is done on the pound of liquid and is added to its energy store. The sum of these three terms represents the energy of the pound of liquid entering the section.

After the system has reached the steady state, whenever a pound of liquid enters at A another pound is displaced at B according to the principle of the conservation of mass. This pound leaving at B will have an energy content of

$$X_B + \frac{u_B^2}{2g} + P_B V_B \text{ ft.-lb.}$$

where

u_B, P_B, and V_B the velocity, the pressure, and the specific volume, respectively, at the point B.

If there were no increases or losses in energy between the points A and B, the energy content of the pound of liquid entering at A would exactly equal that of the pound of liquid leaving at B, by the principle of the conservation of energy. It has been postulated that energy is added by the pump. Let this be w ft.-lb. per lb. of liquid. Some energy will be converted into heat by friction. It has been assumed that the system is at a constant temperature; hence, it must be assumed that this heat is lost by radiation. Let this loss due to friction be F ft.-lb. per lb. of liquid. The complete equation representing an energy balance across the system between points A and B will therefore be

$$X_A + \frac{u_A^2}{2g} + P_A V_A - F + w = X_B + \frac{u_B^2}{2g} + P_B V_B \qquad (17)$$

If the density of the liquid ρ be expressed as pounds per cubic foot, then

$$V_A = \frac{1}{\rho_A}$$

$$V_B = \frac{1}{\rho_B}$$

and Equation (17) may be written

$$X_A + \frac{u_A^2}{2g} + \frac{P_A}{\rho_A} - F + w = X_B + \frac{u_B^2}{2g} + \frac{P_B}{\rho_B} \qquad (18)$$

Fluid Heads.—Since the terms in Equations (17) and (18) are additive, they must be expressed in the same units. The X terms are measured in feet. Every other term in the equation must, therefore, also be measured in feet. Upon examination of the terms in Equations (17) and (18), it will be found that they meet this requirement. For example, a velocity is measured as ft./sec.; and g, being an acceleration, is measured as ft./sec.² The units of the fraction $\frac{u^2}{2g}$ are then $\frac{\text{ft.}^2/\text{sec.}^2}{\text{ft./sec.}^2}$ which reduces to feet. Likewise, the PV term is measured as lb./ft.² × ft.³/lb., or again equal dimensionally to feet. The terms w and F must necessarily be expressed in feet to correspond to the units of the other terms.

Consider a column of liquid 1 ft. square at the base and X ft. high. If the density of the liquid is ρ lb. per cu. ft., then the pressure on the base is equal to the volume of the fluid multiplied by its density. But since the cross-section of the column is of unit area, the volume is numerically equal to the height X, and therefore

$$P = \rho X \tag{19}$$

In other words, a pressure may be measured by the height of a column of fluid of known density, and such a height is termed *head* in discussions of hydraulics.

Since the terms in Equations (17) and (18) are all linear, they are equivalent to pressures, and the various terms are often called *heads*. The X terms are called *potential heads*, the $u^2/2g$ terms are called *velocity heads*, the PV or P/ρ terms are called *pressure heads*, the term F is called *friction head*, and w is the head developed by the pump.

Two things must be noted with regard to Equation (19). In the first place, the units in which the pressure is measured depend upon the units chosen for X and ρ. It is most convenient to measure X in feet, and ρ in pounds per cubic foot, and this gives P in pounds per square foot. In the second place, the term "head" has no meaning as far as pressure is concerned unless the density of the fluid is given.

Equation (17) does not fit all possible cases. If there is any other energy change between points A and B in addition to frictional losses and work furnished by a pump, a term must be inserted to include this form of energy. For example, if heat is

added to or withdrawn from the system between A and B, this must be accounted for by inserting an appropriate term in Equation (17). The F term in Equation (17) is a special case of this method. If the fluid flowing through the system is a gas, the density of the gas will change as it passes through the system because of pressure changes. When a gas expands it loses energy that appears as work, and this work is measured by $\int_{V_A}^{V_B} PdV$. If density changes takes place in the system over which Bernoulli's equation is written, this integral must be evaluated and inserted in the equation just as were the terms w and F. In order to

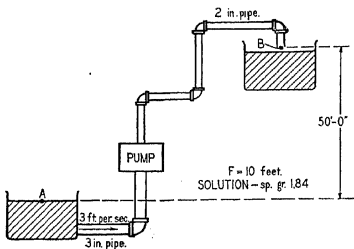



Fig. 9.—Data for Example 5.

evaluate this integral, the relationship between P and V must be known over the entire range of the system to which the equation is applied; and since this relationship can be of many forms, a general rule cannot be given, and the term must be evaluated in accordance with the conditions of the problem. Such cases, in general, fall outside the scope of this book.

Although in Fig. 8 Bernoulli's theorem is written over the two ends of the apparatus, it can be written over any part or parts of the system, and such an equation will be perfectly valid. In general, the equation is written between the two points in the system about which the most information is available. The use of the equation is shown in the following numerical example:

Example 5 (see Fig. 9).—A pump draws a solution, sp. gr. 1.84, from a storage tank through a 3-in. pipe. The velocity in the suction line is 3 ft.

per sec. The pump discharges through a 2-in. pipe to an overhead tank. The end of the discharge line is 50 ft. above the level of the solution in the feed tank. Friction losses in the entire system are 10 ft. of solution. What pressure must the pump develop in pounds per square inch? What is the theoretical horsepower of the pump?

Solution.—In Equation (18) take point A at the surface of the liquid in the feed tank and point B at the end of the discharge pipe. Take the datum plane for elevations through point A. Then

$$X_A = 0$$
$$X_B = 50 \text{ ft.}$$
$$u_A = 0$$
$$u_B = 3 \times \frac{3.068^2}{2.067^2} = 6.61 \text{ ft. per sec.}$$
$$F = 10 \text{ ft.}$$
$$P_A = P_B \text{ (both at atmospheric pressure).}$$
$$\rho_A = \rho_B = 1.84 \times 62.4 = 115 \text{ lb. per cu. ft.}$$

Substituting in Equation (18) gives

$$-10 + w = 50 + \frac{6.61^2}{2 \times 32.2}$$
$$w = 60.68 \text{ ft. of solution of sp. gr. 1.84.}$$

By Equation (19) the pressure corresponding to w may be determined in pounds per square foot. The pressure in pounds per square inch is $P/144$. Hence,

$$\text{Pressure} = 60.68 \times 115 = 6{,}978 \text{ lb. per sq. ft.}$$
$$= 48.5 \text{ lb. per sq. in.}$$

Since

$$\frac{\text{lb.}}{\text{ft.}^2} \times \frac{\text{ft.}^3}{\text{sec.}} = \frac{\text{ft.-lb.}}{\text{sec.}}$$

it is necessary only to multiply the pressure in pounds per square foot by the volume pumped per second to obtain the power consumed. The inside cross-sectional area of the 3-in. pipe is 7.393 sq. in. or 0.0513 sq. ft. At a velocity of 3 ft. per sec., the volume pumped would be $0.0513 \times 3 = 0.1539$ cu. ft. per sec. One horsepower = 550 ft.-lb. per sec.; hence,

$$\text{Horsepower} = \frac{6{,}978 \times 0.1539}{550} = 1.95$$

The power might also have been calculated by the relation

$$\text{ft.} \times \frac{\text{lb.}}{\text{sec.}} = \frac{\text{ft.-lb.}}{\text{sec.}}$$

since the head in feet is known, and the flow in pounds per second could be calculated.

MECHANISM OF FLUID FLOW

When fluids move through a closed channel of any cross-section, either of two different types of flow may occur according to the conditions. These forms are most easily visualized by referring to a classic experiment, first performed by Osborne Reynolds. In Reynolds' experiment, a glass tube was connected to a reservoir of water in such a way that the velocity of the water flowing through the tube could be varied at will. In the inlet end of the tube a nozzle was inserted through which a fine stream of colored water could be introduced. The apparatus was essentially as shown in Fig. 10.

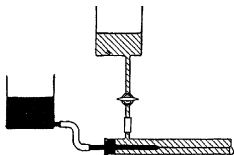



Fig. 10.—Reynolds' experiment.

Reynolds found that when the velocity of the water was low the thread of color maintained itself throughout the tube. By putting in more than one of these jets at different points in the cross-section, it could be shown that in no part of the tube was there any mixing, and the fluid flowed in parallel straight lines.

As the velocity was increased, it was found that at a definite velocity the thread disappeared and the entire mass of liquid was uniformly colored. In other words, the individual particles of liquid, instead of flowing in an orderly manner parallel to the long axis of the tube, were now flowing in an erratic manner so that there was complete mixing.

These two forms of fluid motion are known as *viscous flow* on the one hand, and *turbulent flow* on the other hand. The velocity at which the flow changes from one form to the other is known as the *critical velocity*.

The Reynolds Number.—Reynolds, in a later study of the conditions under which the two types of flow might occur, showed that the critical velocity depended on the diameter of the tube, the velocity of the fluid, its density, and its viscosity. Further, Reynolds showed that these four factors must be combined in one way and one way only, namely, $Du\rho/\mu$, where D is the inside diameter of the pipe, u the average velocity of the liquid,

ρ its density, and μ its viscosity. This function $Du\rho/\mu$ is known as *the Reynolds number* and is of importance in hydrodynamic discussions. It has a fundamental bearing on many of the problems which the chemical engineer must face. Since Reynolds' time much additional work has been done and it has been shown that, for straight circular pipe, when the value of the Reynolds number is less than 2,100 the flow will always be viscous. When the value of the Reynolds number is over 4,000 the flow will always be turbulent, except in very special cases. In the range between these values the flow may be viscous or it may be turbulent according to the details of the apparatus, and in no case can a prediction be made as to whether a given apparatus will give viscous or turbulent flow in this range. Fortunately, however, this range is not often important in engineering work. In giving the above values for the Reynolds number D is measured in feet, u in feet per second, ρ in pounds per cubic foot and μ in pounds per foot per second (see page 133).

Distribution of Velocities.—By measuring the velocities in a circular pipe at different distances from the center, and at a reasonable distance from the entrance to the pipe, it has been shown that in both viscous and turbulent flow, the fluid in the middle of the pipe is moving faster than the fluid next the walls. If velocity be plotted against distance from the walls, curves such as shown in Fig. 11 are obtained. In viscous flow the curve is a true parabola, sharply pointed in the middle and tangent to the walls of the pipe. The average velocity over the whole cross-section is 0.5 times the maximum. In turbulent flow, on the other hand, the curve is somewhat flattened in the middle, and the average velocity is ordinarily about 0.8 times the maximum. The precise relation between the average velocity (u) and the maximum velocity (u_{max}) is shown as a function of the corresponding Reynolds numbers in Fig. 12. It should be noted, however, that these relations hold only in straight sections of pipe where the flow is steady and isothermal. Changes in direction, temperature, or cross-section distort the shape and proportions of the velocity-distribution curves.

It should also be noted that these curves of velocity distribution become tangent to the pipe wall and indicate a velocity approaching zero as the pipe wall is approached. The more refined the methods used for measuring the velocity distribution,

the more clearly this decrease of velocity near the wall is demonstrated. In other words, at the actual surface of the wall the

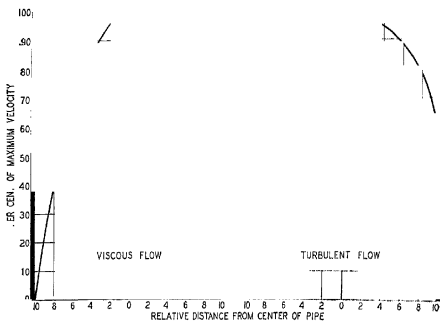



FIG. 11.—Distribution of fluid velocity across a pipe.

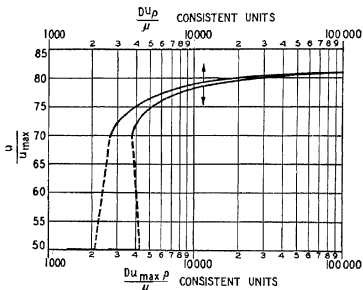



FIG. 12.—Relation between maximum velocity and average velocity.

velocity must be zero. At small distances from the wall the velocity is sufficiently low to be below the critical, and hence there must be a film of fluid moving in viscous flow, even though

the flow in the main body of the stream is fully turbulent. This viscous or creeping film has been demonstrated experimentally and will appear many times in the later discussions in this book.

It should be noted that, although a considerable fraction of the total velocity change from center to tube wall occurs in the film, it is by no means true that the velocity change in the turbulent core is negligible.

VISCOSITY

The term *viscosity* has been used several times in the preceding discussion. It is desirable to define this property and to give some concrete information regarding the methods by which it is measured. In hydrodynamic theory, fluids are often idealized and considered to have no viscosity. Such a fluid offers no

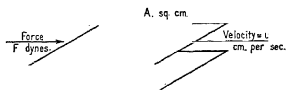



Fig. 13.—Definition of viscosity.

resistance to shear forces. That is, it requires no force to slide one layer of such a fluid past another. An analogy to such a perfect fluid is a deck of ideal playing cards that would slide one upon the other with no frictional resistance. In this case if the top card were displaced by a rubbing motion, none of the other cards would move. All actual fluids, however, exhibit to a widely varying extent some resistance to such shearing forces. An analogy to an actual fluid is a deck of actual playing cards, where if the top card is moved, all other cards will slide to some extent beneath it.

Units of Viscosity.—Consider two layers of a fluid L cm. apart (Fig. 13). Suppose that each of these two layers has an area of A sq. cm. Suppose the top layer is moving parallel to the bottom layer at a velocity of u cm. per sec. relative to the bottom layer. If the fluid were perfect, this constant velocity u could be maintained without expending any energy or applying any force. For any actual fluid, however, a force of F dynes is required to maintain the velocity u. It has been found experimentally that the force F is directly proportional to the velocity u, to the

area A, and inversely proportional to the distance L. This statement is expressed mathematically as

$$F = \frac{\mu u A}{L} \tag{20}$$

where μ is a proportionality constant. Equation (20) can be used to define the unit of viscosity. Upon solving this equation for μ, it is found that

$$\mu = \frac{LF}{uA} \tag{21}$$

The dimension of L is length; that of force F is mass times acceleration, or $\frac{\text{mass} \times \text{length}}{(\text{time})^2}$; that of velocity u is $\frac{\text{length}}{\text{time}}$; and that of area A is $(\text{length})^2$. If these dimensions are substituted in Equation (21) and the cancellations performed, it is found that the dimensions of viscosity are $\frac{\text{mass}}{\text{length} \times \text{time}}$. In the metric system the unit of viscosity is logically defined as grams divided by centimeter seconds. This unit is known as the *poise* and is named after the French scientist Poiseuille, who carried out fundamental investigations on viscosity. It so happens, however, that this unit is inconveniently large for most fluids. For example, water at a temperature of 68.6°F. is found experimentally to have a viscosity of 0.0100 poise. For this reason it is customary to express viscosity in *centipoises;* a centipoise being 0.01 poise, just as a centimeter is 0.01 m. For this reason, also, it is often customary to express viscosities as viscosity relative to water at 68.6°F. This so-called *relative viscosity,* however, is numerically the same as the absolute viscosity expressed in centipoises.

In English units, viscosity may be expressed as $\frac{\text{lb.}}{\text{ft.} \times \text{sec.}}$ This unit has no name. Since 1 g. = 1/453.6 lb. and

$$1 \text{ cm.} = \frac{1}{30.48} \text{ ft.},$$

these may be substituted in the definition of the poise:

$$1 \text{ poise} = 1 \text{ English unit} \times \left(\frac{30.48}{453.6}\right) = 0.0672 \frac{\text{lb.}}{\text{ft.} \times \text{sec.}}$$

The common unit, the centipoise, equals 0.000672 English units. One English unit equals 1,488 centipoises. The English unit is sometimes used with the dimensions lb./(ft. × hr.) To convert centipoises to this unit, multiply by 2.42.

Determination of Viscosity.—It has been shown theoretically and experimentally that when fluids move in viscous flow in a tube of circular cross-section, this flow takes place according to the Hagen-Poiseuille equation.

$$\Delta P = \frac{32Lu\mu}{gD^2} \tag{22}$$

where

ΔP = the drop in pressure in pounds per square foot.
L = the length of the tube in feet.
μ = the viscosity in pounds per foot per second.
g = the acceleration of gravity in feet per second per second.
u = the velocity in feet per second.
D = the diameter of tube in feet.

If the viscosity be known, this equation permits the calculation of the pressure drop due to friction in viscous flow. The Hagen-Poiseuille equation is, however, much more useful for determining viscosity when the other terms are known.

Viscometers.—Since it has been shown that turbulent flow passes into viscous flow where the Reynolds number has a value of 2,100 to 4,000, for the determination of viscosity it will be necessary to insure viscous flow by having the constants of the apparatus so chosen that the value of the Reynolds number will be less than that corresponding to the critical velocity. This is most easily done by choosing a passage with so small a diameter that viscous flow is assured. A viscometer, therefore, consists essentially of a capillary tube through which a fluid may be made to flow under such conditions that the constants in Equation (22) can be evaluated. It usually consists of a glass capillary with a bulb at one end, and this bulb is calibrated at two points with a known volume between them. The bulb and the capillary are filled with the liquid to be investigated, and a known pressure is applied until this predetermined volume has been forced through the capillary. Knowing the time necessary for this flow and the volume that has been displaced, the average velocity is determined. The length and diameter of the capillary may be determined by actual measurement but are more easily evaluated by calibration with a liquid of known viscosity. All the terms in Equation (22) except μ are then known, and μ can be calculated, although the use of an actual apparatus involves certain corrections that will not be discussed here.

For liquids of a viscosity of the order of magnitude of that of water, it is necessary to use fine capillaries and rather elaborate apparatus. For very viscous liquids, such as oils, the diameter of the capillary may be larger so that the liquid will flow through it under gravity head. Viscometers of this type are quite common. Such an instrument consists of a vessel with a short capillary tube in the bottom, and surrounded by a constant-temperature bath. A definite volume of the liquid, the viscosity of which is to be determined, is put in the vessel, and a calibrated receiver is placed below the tube. By removing a stopper from the tube and determining the time necessary to fill the calibrated receiver, a figure is obtained that is a function of viscosity. Two common viscometers for this purpose are the Engler and Saybolt viscometers, and their readings are usually expressed in seconds rather than in absolute viscosities. Readings on the Saybolt viscometer may be converted to absolute viscosities by Equation (23):

$$\frac{\mu}{s} = 0.22\theta - \frac{180}{\theta} \tag{23}$$

where

μ = viscosity in centipoises.
s = specific gravity.
θ = Saybolt reading in seconds.

Data for the viscosities of various fluids are given in Appendices II and III.

FRICTION LOSSES

In Bernoulli's equation, a term was included to represent the loss of energy due to friction in the system. These frictional losses may be of many kinds. An important engineering problem is the calculation of these losses, not only for water but also for any fluid, from its conditions of flow and physical properties. It has been shown that the fluid can flow in either of two ways—viscous or turbulent. For isothermal viscous flow Equation (22) can be used to calculate friction drop. In practice, however, fluids are rarely handled in viscous flow. Since the two methods of flow are so widely different, a different law of frictional resistance is to be expected in the case of turbulent flow from that which is the case in viscous flow. On the other hand, it will be shown that both cases may be handled by one relationship in such a way that it is not necessary to make a preliminary calculation to determine whether the flow is taking place above the critical velocity or below it.

The friction loss of a fluid flowing through a pipe is but a special case of a general law of the resistance between a solid

and fluid in relative motion. Consider a solid body, of any desired shape, immersed in a stream of fluid. Let the length of this body, measured perpendicular to the general direction of flow of the fluid, be D and let the area of contact between the solid and the fluid be A. If the velocity of the fluid past the body be small in comparison to the velocity of sound, it has been found experimentally that the resisting force depends only on the roughness, size, and shape of the solid and on the velocity, density, and viscosity of the fluid. Through a consideration of the dimensions of these quantities[1] it can be shown that

$$\frac{F}{A} = \frac{\rho u^2}{g} \cdot \phi'\left(\frac{Du\rho}{\mu}\right) \tag{24}$$

where

F = the total resisting force.
u = the velocity of the fluid past the body.
ρ = the density of the fluid.
μ = the viscosity of the fluid.
g = the acceleration of gravity.
ϕ' = some function whose precise form must be determined for each specific case.

Here again the Reynolds number $Du\rho/\mu$ appears. The form of the function represented by ϕ' depends upon the geometric shape of the solid and its roughness.

Friction in Pipes.—In the particular case of a fluid flowing through a circular pipe of length L, the total force resisting the flow must equal the product of the area of contact between the fluid and the pipe wall, and the F/A of Equation (24). The pressure drop will equal this product divided by the cross-sectional area of the pipe, since pressure is measured in force per unit area.

$$\Delta P_f = \frac{F}{A}\left(\frac{L\pi D}{\pi D^2/4}\right) = \frac{4\rho u^2 L}{gD} \cdot \phi'\left(\frac{Du\rho}{\mu}\right) \tag{25}$$

This may be written in the following form:

$$\frac{\Delta P_f D}{u^2 L\rho} = \phi\left(\frac{Du\rho}{\mu}\right) = \frac{\Delta H_f D}{u^2 L} \tag{26}$$

[1] BRIDGEMANN, "Dimensional Analysis," pp. 83–86, Yale University Press, 1922.

where

ΔP_f = pressure drop due to friction in pounds per square foot.
F/A = resisting force in foot-pounds per square foot of contact area.
L = length of pipe in feet.
D = inside diameter of pipe in feet.
ρ = density of fluid in pounds per cubic foot.
u = average velocity of fluid in feet per second (total flow in cubic feet per second divided by cross-section of pipe in square feet).
μ = viscosity of fluid in English units (pounds per foot per second).
g = acceleration of gravity (32.2 ft. per sec. per sec.).
ΔH_f = loss in head due to friction in feet ($= \Delta P_f/\rho$).

In making this simplification, the factors 4 and g of Equation (25) have been combined with ϕ' to make a new function ϕ. The factor of roughness cannot be evaluated numerically, and its effect appears in variations in ϕ. Equation (26) expresses the fact that the pressure drop due to friction, multiplied by $D/\rho u^2L$, is a function only of the Reynolds number and pipe roughness. Experiments have shown that for clean smooth circular pipe (brass, copper, or glass) this function may be expressed by the equation[1]

$$\frac{\Delta H_f D}{Lu^2} = 0.0000870 + \frac{0.00777}{Re^{0.32}} \qquad (27)$$

and for clean new standard iron pipe the equation is[1]

$$\frac{\Delta H_f D}{Lu^2} = 0.0001906 + \frac{0.01171}{Re^{0.38}} \qquad (28)$$

where Re equals the Reynolds number.

A plot of Equations (27) and (28) is given in Fig. 14. The data on which these equations are based scatter to such an extent that the results of Equation (27) are accurate only to ± 5 per cent, and those of Equation (28) to ± 10 per cent. For this reason relatively few coordinates have been drawn in Fig. 14, to prevent reading the figure closer than the accuracy of the data warrants. Equations (27) and (28) must not be used when the value of the Reynolds number is below 4,000.

[1] Drew, Koo, and McAdams, *Trans. Am. Inst. Chem. Eng.*, **28**, 56–72 (1932).

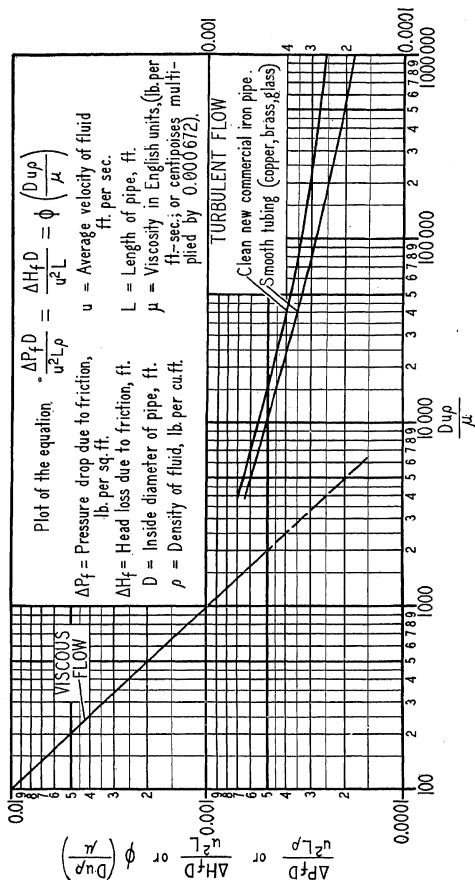



FIG. 14.—Friction of fluids in pipes.

To calculate the pressure drop, it is necessary only to determine the value of the Reynolds number, to read from the correct graph the value of $\Delta P_f D/\rho u^2 L$ or $\Delta H_f D/u^2 L$ and calculate ΔP_f or ΔH_f from this.[1]

The general law for fluid-solid resistance expressed by Equation (24) undergoes certain modifications in certain velocity ranges. For example, for low values of the Reynolds number (where the flow is viscous), the function $\phi'(Du\rho/\mu)$ becomes equal to $\mu/Du\rho$ and Equation (24) degenerates into

$$\frac{F}{A} = \frac{u\mu}{gD} \tag{29}$$

When the force per unit area is translated into pressure drop in a pipe, Equation (22), the Hagen-Poiseuille equation, results, except for the numerical constant. It will be noted on Fig. 14 that a single straight line with a slope of -45 deg. is drawn for lower values of the Reynolds number. This straight line represents the Hagen-Poiseuille equation and gives the friction drop under conditions of viscous flow, that is, for values of the Reynolds number below that corresponding to the critical velocity. This line is straight and at a slope of -45 deg. because Fig. 14 is plotted on log-log coordinates, and under these conditions the slope must be -1. Figure 14 can therefore be used for both types of fluid flow.

Examination of the turbulent-flow lines of Fig. 14 shows that as the value of the Reynolds number increases, the lines become nearly horizontal. For these conditions the friction drop is substantially independent of the Reynolds number (and therefore of viscosity) and ultimately becomes proportional to the square

[1] The above method for the determination of fluid friction is the method originated by Reynolds and carried to its logical conclusion by Stanton. Many engineers use essentially the same method under the name of *Fanning's equation.* Fanning's equation is usually written

$$\Delta P_f = \frac{2fL\rho u^2}{gD} \tag{29a}$$

By comparing this with Equation (26) it will be seen that

$$\phi\left(\frac{Du\rho}{\mu}\right) = \frac{2f}{g} \tag{29b}$$

from which the relation between the ϕ-function as used in this text and f, the *Fanning friction factor,* may be determined.

of the velocity. The change in conditions from low values of the Reynolds number to high values can be visualized as follows: For conditions of viscous flow, the resistance is due entirely to the slip of the fluid along the pipe without the formation of eddies. As the critical velocity is passed, however, eddies form; and for intermediate values of the Reynolds number, the conditions illustrated in Fig. 11 and discussed on pages 28 to 31 exist. Here, it has been shown, there is a core of liquid in turbulent flow; and a slow-moving film of liquid, next the wall, in viscous flow. The total pressure drop due to friction is the sum of the losses in the turbulent core and in the viscous film. Near the critical velocity the latter effect predominates. For high values of the Reynolds number the viscous forces are small as compared to the turbulent forces, the entire friction loss is due to the kinetic energy of the turbulent fluid (measured by the square of the velocity), and the curve flattens out to a constant value.

The above discussion considers only the friction losses when a fluid is passing through straight pipe. When there are disturbances due to changes in section or in direction, other losses occur which must be considered separately. It must be understood that these losses are permanent, since they are due to the conversion of kinetic or potential energy into heat. They do not replace the terms for velocity and pressure heads in Bernoulli's equation.

Enlargement Losses.—If the cross-section of the pipe enlarges so gradually that the fluid adapts itself to the changed section without additional disturbances, there are no energy losses at this point. If the change is sudden, it results in additional losses due to eddies which are greater at this point than in a straight pipe. For sudden enlargement the loss is represented by

$$\Delta H_e = \frac{(u_1 - u_2)^2}{2g} \tag{30}$$

where

ΔH_e = the loss in head in feet.

u_1 = the velocity in feet per second at the smaller cross-section.

u_2 = the velocity in feet per second at the larger cross-section.

Contraction Losses.—When the cross-section of the pipe is reduced suddenly, the losses due to additional eddying are expressed by

$$\Delta H_c = \frac{Ku_2^2}{2g} \tag{31}$$

where

u_2 = the velocity in the smaller cross-section.
K = a constant, depending on the relative areas of the two sections.

Values for this constant are given in Fig. 15.

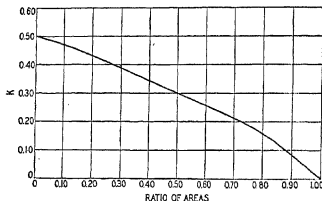



FIG. 15.—Contraction coefficients.

Losses in Fittings.—Every change in direction of the pipe introduces an additional loss due to additional turbulence. The data for the losses in commercial fittings are far from complete, but Appendix I gives approximate values expressed as equivalent lengths of straight pipe. Recent work has shown that this equivalent length is a function of the Reynolds number, increasing as the Reynolds number increases. Data showing this last feature are too limited as yet to warrant their inclusion.

Example 6.—Figure 16 represents an elevated tank connected to a pipe line. The system contains water at 180°F. What must be the height of the water surface in the tank to produce a discharge of 100 gal. per min.?

Solution.—Bernoulli's theorem may be written between points A and B. The datum plane for elevation may be taken through point B. Since no work is done on or by the liquid, except to overcome friction, Equation (18) becomes

$$X + \frac{p_A}{\rho} + \frac{u_A^2}{2g} - F = \frac{p_B}{\rho} + \frac{u_B^2}{2g}$$

The diameter of the tank may be assumed large enough so that u_A is zero; and since the pressures at A and B are both atmospheric, the pressure terms are equal and may be dropped. The above equation reduces to

$$X - F = \frac{u_B^2}{2g}$$

In this case, F includes (1) the contraction loss at the exit of the tank, (2) friction in the 4-in. pipe, (3) contraction loss between 4-in. and 2-in. pipe, and (4) friction in the 2-in. pipe. These items are calculated as follows:

1. *Contraction Loss at Tank.*—From Appendix IV the inside cross-section of 4-in. pipe is found to be 12.730 sq. in.; hence the velocity in the 4-in pipe is

$$\frac{100 \times 231}{60} \times \frac{1}{12.730} \times \frac{1}{12} = 2.52 \text{ ft. per sec.}$$

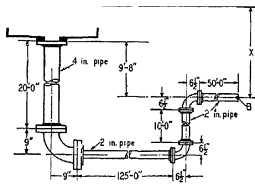



FIG. 16.—Data for Example 6.

The area of the tank is so large in proportion to the area of the pipe that the ratio of the areas is practically zero, and hence from Fig. 15 the constant in Equation (31) is 0.5. Substituting in Equation (31) gives

$$\Delta H_c = \frac{0.5 \times 2.52^2}{2 \times 32.2} \quad 0.05 \text{ ft.}$$

2. *Friction in 4-in. Pipe.*—This involves the use of Equation (26) as plotted in Fig. 14. In Equation (26) (see Appendices I, II and X).

$$D = 4.026 \text{ in.} = 0.3355 \text{ ft.}$$
$$u = 2.52 \text{ ft. per sec.}$$
$$\rho = 60.58 \text{ lb. per cu. ft.}$$
$$\mu = 0.347 \text{ centipoise} = 0.000233 \text{ English units}$$
$$L = 20 + 7.94 = 27.94 \text{ ft.}$$
$$\frac{Du\rho}{\mu} = \frac{0.3355 \times 2.52 \times 60.58}{0.000233} \quad 219{,}700$$

From Fig. 14, where $Du\rho/\mu = 219{,}700$;

$$\frac{\Delta H_f D}{u^2 L} = 0.00030$$

Substituting,

$$\frac{\Delta H_f \times 0.3355}{(2.52)^2 \times 27.94} = 0.00030$$
$$\Delta H_f = 0.16 \text{ ft.}$$

3. *Contraction Loss.*—The ratio of the areas of the two pipe sizes is (Appendix IV) 3.355/12.730 = 0.263. From Fig. 15 the constant in Equation (31) is 0.40. The velocity in the 2-in. pipe will be the velocity in the 4-in. pipe divided by the ratio of the areas, or 2.52/0.263 = 9.58 ft. per sec. Substituting in Equation (31) gives

$$\Delta H_c = \frac{0.40 \times 9.58^2}{2 \times 32.2}$$
$$\Delta H_c = 0.57 \text{ ft.}$$

4. *Friction in 2-in. Pipe.*—The length is the straight pipe plus the equivalent of two ells, or

$$125 + 10 + 50 + (2 \times 3.55) = 192.1 \text{ ft.}$$

$$\frac{Du\rho}{\mu} = \frac{2.067/12 \times 9.58 \times 60.58}{0.000233} = 428,400$$

From Fig. 14,

$$\frac{\Delta H_f D}{u^2 L} = 0.00027$$
$$\Delta H_f = 27.65 \text{ ft.}$$

The total friction loss is

	Feet
Contraction from tank	0.05
Friction in 4-in. pipe	0.16
Contraction from 4 to 2 in.	0.57
Friction in 2-in. pipe	27.65
Total	28.43

Substituting in Bernoulli's theorem, as stated above,

$$X - 28.43 = \frac{9.58^2}{2 \times 32.2}$$
$$X = 29.85 \text{ ft.}$$

and the water is 29.85 − 9.67 or 20.2 ft. above the bottom of the tank.

Example 7.—Water at 40°F. is to flow through 1,000 ft. of horizontal iron pipe at the rate of 150 gal. per min. A head of 20 ft. is available. What must be the pipe diameter?

Solution.—In Equation (26), D, the unknown, appears on both sides of the equation, and hence a direct mathematical solution is impossible. A

satisfactory answer may be obtained, however, by the method of trial and error. By this method a pipe diameter is chosen at random, the lost head calculated, and the result so obtained is compared with the head available to determine whether the assumed diameter was too large or too small.

Assume 4-in. Pipe:

$$\rho = 62.4 \text{ lb. per cu. ft.}$$
$$\mu = 1.55 \text{ centipoises} = 0.001042 \text{ English units}$$
$$D = 4.026 \text{ in.} = 0.3355 \text{ ft.}$$
$$L = 1{,}000 \text{ ft.}$$
$$u = \frac{231 \times 150}{60} \times \frac{1}{12.73} \times \frac{1}{12} = 3.78 \text{ ft. per sec.}$$
$$\frac{Du\rho}{\mu} = 75{,}970$$
$$\frac{\Delta H_f D}{u^2 L} = 0.00035$$
$$\Delta H_f = 14.91$$

The velocity head $u^2/2g$ is 0.22 ft., making a total head lost of 15.13 ft. This is less than the head available; hence the 4-in. pipe is larger than necessary.

Assume 3½-in. Pipe:

$$u = \frac{231 \times 150}{60} \times \frac{1}{9.886} \times \frac{1}{12} = 4.87 \text{ ft. per sec.}$$
$$\frac{Du\rho}{\mu} = 86{,}200$$
$$\frac{\Delta H_f D}{u^2 L} = 0.00034$$
$$\Delta H_f = 27.23 \text{ ft.}$$
$$\frac{u^2}{2g} = 0.37 \text{ ft.}$$

The total lost head is therefore 27.6 ft., or more than the available head. Hence, 3½-in. pipe is too small, and 4-in. pipe must be used.

Application to Gases.—The pressure drops determined by Fig. 14 hold good, not only for liquids but also for gases when the proper values of the constants are employed. In discussing Bernoulli's equation it was pointed out that when gases flow through a pipe, there may be a term involving the work of expansion or compression. In applying Fig. 14 to the flow of a gas through a pipe, the results, if calculated on the basis of the final density, will be reasonably accurate if the pressure drop is not over 10 per cent of the initial pressure. If the pressure drop determined by using Fig. 14 is greater than this, the simple form of Equation (26) cannot be used and recourse must be

had to more elaborate equations that take account of the work term.[1]

Sections Other than Circular.—In applying Equation (26) to flow of fluids in channels that are not circular in cross-section, the problem arises of what to use for the diameter. In these cases it is customary to use an equivalent diameter,[2] defined as $\frac{4 \times (\text{cross-section area of channel})}{\text{wetted perimeter of channel}}$. Thus, for an annular space with diameters D_1 and D_2 the equivalent diameter would be $\frac{4\pi(D_1^2 - D_2^2)}{4\pi(D_1 + D_2)}$ or $D_1 - D_2$ for the D in Equation (26). It will be noted that if the above rule is applied to a channel of circular cross-section, the equivalent diameter is the diameter of the pipe, as it should be if the definition of equivalent diameter is sound.

MEASUREMENT OF FLUIDS

For the control of industrial processes, it is desirable to know the amount of materials used. Since these materials are in the form of liquids or solutions wherever possible, it becomes of prime importance to be able to measure the rate at which a fluid is flowing through a pipe or other channel. Methods of measuring fluids may be classified as follows:

1. Direct weighing or measuring.
2. Hydrodynamic methods.
   a. Orifice.
   b. Venturi meter.
   c. Pitot tube.
   d. Weirs.
3. Direct displacement.
   a. Disc meters.
   b. Current meters.
4. Miscellaneous.
   a. Thomas meter.
   b. Dilution methods.

The methods in the first class involve primarily mechanisms for weighing, but such mechanisms do not fall within the scope

[1] See PERRY, "Chemical Engineers' Handbook," p. 721, McGraw-Hill Book Company, Inc., 1934. Future references to "Perry" refer to this work.

[2] In hydraulics the term *hydraulic radius* is often used. The above definition of the equivalent diameter is four times the hydraulic radius.

of this book. It is not convenient to weigh a gas, but it can be measured directly by introducing it into a bell immersed in a liquid. The volume of the bell per unit displacement is determined by calibration, and the height to which it rises measures the volume of gas collected. Such devices are so simple that they need no further discussion.

Orifices.—When used for measuring the flow of fluids, an orifice is considered to be a thin plate containing an aperture through which a fluid issues. It may be placed in the side or

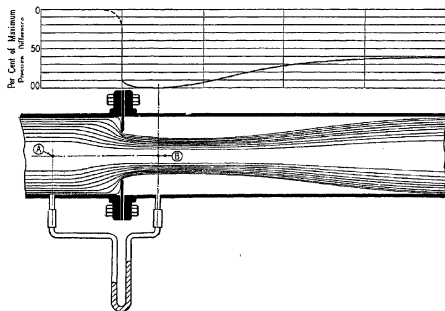



Fig. 17.—Orifice meter.

bottom of a container, but this discussion will be based on the supposition that the plate is introduced into a pipe line.

Figure 17 illustrates such an arrangement to be used for measuring the flow of a fluid. If the edge of the orifice is sharp, the fluid does not at once lose the velocity which it acquired in passing through the orifice. If the two points A and B are chosen, and Bernoulli's equation written between these two points, the following relationship holds:

$$X_A + \frac{u_A^2}{2g} + \frac{P_A}{\rho_A} - F + w = \frac{u_B^2}{2g} + \frac{P_B}{\rho_B} + X_B \qquad (18)$$

Let the section of pipe be horizontal so that the elevation of the points A and B are the same. Then the two X terms are identi-

cal and disappear. As a first approximation, friction losses will be considered inappreciable, and therefore the F term equals zero. Assume that the fluid is a liquid, and hence ρ_A is sensibly equal to ρ_B. Similarly, since no work is done on the liquid or by the liquid, $w = 0$. Equation (18) may then be rewritten as follows:

$$u_B^2 - u_A^2 = \frac{2g}{\rho}(P_A - P_B) \tag{32}$$

Since $P_A - P_B = \Delta P$, and since $\Delta P/\rho = \Delta H$, Equation (32) may be written

$$\sqrt{u_B^2 - u_A^2} = \sqrt{2g\Delta H} \tag{33}$$

If the pipe to the right of the orifice plate in Fig. 17 were removed so that the liquid issued as a jet from the orifice, the minimum diameter of the stream would be less than the diameter of the orifice. This point of minimum cross-section is known as the *vena contracta*. Even if the pipe is full of liquid on both sides of the orifice the vena contracta still exists and is surrounded by swirling liquid. Point B was chosen at the vena contracta. In practice the diameter of the stream at the vena contracta is not known, but the orifice diameter is known. Hence Equation (33) may be written in terms of the velocity through the orifice, if a constant be inserted in Equation (33) to correct for the difference between this velocity and the velocity at the vena contracta. There is some loss by friction, and this also may be included in the constant. Equation (33) then becomes

$$\sqrt{u_o^2 - u_A^2} = C_o\sqrt{2g\Delta H} \tag{34}$$

where u_o is the velocity through the orifice.

If a manometer is connected to the points A and B, as shown in the figure, the pressure at B must be less than the pressure at A, and this difference can be read directly from the manometer. Since the ratio of the area of the orifice to the area of the pipe is known, it follows that the ratio between u_o and u_A is known, and therefore Equation (34) can be solved for either of the velocities. When u_A and the cross-section of the pipe are both known, the volume of liquid handled per hour follows directly.

The constant C_o depends on the ratio of the orifice diameter to the pipe diameter, on the position of the orifice taps, and on the value of the Reynolds number for the liquid flowing in the pipe. In Fig. 18 are plotted several curves. The upper set of curves show the range in allowable positions for the downstream pressure tap. The position of the upstream tap is not of great importance, but it is usually one pipe diameter from the orifice. The second set of curves show the value of the orifice constant C_o for different pipe diameters. It will be noticed that

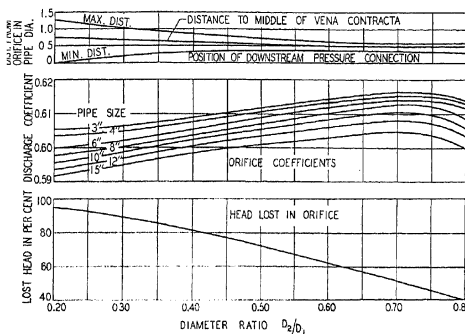



FIG. 18. Orifice characteristics.

over a wide range of conditions the use of 0.60 to 0.61 for C_o will be reasonably accurate for most orifices. The curves of Fig. 18 should not be used for values of the Reynolds number below 15,000 to 20,000 referred to the orifice.

The orifice is a very simple device and is easily installed. Its principal disadvantage is its large power consumption, due to losses in eddies on the downstream side. The orifice always results in a permanent loss of pressure, which increases as the ratio of pipe diameter to orifice diameter increases and may easily be 90 per cent of the pressure drop across the orifice. This relation is shown in the bottom curve of Fig. 18.

Venturi Meters.—It has been shown that the principal disadvantage of the orifice is the power lost due to the sudden contraction, with consequent eddies on the downstream side. If the change in velocity were made gradual so that there were no shocks, the same relations would hold that hold for the orifice, but the energy losses would be less. The Venturi meter as shown in Fig. 19 consists of two tapered sections inserted in the pipe line, with the tapers smooth enough and gradual enough so that there are no serious losses.

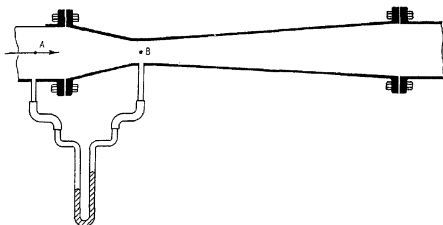



Fig. 19.—Venturi meter.

If point A be taken in the pipe and point B at the contracted section or throat of the Venturi meter, Equation (33) may be written for a frictionless Venturi meter as it was written for the orifice. Since there are practically no losses due to eddies, however, and since the cross-section of the high-velocity part of the stream is accurately defined, the Venturi meter more nearly conforms to the theoretical equation than does the orifice. For the Venturi Equation (33) may be written

$$\sqrt{u_B^2 - u_A^2} = C_v\sqrt{2g\Delta H} \qquad (35)$$

where u_B is the velocity at the throat of the Venturi. This is the same as Equation (34). In the case of the Venturi meter, the value of the coefficient C_v is 0.98; and the head permanently lost is from $\frac{1}{8}$ to $\frac{1}{10}\Delta H$.

For both the Venturi and the orifice, if the diameter of the smaller section is $\frac{1}{5}$ the pipe diameter or less, u_A^2 is so small as

compared to u_B^2 that the former may be neglected and the equations become

$$u_o = C_o\sqrt{2g\Delta H} \quad (36)$$
$$u_v = C_v\sqrt{2g\Delta H} \quad (37)$$

When it is not convenient to install an orifice or a Venturi small enough to satisfy this condition, the same results may be obtained by enlarging the pipe line above the measuring devices. This enlarged section must be long enough to eliminate any eddies or other disturbances and give substantially steady flow in the enlarged portions.

In comparing the Venturi meter with the orifice meter, both the cost of installation and the cost of operation must be considered. The orifice is cheap and easy to install. The Venturi meter is decidedly expensive, as it must be carefully proportioned and fabricated. A home-made orifice is often entirely satisfactory, whereas a Venturi meter is practically always purchased from an instrument dealer. On the other hand, the head lost in the orifice for the same conditions as in the Venturi is many times greater. The power lost is proportionately greater, and when an orifice is inserted in a line that is carrying fluid continuously over long periods of time, the cost of this lost power may be out of all proportion to the saving in first cost. Orifices are therefore used for testing purposes or other cases where the power loss is not a factor, as in steam lines. Venturi meters are used for permanent installations. It should be noted that for a given pipe diameter and a given diameter of orifice opening or Venturi throat, the reading of the Venturi meter for a given velocity is to the reading of the orifice as $(0.61/0.98)^2$ or 1:2.58.

In deriving the equations for the orifice and Venturi meter, liquids have been postulated. If the fluid is a gas, Bernoulli's equation properly applied leads to a solution of the problem. The use of Bernoulli's equation for such cases, however, involves the integration of the work term in Equation (18). It is beyond the scope of this book to carry out such derivation in detail. The result, however, of such an integration is simply stated. For values of ΔH less than 20 per cent of the upstream pressure head, Equations (34), (35), (36), and (37) apply to the flow of gases subject to one condition—that the difference in head

ΔH be expressed in terms of the downstream density. In such cases the error is not over 10 per cent. For cases where the pressure drop is greater than 20 per cent of the upstream pressure head, more complicated equations must be used. If ΔH is so high, however, the meter is probably poorly designed for the service, and the power loss will be excessive.

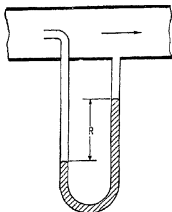



Fig. 20.—Principle of Pitot tube.

Pitot Tube.—Suppose that two tubes be inserted as shown in Fig. 20 into a pipe carrying a fluid. If the tube at right angles to the flow be properly designed, it will measure the pressure head only. The tube that points upstream will measure the pressure head plus the velocity head. The reading R of the manometer will therefore measure the velocity head, and

$$\Delta H_P = \frac{u^2}{2g} \tag{38}$$

where ΔH_P is the head of the fluid whose flow is to be measured that corresponds to R.

It should be noted that whereas the orifice and Venturi meter measure the *average* velocity of the whole stream of fluid, the Pitot tube measures the velocity at *one point* only. As discussed on page 29, this velocity varies over the cross-section of the pipe. Consequently, to obtain the true average velocity over the whole cross-section, one of two procedures must be used. The Pitot tube may be inserted at the center of the pipe and the average velocity calculated from this maximum by means of Fig. 12. If this procedure is adopted, care must be taken to insert the Pitot tube at least 100 pipe diameters from any disturbance in the flow, so that the velocity distribution may be normal. The other procedure is to make the Pitot tube adjustable. Readings are then taken at different points in the cross-section, and the mean velocity found by graphic integration. This latter procedure is the one usually used for low-pressure gases in large ducts.

The simple form of the Pitot tube shown in Fig. 20 is not practical. The tubes themselves cause too much disturbance, and there are apt to be eddies within the pressure tube that disturb its readings. Different forms of construction are shown in Fig. 21. These are more or less approximations. Since the Pitot tube is oftenest used on gases, if the tubes are made small in order to avoid disturbances, they are apt to become clogged with dust. A perfect Pitot tube should obey Equation (38) exactly, but all actual instruments must be calibrated and a constant correction applied. The disadvantages

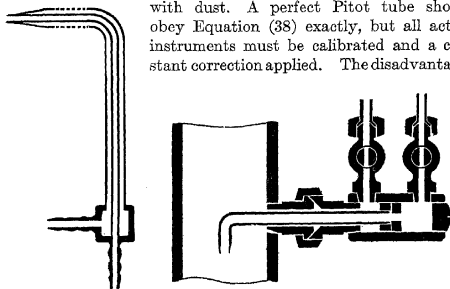



FIG. 21.—Pitot-tube construction.

of the Pitot tube are, first, that it does not give the average velocity directly and, second, that its readings for gases are extremely small. When used on low-pressure gases, some form of multiplying gage such as shown in Fig. 6*b* or 7 must be used.

Weirs.—Weirs are applicable only to liquids flowing in open channels, not in closed pipe lines. The weir consists of a partition or obstruction with a notch cut in it to carry the flow of liquid. The amount of flow is determined by observing the level of the liquid above the edge of the opening in the weir, at a point far enough upstream to be free from the disturbances that occur in the neighborhood of the notch. Various forms of weirs are used with rectangular, V-shaped, and curved-sided notches. For the rectangular weir the flow is given by the Francis formula:

$$\frac{V}{\theta} = 3.33[L - 0.2H]H^{3/2} \qquad (39)$$

where

V = the flow in cubic feet.
θ = the time in seconds.
L = the width of the weir in feet.
H = the height of the liquid over the bottom edge of the weir in feet.

For the V-notch weir the flow is given by

$$\frac{V}{\theta} = 2.505\left(\tan\frac{\alpha}{2}\right)^{0.996} H^2 \tag{40}$$

where α is the angle of the notch and H is measured from the vertex of the notch. A weir with curved sides may be developed in which the rate of flow is directly proportional to the head of the liquid on the weir.

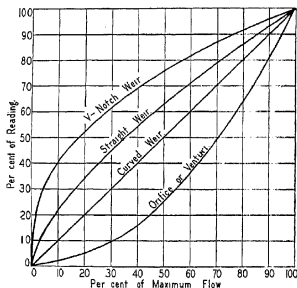



FIG. 22.—Comparison of meter types.

The flow through the orifice or Venturi is proportional to the square root of the head, and therefore the orifice is less accurate for small heads than for large ones. Weirs, on the other hand, are more accurate at small flows than at large, especially in the case of the V-notch weir. The relation between per cent of maximum head on the one hand and per cent of total flow on the other hand is shown in Fig. 22. For wide variations from

small flows to large ones, the V-notch weir is a satisfactory method of measurement. Its principal disadvantage, however, is that it cannot be conveniently incorporated in a pipe line. The pipe must discharge into a vessel containing the weir, and the liquid must be taken out of the vessel from below the weir. This complication limits the applicability of this device.

Flow Meters.—All of the above devices depend for their readings on a difference in pressure. For the purposes of plant control, it is desirable to have continuous records of the flow of fluids in pipe lines. There are many types of recording pressure gages available. It would be possible to attach to any of the above devices a recording pressure gage and from this record determine the average pressure drop, and therefore the average rate of flow. That this would be unsound is apparent from an inspection of Equations (34) to (40) inclusive. In no case except that of the curved-sided weir is the rate of flow proportional to the pressure drop. For the orifice, for instance, the velocity, and therefore the quantity of fluid flowing, is proportional to the square root of the pressure drop. If the pressure fluctuates through an appreciable range, the square root of the average pressure is not equal to the average of the square roots of the individual pressures. It is this latter that is needed to calculate the flow, and obviously the labor of converting a direct record of pressure would be unreasonable. Recording devices have therefore been developed which record, not the pressure drop directly, but the square root of the pressure drop. These records can then be averaged, and by applying a constant factor the total flow over a given period of time can be determined.

Such devices are rather elaborate and are purchased from apparatus houses. The principal element of the *Bailey meter* (Fig. 23) is a mercury well with a float of variable cross-section. One side of the manometer leads to the interior of the container and hence to the mercury surface at A. The other side of the manometer is connected to the under side of the bell B; and as the pressure difference between A and B fluctuates, the bell B tends to rise and fall. The position of the bell B, however, is determined by equilibrium between its total weight on the one hand and the buoyancy of the submerged portions plus the pressure difference on the other hand. The sides of this bell are so curved that its displacement is proportional to the square

root of the pressure difference. The motion of the bell B is then recorded on a chart with the proper constants.

Figure 24 shows the principle of the *Republic flow meter*. One arm of the manometer is connected to the chamber A, whose cross-section is sufficiently great so that the mercury level is

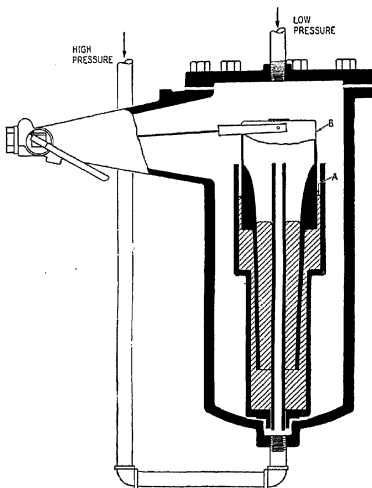



FIG. 23.—Bailey flow meter.

essentially constant. The mercury level will therefore rise and fall in the chamber B in proportion to the pressure drop across the instrument. In the chamber B are mounted a series of resistance spools, R_1, R_2, R_3, etc., and each of them is connected to one of the contact rods C. A constant voltage is applied at E. As the mercury rises it short-circuits the resistances one after the other and therefore changes the current in the circuit. If

the resistance in all the spools is equal and the ends of the contact rods are so spaced as to form a square root curve, the current in the circuit will be proportional, not to the difference in level between A and B, but to the square root of this difference. This current is recorded on a recording ammeter M. The advantage of this type is that the current in the circuit need not be small, and therefore the recording instrument can be made robust. It can also be located at any desired distance from the point where readings are actually taken. The only disadvantage of this

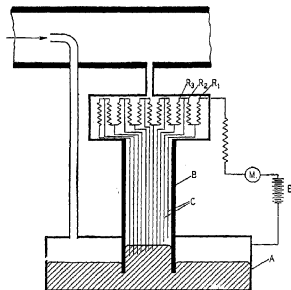



Fig. 24.—Principle of Republic flow meter. A. Mercury reservoir. B. Metering chamber. C. Contact rods. E. Constant voltage supply. M. Recording ammeter. R_1, R_2, R_3, resistance spools.

type is that the record, instead of being a smooth curve, is in a series of steps; but by making a sufficient number of contact rods, the steps may be made so small that they are practically continuous. There are many other similar devices available.

Displacement Meters.—This term covers devices for measur ing liquids, based on the displacement of a moving member by a stream of liquid. These meters may be classified as disc meters and current meters.

Figure 25 shows a typical *disc meter*. The member whose displacement actuates the recording device is a hard-rubber disc A. This disc is mounted in a measuring chamber B which has a conical top and bottom. The disc is so mounted that it is always tangent to the top cone at one point and to the bottom

cone at a point 180 deg. distant. The measuring chamber has a partition C that extends halfway across it, and the disc has a slot to take this partition. The measuring chamber is set into the meter body in such a way that the liquid enters at one side of the partition, passes around through the measuring chamber,

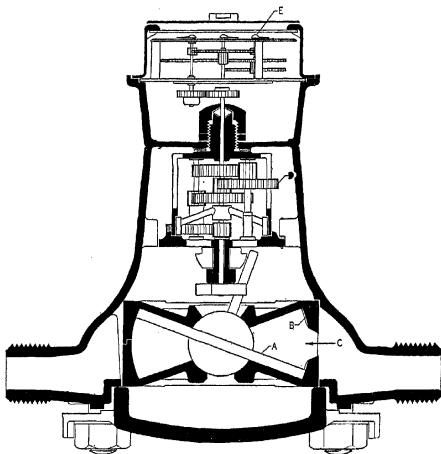



FIG. 25a.—Disc meter, vertical section.

and out on the other side of the partition. Whether the liquid enters above or below the disc, it must move the disc in order to pass, and this motion of the disc results in the axis moving as though it were rotating around the surface of a cone whose apex is the center of the disc and whose axis is vertical. This motion of the axis of the disc is transmitted through a train of gears D to the counting dials E in the top of the meter.

One type of *current meter* is shown in Fig. 26. The turbine wheel and the train of gears which it drives are delicately mounted

so that they move with the minimum of friction. The stream of water entering the meter strikes the buckets on the periphery of the wheel and causes it to rotate at a rate proportional to the velocity of the water passing through the meter. There is a wide variety of types of current meter on the market that differ in the construction of the rotating member. The principle of all of them is the same, namely, a rotating member moving with the least possible friction and so mounted that its speed of rotation is proportional to the rate of flow of liquid.

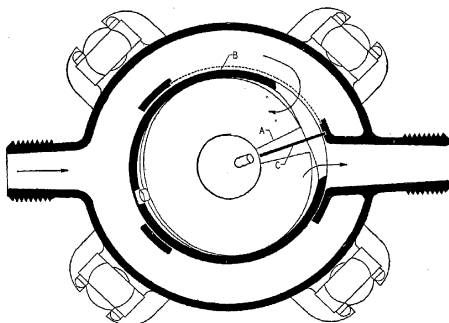



FIG. 25b.—Disc meter, horizontal section. *A*. Disc. *B*. Metering chamber. *C*. Chamber partition. *D*. Gear train. *E*. Dials.

Both these meters are ordinarily made in sizes under 2 in. They may be obtained in larger sizes, but the larger sizes are bulky and expensive. The disc meter is often thought to be more accurate than the current meter for small rates of flow, but it is doubtful if either type is accurate to more than 2 per cent.

The current meter is better adapted for large rates of flow, and the disc meter for small rates. For widely varying flows in large lines the *compound meter* is used. This consists of a small disc meter and a large current meter in parallel. An automatic hydraulically-operated valve connects the disc meter to the line at low water velocities; but when the demand is large it cuts

out the disc meter and cuts in the current meter. The total flow is taken from the sum of the two dials.

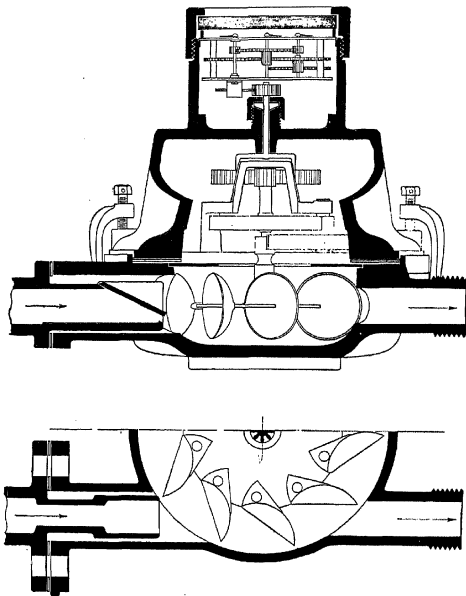



Fig. 26.—Current meter.

Dilution Methods.—In cases where no mechanical device is possible or convenient, a second fluid may be added at a known rate to the stream of fluid to be measured, and the concentration of this second fluid determined by analysis at a point sufficiently

far from the point where it was introduced to insure thorough mixing. For instance, a stream of water flowing in an open channel may be measured by adding to it a solution of common salt of known concentration at a known rate and then determining the salt content of the stream some distance farther down the channel. This method is also applicable to gases, in which case the reagent added may be either carbon dioxide or ammonia, since these are easily determined.

Measurement of Gases.—Gases flowing in small amounts may be measured by mechanical gas meters of the type used on domestic gas supply. These consist of a pair of leather bellows so arranged that one of the bellows is filling while the other is emptying. The movement of the bellows operates a lever, the motion of which is transmitted to a train of gears.

The *wet gas meter* is shown in Fig. 27. It consists of a drum, half submerged in a liquid, with openings (a', b', c', d') around the outside for the outlet of gas, with partitions dividing the inside into a series of radial chambers (A, B, C, D), and with inlets (a, b, c, d) near the center. The gas enters the meter at E. The flow of the gas into a compartment causes it to rise. This results in the rotation of the drum, bringing another compartment below the liquid level so that the gas is displaced by the liquid entering this second compartment. These meters are built in all sizes, up to meters large enough to measure the output of gas-manufacturing plants.

FIG. 27.—Wet gas meter. A, B, C, D, metering compartments. a, b, c, d, compartment inlets. a', b' c', d', compartment outlets. E, gas inlet.

Cycloidal pumps of the type shown in Fig. 72 can be adapted to the measurement of gases by mounting the impellers so that they turn very easily. This type is merely a gas blower run backward, with the gas displacing the impellers instead of the impellers displacing the gas. Even large meters of this type will operate on a pressure drop of one inch of water.

The *Thomas meter* is a special case of the dilution method in which heat is added instead of adding a fluid. Two wire grids are placed some distance apart in the gas line. These grids are so connected that they operate as electric resistance thermometers. Between them is an electric heating coil. The two thermometer grids are connected to a thermostat mechanism which controls the input of energy to the heater so that there is a definite rise in temperature between the two grids. If the heat capacity of the gas is known, it is necessary only to measure the electric input to the heater in order to measure the flow of gas. This electrical input may be recorded on a standard recording wattmeter, which can be calibrated in terms of cubic feet of gas.

Nomenclature of Chapter II

A = area in square feet
C = constant or coefficient
D = diameter in feet
F = friction loss in feet; force in pounds
f = friction factor, no units
g = acceleration of gravity in feet per second per second
H = head in feet
K = constant, in contraction loss formula
L = length, or distance in feet
m = distance in feet
P = pressure in pounds per square foot
R = reading in feet
Re = Reynolds number, no units
u = velocity in feet per second
V = specific volume in cubic feet per pound = $1/\rho$ (in Eq. (39), volume in cubic feet)
w = work added, foot-pounds per pound
X = distance, or head, in feet
sub A, B, refer to materials A and B or points A and B.
sub c = sudden contraction
sub e = sudden enlargement
sub f = due to friction
sub o = orifice
sub P = Pitot
sub v = Venturi
sub 1, 2, = stages or positions 1 and 2 (1 always upstream from 2)
α = angle
Δ = difference in, or loss of
θ = time in seconds
μ = viscosity, pounds per foot per second

ρ = density in pounds per cubic foot = $1/V$

$\left.\begin{matrix}\phi \\ \phi'\end{matrix}\right\}$ = experimental functions, exact form undefined

Problems

1. A differential manometer like Fig. 6*b* has oil of specific gravity 1.025 in the bottom of the tube, and on top of the oil is water. The manometer is filled so that the liquid in the two large chambers is at the same level. If a pressure difference of 0.45 in. of water exists between points 1 and 7, what will the reading of the manometer be?

2. Calculate the velocities below which the flow will be viscous for

a. Water at 60°F. in ⅛-in. standard pipe.
b. Water at 60°F. in 2-in. standard pipe.
c. Water at 200°F. in 2-in. standard pipe.
d. Air at 60°F. and 5 lb. gage in 2-in. standard pipe.
e. Oil of a viscosity of 300 centipoises and a specific gravity of 0.92 in 4-in. standard pipe.

3. Water at 40°F. is to flow out of an elevated tank through 1,000 ft. of standard 4-in. pipe. The level in the tank is 20 ft. above the discharge end of the pipe. What is the rate of flow in gallons per minute?

4. A pump takes water at 50°F. from a large reservoir and delivers it to the bottom of an open elevated tank. The level in the tank averages 160 ft. above the surface of the reservoir. The line is 3-in. standard pipe, and consists of 500 ft. of straight pipe, 6 ells, 2 tees, and 2 gate valves. The pump delivers 150 gal. per min. What is the horsepower consumed if the pump has a mechanical efficiency of 55 per cent?

5. Air at rest, at 70°F. and 30 in. Hg, is forced by means of a fan through a flue 3 ft. in diameter and 100 ft. long at a velocity of 20 ft. per sec. What is the theoretical power required? If the motor has an efficiency of 90 per cent and the fan an efficiency of 20 per cent, how large should the motor be?

6. A standard sharp-edged orifice, 1½ in. in diameter, is inserted in the straight pipe of Problem 4, with the taps in the correct positions. What will be the pressure drop in pounds per square inch across the orifice? How much power is consumed at the orifice? How much is the power at the pump motor increased?

7. A Venturi meter, with a throat 1½ in. in diameter and a coefficient of 0.98, is used to replace the orifice of Problem 6. What will be the Venturi reading, the theoretical power consumption of the Venturi, and the increase in power at the pump motor?

8. What would be the reading of a Pitot tube (coefficient 1.00) inserted at the center of the 3-in. pipe of Problem 4, if the Pitot tube were connected to a manometer containing acetylene tetrabromide (sp. gr. = 2.97)?

9. Repeat Problems 4, 6, 7, and 8 if the pump is handling an oil whose specific gravity is 0.9, viscosity 10.0 centipoises. What would be the readings if orifice and Venturi were changed from 1½ to 2 in.?

10. Plot the curve of discharge in gallons per minute against head in inches, for the flow of oil of sp. gr. 0.87 through a 90-deg. V-notch weir.

11. Brine of specific gravity 1.20, is flowing through a standard 3-in. pipe at a maximum rate of 185 gal. per min. In order to measure the rate of flow, a sharp-edged orifice, connected to a simple mercury manometer is to be installed. The maximum reading of the manometer is to be 400 mm. of mercury. What size orifice should be installed? Express the result to the nearest ⅛ in.

Repeat, assuming that a Venturi meter is used instead of an orifice.

12. Saturated steam at 125 lb. gage is flowing through a standard 3-in. pipe. To calibrate a steam meter, anhydrous ammonia gas is fed into the line from a cylinder of liquid ammonia. In one test when the rate of steam flow was substantially constant for 20 min., 19.6 lb. of ammonia were fed into the line. At a sufficient distance beyond the point of introduction of ammonia to insure complete mixing, a stream of the mixture was withdrawn and condensed. The condensate for the entire 20 min. was mixed, and a sample of 100 cc. was titrated with normal sulphuric acid. It required 57.3 cc. of the acid to neutralize the sample. The average of the steam meter readings for this period was 5,530 lb. steam per hr. What was the per cent error of the steam meter?

CHAPTER III

TRANSPORTATION OF FLUIDS

The transportation of fluids is one of the most important unit operations in chemical engineering. The transportation of material in fluid form is so much more convenient and economical than transporting solids, that wherever possible materials are moved in the form of liquids or solutions. This chapter will deal with the apparatus for moving fluids, as the previous chapter dealt with the mechanics of fluid flow and pipe resistance.

PIPE

The first requisite in transporting a fluid is a channel through which the flow may take place. Hydraulic and mining engineers often employ open channels, but the chemical engineer usually transports his fluids in some form of pipe. Although pipe is made of many special materials for special purposes, iron pipe is used so much more widely than any other form that it will constitute the basis of this discussion.

FIG. 28.—Bell-and-spigot joint for cast-iron pipe.

Cast-iron Pipe.—This is used mainly for underground lines that carry relatively non-corrosive liquids. It is heavier and more expensive than varieties of pipe to be described later, and the joints are less satisfactory. It has an appreciably greater resistance to corrosion than ordinary iron pipe. Cast-iron pipe is regularly made in sizes of 3 in. nominal inside diameter and larger. The usual length is 12 ft. on the straight section. The commonest joint for this class of pipe is the *bell and spigot*, which is shown in Fig. 28. The dimensions of this joint are specified in every detail by the American Water Works Associa-

tion. In assembling this joint, the bottom of the space between the spigot and the bell is usually calked with *oakum*. On top of this, lead is cast; and this lead is then calked into place with blunt chisels, in such a manner that the lead is compressed to fill the groove on the inside of the bell. Sometimes instead of melted lead, a fibrous form of lead called "lead wool" is used; or Portland cement, asbestos rope, or various other materials may be used instead of oakum.

A properly made bell-and-spigot joint will hold at pressures up to 100 lb. per sq. in. and has the advantage that it does not require the two pipes to be in perfect alignment when the joint is made. Ordinarily, however, it is considered unsafe for pressures over a few pounds per square inch, and some of the more elaborate joints not discussed in this book are used for the higher pressures. Flanged cast-iron pipe is made but is not considered good practice, because cast iron is weak except in pure compression.

Iron Pipe.—In speaking of the materials of which pipe is made, usage differs from other branches of engineering. The machine designer, the structural engineer, and almost everyone else who uses iron and steel always understands cast iron when the term "iron" is used without qualification. In the pipe trade, however, cast-iron pipe is always called cast-iron pipe, and when the word "iron" is used without other qualifications, it always means a low-carbon steel. This use of the term *iron pipe* is perfectly definite and standard; so that when one speaks of steel pipe, he usually means a special pipe made of a higher-carbon steel. In the early days of the pipe industry, all pipe was made from wrought iron. Some pipe is still so made. This pipe must be specified as wrought-iron pipe, and even so, much mild-steel pipe still passes for wrought iron.

Iron pipe is made by rolling a flat strip or *skelp* to the proper width and thickness. For pipe under 1½ in. this skelp is then drawn through dies and butt welded in one operation. Pipe over 1½ in. is lap welded by drawing so as to bring the edges over each other, reheating to a welding heat, and running through a pair of welding rolls. The butt weld is less desirable. The standard length of iron pipe is approximately 20 ft. The skelp may also be rolled into a spiral and the edges riveted. *Spiral riveted pipe* is usually made much thinner walled than the standard and is used for temporary construction work, for exhaust

and vent headers, for overhead lines with long unsupported spans, and in other places where its lightness and cheapness are the deciding factors.

The Briggs Standard.—Iron pipe is regularly drawn to the Briggs standard gage. This specification covers, not only the dimensions of the pipe, but also the details of the threads that are used on it. An abridged form of the Briggs specifications is given in Appendix IV. It should be noted particularly that the nominal size is only an approximate size and is neither the inside nor the outside diameter. In the larger sizes the nominal size is nearly the actual inside diameter. The Briggs standard does not include sizes over 12 in. Pipe larger than this is known as *large outside diameter pipe* and is specified by actual outside diameter and wall thickness. The common wall thickness is ⅜ in., but other weights may be obtained. Details of pipe threads are also given in Appendix IV. Pipe threads are finer than standard machine threads of the same diameter and are always tapered. Iron pipe is also made in extra-strong and double extra-strong thicknesses. In such cases the outside diameter and the threads are the same as those of standard pipe, and the extra thickness is on the inside. This is so that all weights of pipe may be threaded with the same standard dies and used with standard fittings.

Tubing.—Copper and brass especially, and, to some extent, iron, nickel, and other metals, are sold in the form of tubing rather than pipe. Pipe is completely specified by giving its nominal size from the Briggs tables. Tubing, on the other hand, is sold on the basis of actual outside diameter and wall thickness. Tubing of a given outside diameter is usually drawn in a variety of wall thicknesses, so that both must be specified. In the case of copper, brass, and iron, the material can be obtained either as pipe or as tubing. The wall thickness of copper and brass tubing is often expressed in terms of the Birmingham wire gage (B.W.G.). Appendix V gives the dimensions of some common sizes of tubing.

FITTINGS

Fittings are used in pipe lines for

1. Joining two pieces of pipe.
2. Changing the direction of the line.

3. Changing the diameter of the line.
4. Connecting branch lines.
5. Stopping the end of a line.

It is obvious that in many cases two or more of these functions may be combined in the same fitting.

The commonest material of which fittings are made is gray cast iron and it is used for probably 90 to 95 per cent of the fittings in ordinary chemical engineering practice. Where vibration is serious and there is danger of cast iron cracking, malleable iron may be used. For high pressures, or severe service, cast-steel fittings are available; and for exceptional cases fittings forged from mild steel are on the market. These latter are relatively expensive and are used only where the service is severe from the standpoint either of pressure or of secondary strains.

Pressure Ratings of Fittings.—Fittings are usually rated as low pressure, standard, extra heavy, and hydraulic. The low pressure is usually rated at 25 lb. per sq. in. for steam or air, the standard at 125 lb., extra heavy at 250 lb., and hydraulic in varying classes from 300 to 10,000 lb. per sq. in. These pressures have no direct relation to the breaking strength of the fittings. It is rare that either pipe or fittings fail due to pressure alone. Most fittings fail either from expansion strains or from shocks such as water hammers. These may not be calculated in advance, and therefore a wide margin of safety exists between the actual bursting strength of a fitting and the pressure for which it is rated. In fact, in many cases where the pressure is no greater than would be allowed for standard fittings, but where strains due to expansion may be serious, extra-heavy fittings may be used.

The use of low-pressure fittings in general practice is to be discouraged because they are easily mistaken for standard fittings and used where they are too light. Their principal field is such service as gas distribution where a large enough stock may be kept on hand so that the slight decrease in cost of low-pressure fittings is justified and where the low-pressure fittings may be kept permanently separated from standard fittings.

Pipe Joints.—Two pieces of pipe may be joined in a variety of ways. Only the most important of these will be mentioned. The joint may be welded, and this method is rapidly gaining

in favor. *Couplings* are short sleeves threaded internally on each end. One coupling is regularly furnished with every length of pipe. Couplings are considered bad practice in sizes over 2 in. but are sometimes used, especially where the joint is made up in the shop and not in the field. One type of *union* is illustrated in Fig. 29. In this case the joint is metal to metal. The better unions have brass rings pressed into them, and these rings are finished to spherical surfaces of different radii so that the joint will be tight even if the pipe is slightly out of alignment. Less desirable types require a ring of packing which needs replacement at intervals. Screwed joints are not considered good practice in sizes above 2 or 2½ in. for several reasons. First, it is difficult to make the pipe thread enter the fitting on large sizes. Second, threads over 2½ in. must usually be cut on a machine in the shop rather than with hand tools. Third, the wrenches needed to tighten threads on pipe 3 in. or larger are often difficult to handle.

FIG. 29.—Union.

Flanges are the universal method of connecting pipe of any size over 2 to 2½ in. They may be classified by methods of facing and methods of attaching. Such a classification is as follows:

Methods of facing	Methods of attaching
Plain face	screwed
Grooved face	shrunk
Raised face	riveted
Polished face	welded
Male-and-female face	swivelled
Tongue-and-groove face	

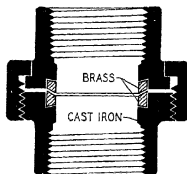



Figure 30 shows various methods of facing flanges. The plain face *a* is obvious. The grooved face is made by taking a light

cut in a lathe over the whole face with a round-nosed tool. This gives a better grip on the packing, but is used only for low pressures and soft packing. The raised-face flange *b* is used regularly on all fittings designed for steam pressures of 250 lb. or higher and

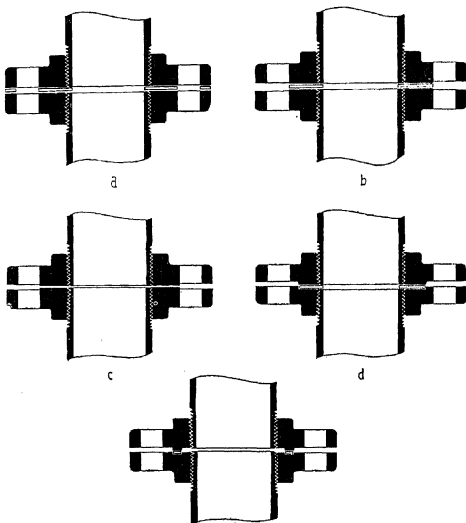



FIG. 30.—Methods of facing pipe flanges. *a*. Plain face. *b*. Raised face. *c*. Ground face. *d*. Male and female. *e*. Tongue-and-groove face.

is often considered good practice for pressures lower than these. The gasket on the plain-faced flange usually extends to the rim of the flange. If considerable tension has to be put on the bolts in order to make the joints tight, the flange may spring enough to make contact around the outer edge without being tight at the

center. As higher pressures usually call for thinner and harder gaskets, this condition is aggravated with such packings. The standard height of the raised face is $\frac{1}{16}$ in. The gasket is cut to cover the raised portion only. In this way the full pressure of the bolts comes on the gasket without any possibility of the edges of the flanges meeting. The ground joint *c* is used only when there is no gasket material available that meets the service conditions. It is very difficult to make tight and is very rarely used. The tongue-and-groove *e* and male-and-female face *d*, are used for pressures so high that there is danger of blowing out the gasket.

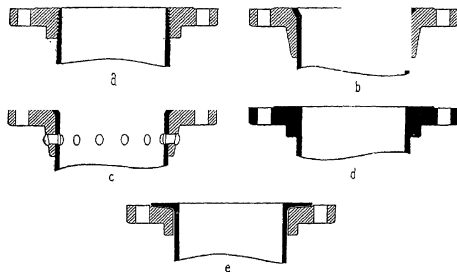



FIG. 31.—Methods of attaching flanges to pipe. *a*. Screwed. *b*. Shrunk and peened. *c*. Shrunk and riveted. *d*. Welded. *e*. Van Stone.

Figure 31 shows methods of attaching flanges to pipe. The standard method *a* is to thread the end of the pipe and screw on the flange. For exceptional service the flange may be shrunk onto the pipe as in *b*, in which case the outer face of the flange has a tapered opening and the pipe is spun out into this taper. In some cases flanges have an extra-long hub and in addition to being screwed or shrunk to the pipe are riveted through this hub (Fig. 31*c*). This additional precaution is practically never used except in marine work, where it is considered an additional safeguard against the joint loosening from vibrations. The flange may be welded to the pipe, either by the torch method or by the ordinary forge-and-hammer method (Fig. 31*d*); cast-iron flanges

cannot be welded to iron pipe, so steel flanges must be used. This is used in high-pressure work or central station practice. Finally, the end of the pipe may be upset after a loose flange has been slipped over it, and then faced off to form the actual contact surface (Fig. 31*e*). This has the advantage that since the flange is loose on the pipe, bolt holes may always be brought into alignment even on complicated assemblies. It has the disadvantage that every individual piece of pipe is a special forging, and repairs are slow and costly. Practically every manufacturer of pipe and fittings has some slight modification of this joint which he calls by a special name. The joint usually passes in common parlance under the name of the *Van Stone joint.*

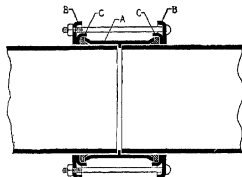



FIG. 32.—Ring joint. *A*. Sleeve. *B*. Rings. *C*. Packing recess.

Ring Joint.—Another type of joint widely used, especially for gas and oil lines, is the ring joint shown in Fig. 32. A loose sleeve *A* is slipped over the joint, and this sleeve is so formed that it functions as a stuffing box on either end. At either end of the sleeve is a ring *B* with a projecting ring *C* which closes the stuffing box and compresses the packing. These joints may be easily and quickly applied, provide for some expansion, and do not require that the pipe be perfectly aligned. The packing ultimately fails and the joint is not self-supporting. Consequently they are seldom used in ordinary plant service but are common on underground lines. This joint is known under a variety of trade names but is perhaps most commonly spoken of as the *Dresser joint.*

Expansion Joints.—All pipe lines of any considerable length in which there may be a change of temperature must be provided with some means for relieving the stresses that are caused by

expansion. This may be done by making a number of turns in the pipe, either with ordinary fittings or with special bends. These turns are so located that no one of them receives more than a fraction of the total stress. Sometimes, for low pressures, a joint consisting of a short length of flexible copper tubing is used. The usual expansion joint, however, is merely a packed slip joint. One type is shown in Fig. 33. In many cases expansion joints have only one stuffing box, instead of two as shown in Fig. 33.

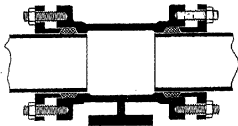



FIG. 33.—Double-ended slip expansion joint.

Fittings.—When a joint fills any of the functions 2 to 5, inclusive, of the outline on page 65, the connection is made by a fitting. For changing directions, these are usually *elbows*. For introducing branches, they are *tees*, *crosses*, or *Y-bends*. For changing the size of the pipe in a straight run, a *reducer* is used; or if this function is to be combined with the function of another fitting, the openings in the fitting may be of unequal sizes, in which cases they are called *reducing fittings*. The most common method, however, of changing the size of a line is by means of *bushings* in small sizes or *reducing flanges* in larger sizes. To close the end of a line, *plugs* or *caps* are used in small sizes and *blank flanges* in large sizes.

A short piece of pipe, threaded at both ends, is called a *nipple*. This term is often, though not necessarily, confined to lengths under 6 in., because such lengths cannot be threaded in an ordinary vise with hand tools but are purchased as such. A *close nipple* is one so short that the threads meet; a *short nipple* is one that has a very short shoulder (approximately ¼ in.) between the threads.

A considerable number of ordinary pipe fittings are shown in Fig. 34, although it must be understood that there is a large variety of fittings available in addition to the ones shown in the figure. Fittings may be threaded and screwed directly to pipe. This is not considered good practice in sizes over 2 to 2½ in. The universal practice in larger sizes is to use flanged fittings which are attached to corresponding flanges on the pipe. Typical flanged fittings are shown in Fig. 35. The flanges of such fittings

are faced to correspond to the method of facing used for plain flanges in the same pipe line.

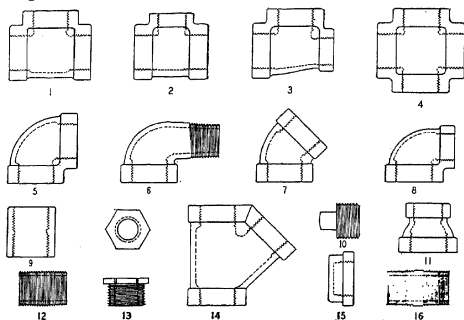



FIG. 34.—Screwed pipe fittings. 1. Tee. 2. Tee reducing on outlet. 3. Tee reducing on run. 4. Cross. 5. Elbow. 6. Street elbow. 7. 45° elbow. 8. Reducing elbow. 9. Coupling. 10. Plug. 11. Reducer. 12. Close nipple. 13. Bushing. 14. Y-branch. 15. Cap. 16. Short nipple.

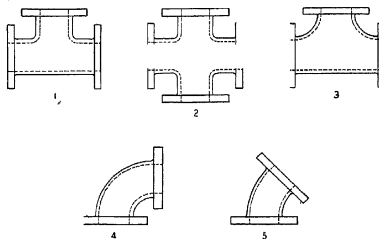



FIG. 35.—Flanged pipe fittings. 1. Tee. 2. Cross. 3. Double-sweep tee. 4. Elbow. 5. 45° elbow.

VALVES

The devices used to control the rate of flow of fluids in a pipe line show an even wider variety than do fittings. For the purpose of this book they will be classified as follows:

Plug cocks.
Globe valves.
  Metal disc.
  Composition disc (Jenkins type)
Gate valves.
  Non-rising stem.
  Rising stem.
  Outside screw and yoke.
Check valves.
  Ball check.
  Swing check.
Reducing valves.

Cocks.—Cocks are the simplest method of regulating the flow of fluids. They consist essentially of a body casting, in which fits a conical plug with a passage through it. This must, of course, be closed by some sort of packing around the stem. A simple type of cock is shown in Fig. 36. Cocks are universally used on small lines for compressed air, rarely for steam or water. Their disadvantage is that if the sides of the plug are too nearly parallel, the plug is easily wedged in the body so firmly that it is difficult to turn, while if the sides of the plug are tapered too much, the pressure in the line acting against this inclined surface has an upward component that tends to push the plug out of its seat. Most cocks tend toward the former difficulty rather than the latter and therefore the universal complaint is that cocks stick and are hard to open. To remedy this, there are a number of special designs on the market, one of the best known of which has lubricant inserted at the stem of the cock. The lubricant is transmitted to the working faces through small holes drilled through the body of the plug. This is satisfactory for ordinary service, but in special cases it is difficult to find greases that will not melt or be dissolved. Another disadvantage of cocks is that when the bore is cylindrical, as is nearly always the case, the area of the opening in the pipe changes very rapidly with a slight amount of rotation when the cock is just opened and changes

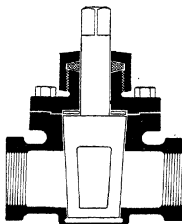



FIG. 36.—Plug cock.

practically not at all when the cock is nearly open. Consequently, it is difficult to regulate flow with a cock, especially at low rates of flow that call for fractional openings. Cocks are usually used, therefore, where they will be either wide open or completely closed.

Globe Valves.—Typical globe valves are shown in Figs. 37 and 38. The essential feature of these valves is a globular body with a horizontal internal partition, having a circular passage-

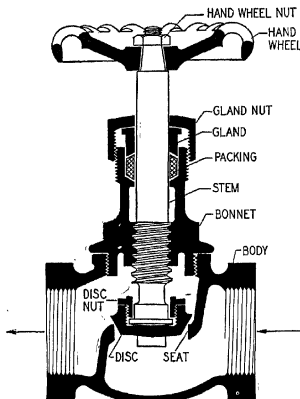



Fig. 37.—Globe valve.

way in which is inserted a ring called the *seat*. Although there are a large number of globe valves varying in cost and ability to withstand pressure, the main differences pertain to the construction of the valve disc and valve seat. The cheaper valves have no separate seat ring, but the better valves have this feature for ease in renewal. The two main types of valve discs are the metal disc shown in Figs. 37 and 38*a* and the composition disc shown in Fig. 38*b*. The globe valve is ordinarily used in smaller sizes. It is generally considered poor practice to use a globe valve in a size larger than 2 in.

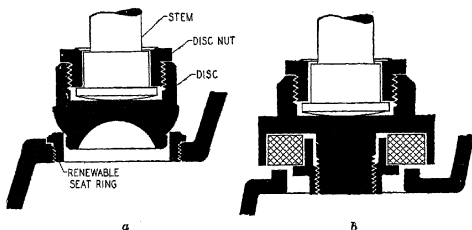



FIG. 38.—Globe valve discs. *a*. All-metal disc. *b*. Jenkins-type disc.

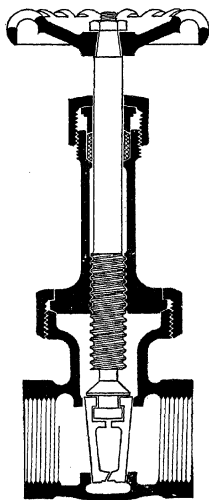



FIG. 39.—Rising stem gate valve.

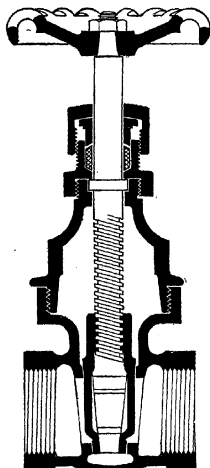



FIG. 40.—Non-rising stem gate valve.

Gate Valves.—Gate valves are universally used in the larger sizes and can be obtained in a wide range of costs, materials, construction, and mechanical details. The rising stem gate valve shown in Fig. 39 illustrates the general type of construction. The non-rising stem valve is shown in Fig. 40. The distinctive feature of this valve is the fact that the thread of the valve stem engages the gate, and the gate rises and falls without the stem rising and falling through the stuffing box. The non-rising stem valve has the advantage of requiring less overall length when open than does the corresponding rising stem valve. On the other hand, one can tell at a glance whether or not a rising stem valve is open. Here, again, a wide variety of gates and seats is obtainable, depending largely upon the cost and the service to which the valve is to be put.

The severe pressure on the gates is a controlling factor in large valves. This not only requires heavy gate construction but also makes it difficult to open the valve manually. In such cases large gate valves are often equipped with small by-pass valves which allow equalization of pressure on the two sides of the gate before opening the main valve.

A third type of gate valve shown in Fig. 41 is known as the *O. S. & Y.* (abbreviation for "outside screw and yoke"). The use of this valve is limited to large sizes where it is necessary that the size of the bonnet casting be cut down to reasonable dimensions.

Q. O. Valves.—The gate valves discussed in the preceding section have threaded stems, which require a number of turns to close the valve completely. So-called *Q. O.* valves ("quick opening") have smooth stems and are opened or closed by a lever handle in a single operation. Such valves are convenient but involve the danger of a water hammer.

Water Hammer.—When a pipe contains a column of moving liquid, there is considerable kinetic energy stored in the liquid by virtue of its mass and velocity. If the velocity is suddenly destroyed (by the quick closing of a valve or cock) this energy cannot be absorbed, since the liquid is nearly incompressible, and therefore appears as an instantaneous shock which may represent excessively high pressures. This effect is greater as the pipe line is longer, the velocities higher, and the time of closing the valve shorter. Fairly long lines may contain enough mass

of liquid to produce a water hammer of sufficient intensity to break fittings. Q. O. valves, therefore, should be used only on short lines. Cocks are open to the same objection if they are quickly closed.

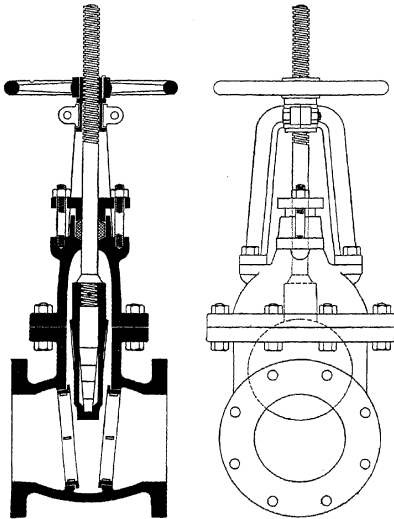



FIG. 41.—*O. S. & Y.* gate valve.

Check Valves.—Check valves are used when unidirectional flow is desired. They are automatic in operation and prevent flow in one direction but allow it in the other. Figure 42 shows two standard type of check valves. The illustrations are self-explanatory.

Reducing Valves.—Reducing valves are used where it is desired to maintain in one part of a system uniform pressures lower

than the pressure in another part of the system. The construction and operation of these valves show an even wider variety than any of the divisions discussed before. The valve shown in Fig. 43 is merely one type of a very large class. This valve is designed for maintaining on the outlet side a constant but lower pressure than the pressure existing on the inlet side. It contains a main valve disc *A* controlling the principal supply of fluid (usually steam), which passes from the left-hand to the right-hand side of the valve. This valve disc is closed by the action of the main valve spring *B* and is opened by pressure applied to the under side of the main plunger *C*. In the upper part of the bonnet there is a thin diaphragm *D* of flexible metal

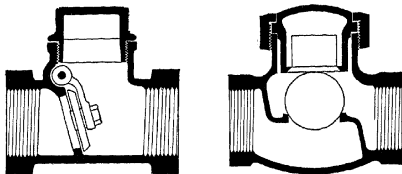



FIG. 42.—Check valves. *a*. Swing check. *b*. Ball check.

which is balanced between the pressure of the regulating spring *E* on its upper surface and the pressure existing on the low-pressure side of the valve, transmitted to its lower surface through port *F*.

Suppose that the pressure at the right of the valve tends to fall. This relieves the pressure on the under side of the diaphragm so that the spring deflects it downward. In so doing, it opens the pilot valve *G*, and this allows a supply of high-pressure steam to flow through a passage *H* in the body to the under side of the plunger. The plunger therefore rises, opens the main valve farther, and supplies additional steam from the left-hand side. As the pressure on the right-hand side increases, the reverse cycle takes place. The diaphragm is deflected upward, the pilot valve closes, the pressure on the under side of the plunger is decreased, and the main valve partly closes. When these valves operate properly, the main valve does not completely open

or completely close but maintains a balanced position and fluctuates up and down as the pressure on the right-hand side tends to fluctuate. Practically all regulating valves operate on a similar principle, namely, a diaphragm with a spring or counterweight exerting pressure on one side, and the pressure which it is

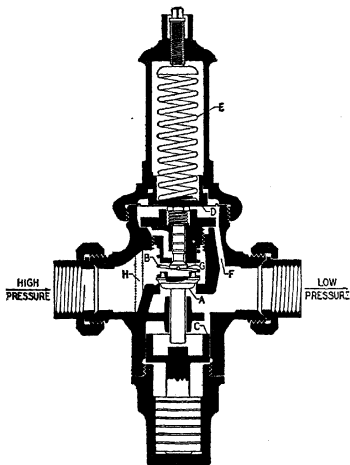



Fig. 43.—Reducing valve. *A*. Main valve disc. *B*. Main valve spring. *C*. Plunger. *D*. Diaphragm. *E*. Regulating spring. *F*. Port from low-pressure side. *G*. Pilot valve disc. *H*. Port supplying steam to plunger.

desired to maintain constant applied to the other side. Fluctuations of the pressure cause corresponding fluctuations in the diaphragm, the movement of the diaphragm opens or closes the pilot valve, and this supplies motive power to open or close the main valve.

Valve Materials.—Valves in pipe sizes of 2 in. and under are usually all brass. In sizes over 2 in. they are usually iron

castings with principal working parts, such as the seat, the contact faces, the disc, and sometimes the stem, of brass or bronze. Such valves are known as *I. B.* (iron body) or *brass-trimmed valves.* For use with solutions that corrode brass, such as ammoniacal or cyanide solutions, all-iron valves can be obtained, but these are not satisfactory for general service because of the rusting of the contact surfaces. For high pressures or severe service, the main castings instead of being cast iron may be high-grade bronze or cast steel; and in exceptional cases the whole body of the valve is a single forging in which the necessary openings are machined. For handling superheated steam at high pressures, the trimmings, instead of being brass, are usually monel metal (an alloy of nickel and copper). The brass-fitted cast-iron valve in sizes above 2 in. and the all-brass valve in sizes of 2 in. and lower are common practice.

GENERAL PRACTICE

This section does not pretend to be a handbook on steam fitting, but it will be of value to mention a few general principles. For the sake of appearance, pipe lines should be run in straight lines with right-angled turns; and the running of pipe at odd angles is usually considered careless work, even though it shortens the line and reduces the number of fittings. It is often considered good practice, where a pipe makes a right-angled turn, to use, instead of an elbow, a tee with one connection closed with a plug or a nipple and cap. This not only facilitates cleaning but also makes it possible to connect into the line without breaking a number of joints. For similar reasons, in long lines of screwed fittings, unions should be inserted at frequent intervals; because if repairs are to be made and a union is not near the point of repair, either the pipe must be taken down to the nearest union or a fitting must be broken. This is an additional advantage of flanged fittings, since any fitting in the line serves the same purpose as a union. In case it becomes necessary to connect two points that cannot be conveniently reached by a series of right-angled bends, the use of two 45-deg. elbows will permit making a connection at any possible angle.

In making sketches, and in drawings in which pipe lines are to be merely indicated, it is customary to use a single line for

a pipe and to indicate, rather than draw, valves and fittings. Standard conventions for such work are given in Fig. 44.

SPECIAL MATERIALS

The chemical engineer is often required to handle materials that are too corrosive to permit the use of iron pipe and fittings. There is no one satisfactory non-corrodible material for all purposes. Most of them have serious disadvantages. Among the materials available may be mentioned the following:

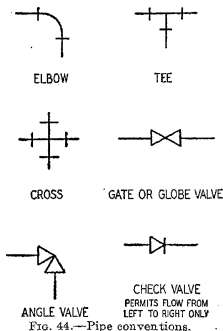
Acid-proof Stoneware.—This material is highly resistant to the action of acids but has a very low strength. It cannot be made in large pieces; it cannot be heated and is easily broken from small temperature changes. A common joint is the bell-and-spigot joint, but pipe with specially designed flanged ends is available.

Glass.—A few sizes of pipe with a specially designed flanged end are furnished by one manufacturer. Glass pipe is stronger and lighter than stoneware and is much more resistant to thermal changes. The variety of forms at present available is so small that it cannot be considered a general material.

Fig. 44.—Pipe conventions.

Duriron.—This is a special high-silicon iron. It is exceedingly resistant to acids, stronger than cast iron, and is made in a variety of shapes and sizes. Its principal disadvantages are that it is extremely brittle, extremely hard, and has a very high coefficient of thermal expansion. Ordinary flanges cannot be used, but designs are available using special flanges. The greatest care must be used in employing this material to prevent strains and consequent breakage due to thermal expansion. It cannot be machined.

Lead.—Lead pipe is available in a large variety of weights and diameters and is especially resistant to solutions containing sul-

furic acid. It is usually joined by *burning* (melting two adjacent pieces with a torch, but in such a way that the pieces unite as they cool). A skillful lead burner can build up practically any kind of equipment with any kind of branches or connections, from lead pipe and sheet. The principal disadvantage of lead is that its elastic limit is very low, and consequently practically every strain, from either mechanical or thermal causes, results in a permanent deformation. As a result, lead work tends to "crawl" so that it must be securely fastened at frequent intervals and supported at all points.

Copper and Brass.—These materials are frequently used in chemical engineering practice, especially copper. Copper is resistant to practically all acids except nitric in moderate concentrations, as long as oxidation is avoided. It has a high strength and is easily worked but is expensive, although its first cost is partly offset by its high scrap value. Copper pipe joints are usually made either by brazing or by turning the end of the pipe over an iron flange. Brass is not especially resistant and is seldom used except in the form of small brass valves.

Lined Apparatus.—Iron apparatus that is to be used for handling corrosive liquids can be lined with lead or with rubber. The lead is usually bonded to the iron with a layer of tin, which alloys with the iron on one side and with the lead on the other. This construction can be used on pipes and valves, but complicated apparatus cannot be easily lead lined. Rubber linings are vulcanized to the steel, and the technique of this has been recently developed to the point where almost any kind of iron apparatus can be provided with a highly resistant rubber lining, perfectly bonded to the iron.

Special Alloys.—There are many varieties of special alloys. In general, these fall into two groups, those in which copper is the base and those in which iron is the base. The copper-base alloys are usually quite complex, containing four or five constituents, and some of them are highly resistant to corrosion. They can be obtained in various forms, including tubing. Those with the iron base are usually alloys of chromium, or nickel, or both. Some of these are known as stainless steels, are very resistant to corrosion, and are now available in practically any form desired. Monel metal is an alloy of approximately 65 per cent nickel, 30 per cent copper, and the balance minor ingredients. It is strong

and easily worked, is quite resistant to dilute solutions, but is not easily available in the form of pipe. Recently, pure nickel has become available at a reasonable price, and its uses are expanding rapidly. It can be obtained in the form of tubing as well as sheets and bars.

PUMPS

A large number of pumps, differing widely in principle and mechanical construction, have been developed to meet a wide variety of operating conditions. No one pump or class of pumps can be considered to be of prime importance with respect to the rest. All the pumps to be described are good practice under suitable conditions at the present time.

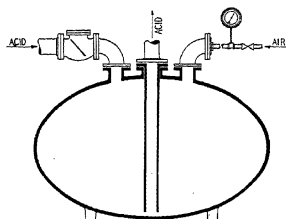



Fig. 45.—Semi-automatic acid egg.

Acid Egg.—Possibly the simplest method of moving a fluid is to displace it from a container by means of air. Apparatus of this type has long been standard in transferring sulfuric acid, and due to this application such apparatus is usually called an acid egg. It is also called a *blow-case* or *montejus*. A simple form consists essentially of a container that can be charged with a batch of fluid to be moved, an outlet pipe leading from the bottom of the container, an inlet pipe, and a pipe through which compressed air can be admitted. The action of such an apparatus is obvious.

A semiautomatic acid egg is shown in Fig. 45. A check valve is placed in the inlet line, and the air line is reduced to half or three-quarter inch pipe. The air-control valve is located at

any convenient point and may be some distance from the egg. A gage is placed in the air line near the air-control valve and between it and the egg. When air is turned on, the gage shows a pressure due to the column of acid in the discharge pipe. The cross-section of the acid outlet is enough larger than that of the air-inlet line so that when the egg empties the air pressure falls and the operator notices this on the gage.

More complicated forms of this apparatus are available which will automatically turn the air on and off as required, and such acid eggs require no attention during their operation. With the development of centrifugal pumps made of acid-resisting materials, the necessity for the acid egg is gradually passing. Its principal disadvantages are not only that it is usually manually operated and intermittent in its action but also that it is wasteful of power because at the end of the cycle the entire egg is filled with air at the discharge pressure, and this is all lost as soon as the discharge pipe empties. The acid egg cannot

FIG. 46.—Diagram of air-jet lift.

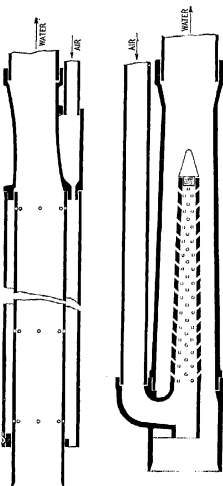



FIG. 47.—Air-jet lift footpieces.

make use of the work that could be done by the air in expanding from the pressure of the egg to atmospheric.

Air Lift.—A more efficient use of compressed air as a fluid transporting agent is represented by the air lift. Figure 46 shows an air lift. In this apparatus the discharge pipe is immersed in the liquid to be pumped, and a jet of compressed air is admitted into the discharge pipe at the submerged end. The pressure and velocity heads of the air released at this point carry the air and slugs of liquid up through the pipe to the discharge end. The manner in which the air is distributed at the bottom of the air lift has a considerable effect on the performance of the apparatus. Figure 47 shows two types of footpiece.

Air-lift Theory.—Although the mechanical construction of the air lift is simple, its action is so complicated that no adequate mathematical theory has ever been developed. Such a theory would involve the frictional resistance of bubbles of air rising through a column of water and is beyond the present state of our knowledge. An empirical formula[1] that has been developed from practice is

$$V_a = 0.8 \frac{H_t}{C \log \frac{H_s + 34}{34}} \tag{41}$$

in which

V_a = cubic feet of free air required to lift one gallon of water.
H_t = total lift, or the total distance from the working surface of the water to the point of discharge.
H_s = running submergence, or the distance from the water level to the point of air inlet.
C = a constant to be taken from the following table:

Lift, feet (H_t), inclusive	Constant
10 to 60	245
61 to 200	233
201 to 500	216
501 to 650	185
651 to 750	156

[1] Ingersoll-Rand Company.

The submergence, expressed as the ratio $\left(\frac{H_s}{H_s + H_t}\right)$, should vary from 0.66 for a lift of 20 ft. to 0.41 for a lift of 500 ft. This formula is said to approximate average practice, but a wide variation in results can be obtained by varying the design of footpieces.

This method of lifting a fluid has an advantage over the acid egg in that the air is utilized at a constant efficiency due to continuous operation. Another prime advantage of this apparatus is the complete absence of moving parts and its extreme simplicity. On the other hand, the air lift requires a great deal of compressed air and the overall mechanical efficiency of air com-

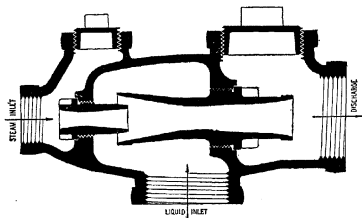



FIG. 48.—Ejector.

pressor and lift is low, though higher than in the acid egg. Thirty per cent mechanical efficiency is probably as high as is usually obtained. Another primary disadvantage is the submergence required.

Ejectors.—Another common method of moving a fluid without using moving parts is by means of an ejector. One type is shown in Fig. 48. The essential feature of the ejector is the expansion of a fluid through a nozzle, the discharge of which is in contact with the fluid to be moved. As the first fluid issues from the nozzle, its velocity head is increased with a corresponding decrease in its pressure head. If this pressure head is less than that of the second fluid at that point, the second fluid will be sucked into the ejector. The simple ejector has the disadvantages of being able to develop only a small head, of being mechani-

cally inefficient, and of diluting the material as it is being transferred. The simple ejector, operated by steam, is used for transferring liquids from one tank to another, and similar cases where the head is low. It is also used to move gases at low heads. On the other hand, refinements in design have produced efficient types that may be used as vacuum pumps.

The simple type of ejector has been developed to a point where it is so efficient mechanically that it can pump a fluid into a space under the same pressure as is present on the actuating fluid. In this form it is called an *injector*. The best example of this type of injector is that used for feeding boilers.

RECIPROCATING PUMPS

In the past, by far the most important method of moving fluids was by the use of some form of reciprocating pump. Such pumps are still of very great importance, although they have found a formidable competitor in the more recently developed rotary pumps of various types.

Reciprocating pumps can be classified in several ways. The principal basis of classification is the construction of the *water cylinder*. Common constructions may be summarized as follows:

- *A*. Piston
- *B*. Plunger
  - *a*. Inside packed
  - *b*. Outside packed
    - (1) Center packed
    - (2) End packed

In addition to this classification, the water cylinder may be *single acting* or *double acting*. Another classification is based on the type of valves, which may be *deck valves* or *pot valves*.

From the standpoint of motive power, reciprocating pumps are classified as *steam pumps* or *power pumps*. They may also be classified as *simplex, duplex, triplex*, etc., according to the number of water cylinders operated on a single drive-mechanism.

Piston Pumps.—Figure 49 shows a simplex double-acting steam-driven deck-valve piston pump. This pump is suitable for heads up to 150 to 200 ft. and for any liquids that are not especially viscous, corrosive, or abrasive. The piston consists essentially of two discs *A* and *B*, with rings of packing *C* between them, so arranged that the outer of these discs can be drawn up to compress the packing. The piston may operate in a cylinder

bored directly in the pump casting, but in the better pumps the piston operates in a removable bronze liner D. The lower row of valves E are suction valves, the upper row F are discharge

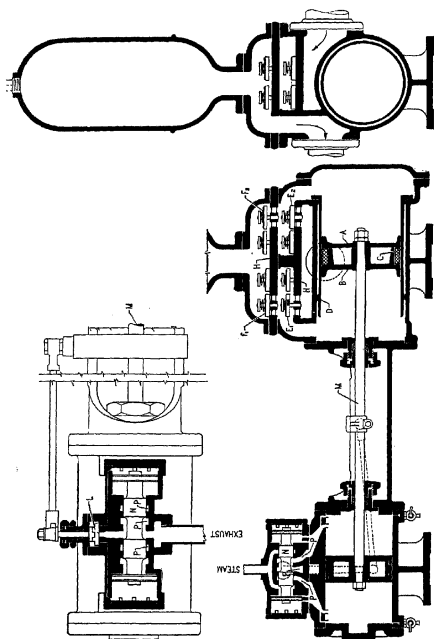



FIG. 49.—Simplex double-acting piston pump. A, B. Water piston. C. Water piston packing. D. Liner. E_1, E_2. Suction valves. F_1, F_2. Discharge valves. H. Valve decks. L. Pilot valve. M. Piston rod. N. Main steam valve. P. Steam ports.

valves. If the piston is moving from left to right, it is obvious that it will create a suction on the left-hand side which will open the left-hand suction valves E_1 and close the left-hand

discharge valves F_1. At the same time a pressure is developed on the right-hand side which will hold closed the right-hand suction valves E_2 and open the right-hand discharge valves F_2. This pump is *double acting,* in that it displaces water on both halves of the cycle, and it will be seen that such a pump requires a minimum of four valves.

The construction of these valves is shown in Fig. 50. A bronze valve seat G is pressed or threaded into the valve deck H and carries a spider supporting the central boss. Into this boss is fastened a stem J that carries the spring that holds valve disc K against the valve seat. The disc may be of hard-rubber composition or of metal, the former being the commoner. Such

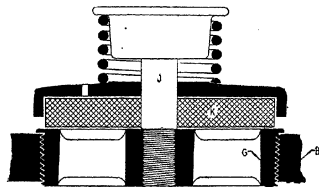



Fig. 50.—Pump valve. *G*. Valve seat. *H*. Valve deck. *J*. Stem. *K*. Valve disc.

valves cannot be made larger than 5 or 6 in. in diameter. In larger valves the total pressure would be too great for a rubber disc to withstand; while at the same time increasing the contact area between valve disc and valve seat would increase the chance of imperfect seating at some point. If the pump is to discharge during one half-stroke more water than can be passed through such a valve at reasonable velocities, several valves will be used. Consequently, instead of the minimum of four valves there will often be a multiple of four, as shown in Fig. 49.

The steam-end construction is subject to considerable modification in different reciprocating pumps, but such differences will not be considered here. In general it can be said that the steam end of reciprocating pumps differs from that of the reciprocating steam-engine in that the steam is not used expansively but is used under full pressure for the entire stroke. This is

necessary because a constant pressure must be maintained on the water end in order that the pump discharge shall remain uniform. Consequently the steam valve gear must be so designed that the steam ports open fully at the beginning of the stroke and remain fully open to the end of the stroke. This usually involves a double valve-mechanism. In the pump shown in Fig. 49, a pilot valve *L*, operated by the piston rod *M*, trips the main valve *N* which moves far enough to uncover completely the steam ports *P*.

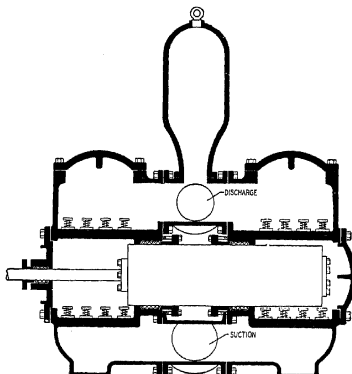



Fig. 51.—Outside-center-packed plunger pump.

Plunger Pumps.—A plunger is differentiated from a piston in that a plunger moves past stationary packing, whereas a piston carries its packing with it. As such pumps as are shown in Fig. 49 become larger, the difficulty of replacing the packing increases. When pumping liquids containing suspended matter that is apt to cut the packing so that replacements are more frequent, it is desirable to have packing more accessible. This has led to the development of the outside-center-packed pump shown in Fig. 51. In this case the pump casting is split in the

middle, and there are two stationary stuffing boxes with a plunger moving through them. This places the packing on the outside of the pump where it is easy to see leaks and easy to make repairs when they become necessary.

Figure 51 also shows how the area of the valve deck increases as the size of the pump is increased. It is obvious that such a pump would not be suitable for high pressures, because the large flat unstayed valve decks of cast iron must, at each stroke of the pump, carry the full difference between suction and discharge pressures. This pressure distributed over such a large area cannot be carried by a material that is as weak in bending as is cast iron. Consequently, for pumps to operate against higher pressures, a different design of valve, called the *pot valve,* is employed.

Outside-end-packed Pumps.—Figures 52*a* and 52*b* show the water end of a duplex outside-end-packed plunger pump with pot valves. In this case the water cylinder is divided into two parts by a partition and the plunger is also in two parts. The left-hand half *A* of the plunger is directly connected to the piston rod *B*, and the right-hand half *C* of the plunger is operated from the other end by means of a yoke *D* and tie-rods *E*. In this case the stuffing boxes that contain the packing are much easier of access than in the outside-center-packed type. Consequently, this type of pump is suitable for higher pressures because the packing may be more easily maintained. Higher pressures, however, preclude the use of deck valves. Consequently, this pump has pot valves. Their construction is obvious from the drawings, and it is merely necessary to call attention to the fact that since each valve is in a separate cylindrical casting, these castings may be made as heavy as desired and can, therefore, stand any pressure that it may be necessary to impose on them.

Since pot valves are required only where high pressures are employed, it is obvious that a rubber composition disc is hardly suitable. Consequently, pot valves usually have metal discs which are often provided with guide vanes to keep them in alignment as shown in Fig. 52. For exceedingly high pressures or viscous liquids, even the metal disc may not be strong enough, and in such cases a loose metal ball is used in place of the valve disc.

Duplex Pumps.—It will be noticed that the pump of Fig. 52 has two water cylinders and was called a duplex pump. This

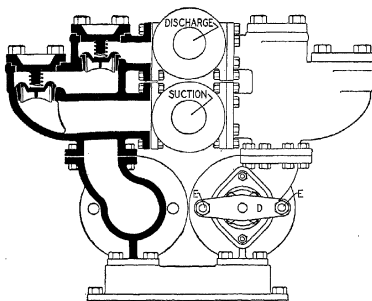



FIG. 52*a*.—Duplex outside-end-packed plunger pump (cross-section).

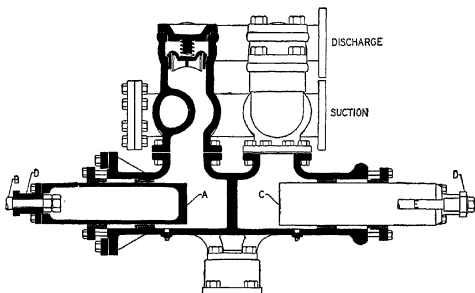



FIG. 52*b*.—Duplex outside-end-packed plunger pump (longitudinal section). *A*, *C*. Water plungers. *B*. Piston rod. *D*. Yokes. *E*. Tie rods.

particular classification is important enough to warrant special mention. The pump of Fig. 49 is a simplex pump, since it has one water and one steam cylinder. The duplex pump can be

considered to be two simplex pumps mounted side by side on the same frame with each pump actuating the steam valve of the other. The steam end of such a pump is shown in Fig. 53. The piston rod *A* carries a cross-head *B* which operates the arm *C*, the rocker shaft *D*, the arm *E*, the connecting rod *F*, and the valve rod *G*. The valve operated by this rod controls the steam supply to the piston that operates the piston rod *J*. In the same way the piston rod *J* operates the valve on the other side of the

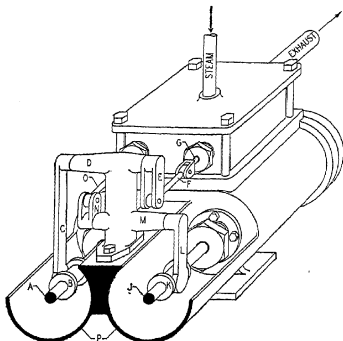



FIG. 53.—Steam end of duplex pump. *A*, *J*. Piston rods. *B*, *K*. Cross-heads. *C*, *L*. Rocker arms. *D*, *M*. Rocker shafts. *E*, *N*. Short rocker arms. *O*, *F*. Connecting rods. *G*. Valve stem. *P*. Yoke casting.

pump through *K*, *L*, *M*, *N*, and *O*. The steam cylinders are bored separately in one casting, the water cylinders in another, and the two are connected by the yoke *P*. Both steam valves are in a common valve chamber above the steam end casting.

Power Pumps.—Reciprocating pumps are most often thought of as having the water cylinder and the steam cylinder on opposite ends of the same piston rod and therefore driven by direct steam pressure. This is not necessary, since it is obvious that any form of power may be utilized to drive the piston rod. The general name *power pumps* is given to all forms of reciprocating pumps in which the piston is actuated by some other force than

direct steam pressure. This usually involves connecting the piston to a crank shaft which is rotated, usually through reduction gears, by a belt from a line shaft, an electric motor, or any other convenient form of power.

If the piston is to be operated from a crank shaft, any number of water cylinders may be connected in parallel and their pistons located at different points on the shaft. This has the advantage

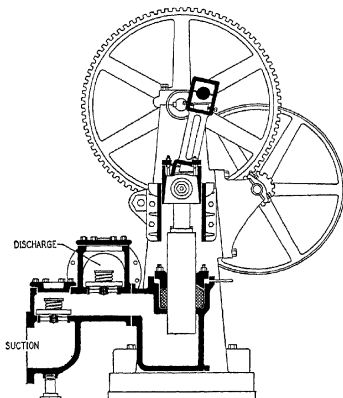



Fig. 54.—Vertical single-acting triplex power pump.

of making the discharge of the pump more uniform and free from pulsations, and it also means that if there are several cylinders in parallel each cylinder can be smaller, and therefore easier to build for high pressures and easier to keep packed.

A very common form of such pumps is the vertical single-acting triplex power pump (Fig. 54). Three cylinders with their suction and discharge valves as shown in this section are arranged side by side, and their pistons connected at points 120 deg. apart on the crank shaft. Such a pump as shown is *single acting*. In other words, it discharges only on the downstroke. Only two

valves are required, therefore, for each plunger. Vertical triplex pumps are not necessarily single acting but are very often so built.

Diaphragm Pumps.—The diaphragm pump is shown in Fig. 55. It is ordinarily thought of as a very cheap pump for the crudest and most temporary service. For the chemical engineer, however, it is the most satisfactory pump available for handling liquids with large amounts of solids in suspension under low

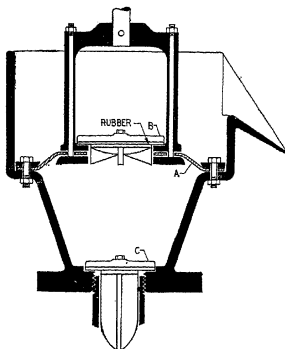



Fig. 55.—Diaphragm pump. *A*. Diaphragm. *B*. Discharge valve. *C*. Suction valve.

heads. It also has the advantage of permitting regulation of the rate of discharge. Instead of a piston or plunger, it employs a flexible diaphragm *A*, with a flap discharge valve *B* in the center. It also has a suction valve *C*. Since it has no moving parts except the flexible diaphragm and the valve, since its construction is rugged and simple, and repairs are easily made, it is suited for the most severe service. By operating the diaphragm from an adjustable eccentric, the stroke may be varied and the discharge controlled within accurate limits.

Reciprocating-pump Theory.—In the design of reciprocating steam pumps, two points are of major interest. The first is the

size of water cylinder necessary to give the desired discharge; the second is the size of steam cylinder necessary to generate the desired pressure. Details of mechanical design are not within the province of this book but belong to the field of the mechanical engineer.

The theoretical displacement of the piston during one stroke is obviously the area of the piston multiplied by the length of the stroke. A "stroke" is the distance traversed between two successive changes in direction. The number of strokes per minute should not be over 100 to 125 for pumps of less than 10-in. stroke, because higher speeds result in excessive wear on valves and valve springs. For pumps of 10-in. stroke or longer, a piston speed of 100 ft. per min. is customary. The theoretical displacement of a double-acting pump in cubic feet per minute will be the product of the piston speed in feet per minute and the area of the piston in square feet. The theoretical displacement of a single-acting pump will be half of this. Of course, this theoretical displacement is never reached. There are losses due to slippage past the piston from imperfect packing, due to leaky valves, and due to failure of the valves to close instantly when the piston reverses its direction of travel. All of these factors result in an actual discharge of 50 to 90 per cent of the theoretical displacement. This fraction is usually spoken of as the *volumetric* or *water-end efficiency*. The lower figure represents poorly packed pumps working at high speeds; the larger figure represents large, slow-speed pumps with the packing and valves maintained in first-class condition. Seventy-five per cent is perhaps a fair average for estimating purposes.

The steam pressure in pounds per square inch multiplied by the area of the steam piston in square inches represents the total force acting on the piston rod. If the pump were a perfect machine operating without friction, this would also be the total pressure developed on the water piston. This total pressure divided by the area of the water piston in square inches would give the theoretical maximum water-end pressure in pounds per square inch. Under these conditions, however, the total force on the steam piston would be exactly equal to the total force on the water piston, and therefore the pistons would be stationary. In order to do work on the liquid and to overcome pump friction, the total pressure on the steam piston must be greater

than that desired on the water piston. The ratio of the theoretical pressure on the steam piston to the pressure actually needed is known as the *steam-end* or *pressure efficiency* and varies from 60 to 80 per cent.

The rate of discharge from one end of the piston of a reciprocating pump is zero at the beginning of a stroke and rises to a maximum as the piston reaches full speed. The discharge from a single-cylinder double-acting pump would be such a curve as

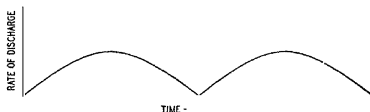



FIG. 56.—Discharge curve of single-cylinder, double-acting pump.

shown in Fig. 56. To remove pulsations in the line with their consequent losses, the duplex pump is often recommended, since its discharge curve should theoretically be the sum of two such curves as Fig. 56, half a stroke apart. It is obvious that the larger the number of cylinders the smoother will this curve be. For example, the theoretical discharge from a triplex pump is indicated in Fig. 57. In pumping at high pressures the dangers of shocks and pulsations are aggravated, and it is obvious that

FIG. 57.—Discharge curve of triplex single-acting pump.

for high pressures the triplex pump is more suitable than the single or duplex. As a matter of fact, pumps to deliver very high pressures often have five cylinders on a single crank shaft to make the discharge curve still more uniform.

It will be noted that the pumps of Figs. 49 and 51 have large air domes on the water end. These are to minimize the effect of such fluctuations as are indicated in Fig. 56. The air will be compressed when the water piston is accelerating and will expand

when it is decelerating. Air chambers are often installed on the suction side also, especially if the suction pipe is long.

ROTARY PUMPS

The most marked development in pumping practice in the past two decades has been the rapidity with which rotary pumps of all types have tended to supersede reciprocating pumps. The types that have been developed show fully as wide variations as the various types of reciprocating pumps.

Rotary pumps may be classified as either positive-displacement pumps or centrifugal pumps. These latter may be subdivided into volute and turbine pumps. Centrifugal pumps may also be classified as single stage or multistage; open impeller or closed impeller; and single suction or double suction. These last four bases of classification are coordinate (*i.e.*, any combination is theoretically possible).

Rotary Positive-displacement Pumps.—One type of rotary positive-displacement pump is the *gear pump* shown in Fig. 58. This pump consists essentially of two gears which mesh with each other and which run in close contact with the casing. Slugs of liquid are caught between the gear teeth and the casing, carried around next to the casing, and forced out through the discharge pipe. Such a pump handles viscous or heavy liquids, develops high pressures, and gives a discharge nearly free from fluctuations and independent of discharge pressure. The number of teeth varies from two or three on each wheel to a considerable number as shown in the figure. The two- or three-lobed pumps are usually known as *cycloidal pumps*, although in the strict sense of the word the gear pumps are also cycloidal. A cycloidal pump adapted for transporting gases is shown in Fig. 72, page 115.

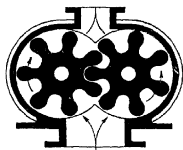



FIG. 58.—Gear pump.

There is a wide variety of rotary positive-pressure pumps other than the gear type. Most of them cannot be mentioned in this work, but to give an illustration of one type of construction employed, the *Kinney pump* is illustrated in Fig. 59. A rotary cam A is mounted eccentrically in the working chamber. This

slides inside the wearing ring *B* which, as the cam rotates, is brought in contact with the lining of the working chamber, the point of contact traveling around from inlet to outlet. Riding on top of the wearing ring is a sliding valve *C* which has an opening *D* at the right, near the bottom, connecting to a passage *E* up

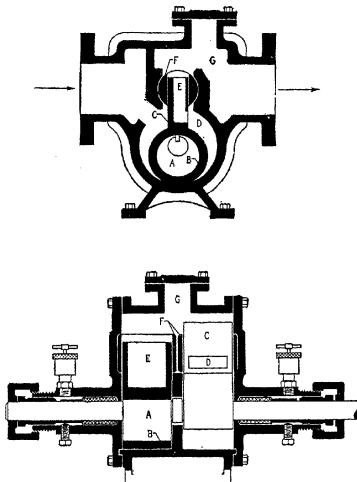



Fig. 59.—Rotary positive-pressure pump (Kinney). *A*. Cam. *B*. Wearing ring. *C*. Discharge valve. *D*, *E*. Discharge ports. *F*. Discharge valve slide pin. *G*. Discharge chamber.

through the valve. This valve slides vertically through a slide pin *F*. As the rotating cam moves to the right from the position shown, the valve permits the liquid on the right of the wearing ring to be forced upward through the valve into the discharge chamber *G*. The pump contains two rotating cams and two sliding valves at 180 deg. to each other, but both discharging into a common chamber.

Since the performance of all positive-displacement rotary pumps depends on maintaining a running fit between the rotating member and the casing (and, in the case of gear pumps, between the gears also), it is not possible to use these pumps on liquids that carry solids in suspension. Although pumps of the cycloidal or rotary positive-pressure type can be used on any clear liquid, they are most commonly used on viscous liquids and will handle quite stiff pastes. Semifluids, waxes, and similar materials can be handled with these pumps if the speeds are not too high. If the discharge valve of such a pump is closed, while the pump is running, the pressure developed will either stop the pump or cause breakage. The discharge rate of these pumps is directly proportional to their speed provided the contact between rotating parts and casing is good. These last two characteristics sharply differentiate these pumps from centrifugal pumps.

Centrifugal Pumps

The general class of centrifugal pumps is probably the most important type of pump at present. This class has been developed to the point where some form of centrifugal pump can be found for practically every service. Centrifugal pumps are of two distinct types, volute pumps and turbine pumps. The distinction between these will be more apparent as the pumps are described than it would be from formal definitions.

Volute Pumps.—The simplest form of centrifugal pump is the single-stage, single-suction, open-runner volute, shown in Fig. 60. The most important member of the centrifugal pump is the impeller or runner *A*. This consists essentially of a series of curved vanes extending from a hub *B*. The simplest form of impeller, the open single suction, is shown in Fig. 61*a*. This is mounted in the casing *C* of the pump of Fig. 60, in such a way that the two halves of the casing are as nearly as possible in contact with the surface of these vanes. Water entering at the suction connection is thrown outward by the rotation of the vanes. As the liquid leaves the vanes and enters the volute *D* of the casing, its velocity is decreased. According to Bernoulli's theorem, therefore, its pressure must be correspondingly increased, and this increase in pressure is a measure of the head developed by the pump.

The open-runner, single-suction pump is the cheapest of all centrifugal pumps, but it is also the least efficient. There are two main power losses in this type of pump. First, the water, which is thrown out radially by the vanes, must suddenly change its direction as it enters the volute. Any such sudden change of direction involves turbulence which consumes power in the form of friction. Second, these are cheap pumps and therefore not accurately finished. The fit between the impeller and the casing

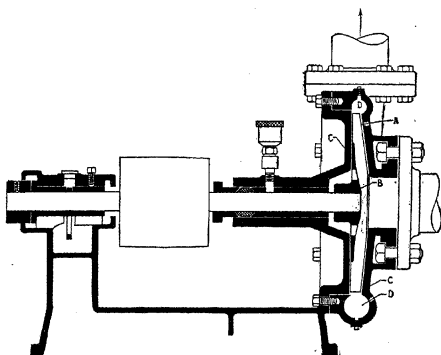



FIG. 60.—Single-stage, single-suction, open-impeller, volute pump. *A*. Impeller. *B*. Impeller hub. *C*. Casing. *D*. Discharge volute.

is usually poor, and therefore there is leakage from the discharge side back to the suction side.

To prevent this loss by leakage from the discharge side to the suction side, the closed impeller shown in Fig. 61*b* has been developed. In this case the vanes of the impeller are enclosed between two sheets of metal. Consequently, to prevent back leakage, it is necessary only that a close fit be maintained between the outer circumference of the impeller and the entrance to the volute, or between the hub of the impeller and the corresponding point on the casing. These clearances can be maintained by

inserting renewable wearing rings and the back leakage thus reduced to a minimum.

In such a pump as Fig. 60 the liquid exerts an unbalanced hydraulic pressure that tends to pull the impeller away from the shaft. This produces an end thrust on the bearings which is not easily taken up on the type of bearing usually furnished for such a cheap pump. To avoid this end thrust the double-suction impeller was developed. This consists essentially of two such runners as in Fig. 61*b*, placed back to back and united in one casting. Figure 62 shows a double-suction closed-impeller volute pump. From this diagram it will be seen how renewable

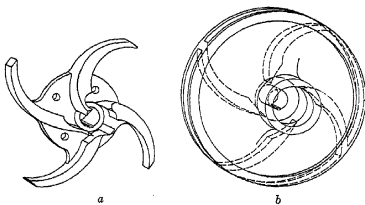



FIG. 61.—Pump impellers. *a*. Open impeller. *b*. Closed impeller.

wearing rings can be inserted at the proper points in order to eliminate back leakage. Such a pump will have much less end thrust on the bearings than the pump of Fig. 60. Most of the better volute pumps are of the double-suction, closed-impeller type. This type is more efficient than the pump of Fig. 60 but is correspondingly more expensive.

If the suction side of a centrifugal pump is under a pressure less than atmospheric, air may be drawn into the pump and greatly decrease or entirely stop the discharge. To prevent this the better centrifugal pumps have the glands sealed by diverting a small amount of liquid under the pressure of the discharge, through seal pipes to lantern rings in the packing.

Turbine Pumps.—In the discussion of volute pumps it was indicated that the principal energy loss was due to turbulence that occurs at the point where the liquid changes its path from radial flow (due to the action of the impeller) to tangential flow in

the discharge volute. Turbine pumps, as a class, are distinguished by the insertion of a *diffusion ring* whose function is to cause the liquid to make this change in direction smoothly and without shocks or eddies. Figure 63 shows a single-stage turbine pump. It will be noticed that in Fig. 63 the diffusion ring A contains passages B which change gradually in cross-section and in direction, so that the liquid issuing from the tip of the impeller C is caught in these passages and turned gradually and smoothly into the discharge volute D. The diffusion ring is stationary.

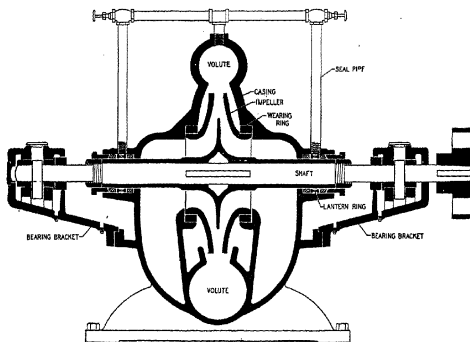



FIG. 62.—Single-stage, double-suction, closed-impeller, volute pump.

The design of the impeller and the general construction of the pump are exactly similar to the corresponding features in volute pumps.

The impellers of turbine pumps are usually single-suction impellers. As pointed out on page 102, this results in an end thrust on the shaft. In the pump of Fig. 63 this is partly overcome by holes E in the impeller, so that the hydraulic pressure behind the impeller is partly equalized with the pressure in front of the impeller. The rest of the end thrust is taken up in the marine-type thrust bearing F. Seal pipes G and lantern rings H are also shown.

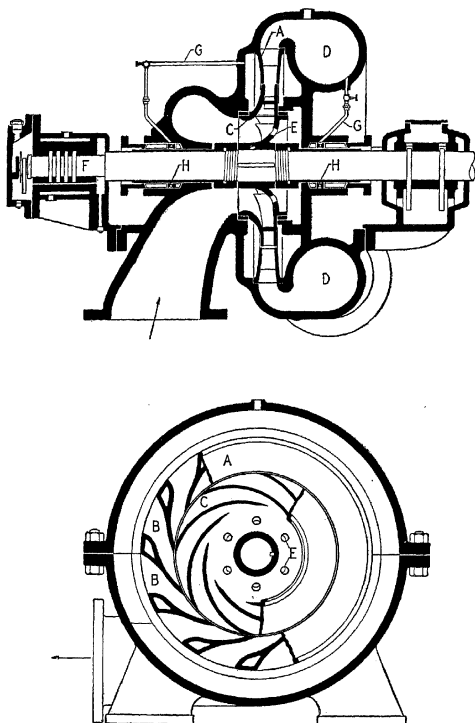



Fig. 63.—Single-stage, single-suction, turbine pump. *A*. Diffusion ring. *B*. Ports in diffusion ring. *C*. Impeller. *D*. Discharge volute. *E*. Balance ports. *F*. Thrust bearing. *G*. Seal pipes. *H*. Lanterns.

The maximum head that it is practical to generate with a single impeller is about 75 to 100 ft. If higher heads than this are desired, two or more impellers must be placed in series. The losses in a volute pump are serious enough so that if the liquid were discharged from one impeller to passages leading to another impeller, the overall efficiency of the pump would be quite low, and accordingly multistage volute pumps, although they are made, are uncommon. The turbine pump, however, provides means for taking the liquid from the tip of one impeller and delivering it with the minimum of losses to the inlet of the next impeller and therefore makes it possible to design a multistage

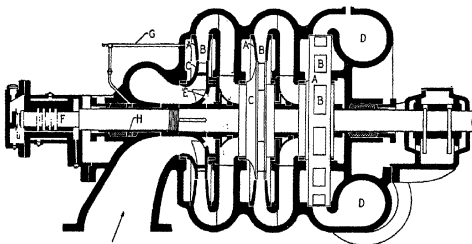



Fig. 64.—Three-stage, single-suction, turbine pump. *A*. Diffusion rings. *B*. Ports in diffusion rings. *C*. Impellers. *D*. Discharge volute. *E*. Balance ports. *F*. Thrust bearing. *G*. Seal pipes. *H*. Lanterns.

pump of reasonable efficiency. Figure 64 shows a multistage turbine pump in which heads up to 300 to 350 ft. may be generated. In Fig. 64 the first stage is shown as a section through both impeller and diffusion ring; the second stage shows the impeller in elevation and diffusing ring in section; and the third stage shows the diffusion ring in elevation. The parts are lettered to correspond to the description of Fig. 63. These pumps are usually relatively expensive and are practically impossible to build in special metals. Consequently, when a chemical engineer wishes to handle corrosive liquids at relatively high pressures, he is usually forced to turn to some form of reciprocating pump, and the turbine pump is reserved for clear, non-viscous, and non-corrosive liquids.

Self-priming Pumps.—One of the principal disadvantages of the centrifugal pump is that, since it depends on the inertia of a liquid to develop a pressure, it cannot pump if air enters the impeller. This is called *air binding.* Ordinary centrifugal pumps must be provided with some means for filling the suction line and the pump casing with liquid before the pump will discharge. This may be done by providing a check valve in the suction line, so that the casing and suction line will not drain

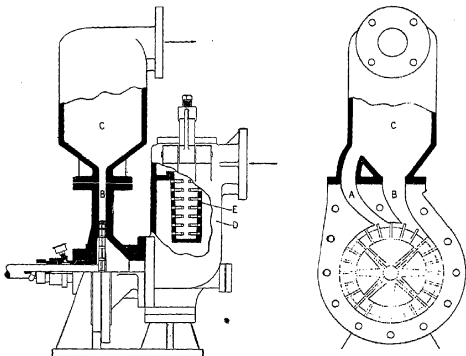



FIG. 65.—LaBour self-priming pump. *A*. Return port. *B*. Discharge port. *C*. Separating chamber. *D*. Suction chamber. *E*. Strainer.

when the pump is shut down. Another method is to provide a small priming pump, operated either by hand or by a steam jet, to remove air from the casing. Wherever possible, centrifugal pumps should be so located that the pump suction is under a positive head, and thus the necessity for priming is eliminated.

There are certain pumps especially designed for the prevention of air binding. One such pump is the *LaBour pump.* This is shown in Fig. 65. The impeller is of a somewhat unusual shape and is not surrounded by a volute. Liquid is caught between the blades of the impeller and escapes through the discharge ports *A* and *B*. The discharge connection *A* is the return from

the separating tank C. When there is air in the pump, it passes into the separating tank and out through the separating-tank discharge connection. Any liquid that follows it drains back through the return into the pump casing. When the casing is entirely free from air, the pressure in the two discharge ports is substantially equal so that there is little circulation back to the pump from the separating tank. The container D on the suction side has sufficient capacity so that when the pump is stopped and the liquid in the separating tank drains back into the pump casing, it will not be lost through the suction line. This particular container is provided with a suction strainer E.

FIG. 66.—Forces in centrifugal-pump impeller.

Centrifugal-pump Performance.—The action of a liquid in a centrifugal pump is more complex than in a reciprocating pump. In Fig. 66, let B be the section of one vane of the impeller which is rotating in the direction shown by the arrow. A particle of water leaving the tip of the vane is traveling at a velocity U tangentially, due to the speed of the impeller itself; and at a velocity W parallel to the tip of the impeller due to its slipping along the surface of the vane from centrifugal force. The resultant of these two velocities is V. whose direction and magnitude are obviously determined by the relative values of U and W.

In the volute of the pump, the velocity head corresponding to V is converted more or less completely to pressure head by the change in cross-section in accordance with Bernoulli's theorem. The pressure developed by a centrifugal pump will vary with the angle of the vane, with the velocity of the tip of the impeller, with the friction losses within the impeller, and with the completeness of conversion of velocity head into pressure head in the pump casing. The exact interrelationship of these variables cannot be predicted mathematically, and consequently the form

of centrifugal-pump impellers and their performance are developed partly by calculation and partly by making a variety of impellers and subjecting them to actual test.

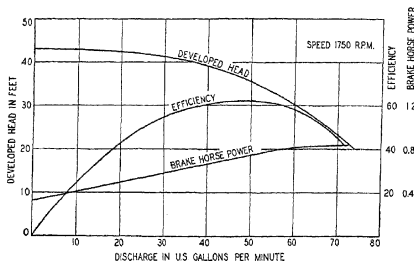



FIG. 67.—Characteristic curves of a low-head centrifugal pump.

The performance of any particular centrifugal pump is best expressed by means of curves. The manufacturer usually supplies with every pump a copy of its *characteristic curves.*

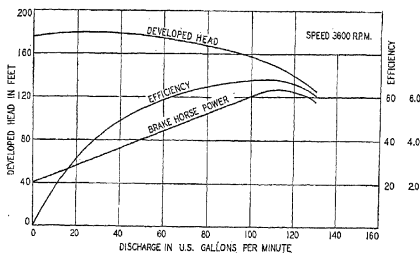



FIG. 68.—Characteristic curves of a high-head centrifugal pump.

Two such sets of curves for constant speed are shown in Figs. 67 and 68. Figure 67 represents the performance of a low-speed, low-head pump, while Fig. 68 is for a high-speed, high-head

pump. The efforts of the designer are usually directed toward making both head curves and efficiency curves as flat as possible, or at least flat over as wide a range as possible. Centrifugal pumps are usually rated on the basis of head and capacity at the point of maximum efficiency. The size of a centrifugal pump is usually specified as the diameter of the discharge connection.

Increasing the speed of a centrifugal pump usually increases the head for any given rate of discharge and *vice versa*. Speed has less effect on the efficiency. The commonest method of driving a centrifugal pump is by a direct-connected motor, and therefore in practice most centrifugal pumps are constant-speed machines.

The head developed by a centrifugal pump is determined largely by the angle of the vanes and the speed of the tip of the impeller. The volume of the discharge, however, is determined by the cross-section of the passages. Consequently, centrifugal pumps for high heads and small volumes have impellers of large diameter but with narrow slots; while pumps for low heads and large discharges have impellers of small diameter and wide slots.

Reciprocating Pumps vs. Centrifugal Pumps.—The advantages of reciprocating pumps in general over centrifugal pumps may be summarized as follows:

1. They can be designed for higher heads than centrifugal pumps.

2. Their first cost is less than that of the most efficient turbine pumps, capacity for capacity.

3. They are not subject to air binding, and the suction connection may be under a pressure less than atmospheric without necessitating special devices for priming.

4. They are more flexible in their operation than centrifugal pumps.

5. They operate at nearly constant efficiency over a wide range of discharge rates.

The advantages of centrifugal pumps over reciprocating pumps are:

1. In moderate sizes, the efficiency of a good centrifugal pump is apt to be equal to, or somewhat better than, the efficiency of the average reciprocating pump.

2. Centrifugal pumps deliver liquid at uniform pressure without shocks or pulsations.

3. They can be directly connected to motor drive without the use of gears or belts.
4. Valves in the discharge line may be completely closed without injuring them.
5. They will handle liquids with large amounts of solids in suspension.
6. They can be built in a wide variety of corrosion-resisting materials.

The mere statement of the comparisons given above does not tell the whole story. If the reciprocating pump is a steam-driven pump, the value of its exhaust must be considered. Where steam is to be used as so-called *process steam* (that is, low-pressure steam used for heating, evaporating, and similar operations other than power generation), exhaust steam is of practically the same value as steam direct from the boilers. From this point of view a steam pump may be considered simply a reducing valve, and its efficiency is of no consequence so long as there is a use for its exhaust. Theoretically a centrifugal pump could be driven by a small steam turbine under the same circumstances, but steam turbines cannot be built in very small sizes and in any case are much more expensive than a reciprocating steam cylinder doing the same work. Consequently, if a plant has use for exhaust steam, the steam pump is not always an inefficient pump. In some cases where large numbers of pumps are used in widely separated places in the plant, it may be more economical to generate power at a central point by a non-condensing prime mover and send the exhaust steam directly to the point where it is to be used, distributing power to the individual pumps in the form of current. The other alternative, distributing high-pressure steam to widely separated steam pumps and collecting their exhaust into a common header, is obviously less practical for this case. Many intermediate cases arise and no general rules can be laid down. It should be noted, however, that the present tendency is strongly in favor of motor-driven centrifugal pumps wherever they can be adapted to the service in hand, and reciprocating steam pumps are found much less frequently than 15 years ago.

TRANSPORTATION OF GASES

When gases are to be moved, the device used is commonly called a *blower* or a *compressor*, although in principle and design

it may be exactly comparable to some type of pump. The classification of blowers, therefore, is nearly parallel to the classification of pumps.

Ejectors.—Gases may be moved at low pressures, by devices actuated by a steam or water jet, exactly like the types described on page 86 and illustrated in Fig. 48. The efficiency of the simplest forms of these devices is low and the pressures that they can overcome are small. They are usually used for producing a draft, for exhausting gases from an apparatus, and similar cases. They can be made to discharge only against atmospheric pressure or pressures a few inches above atmospheric. Their principal advantage is that they are simple and cheap and need no attention. The highly specialized and efficient types that are used as vacuum pumps are described in Chap. V.

Reciprocating Compressors.—Where gases are to be delivered against pressures of more than a few pounds per square inch, the device usually employed is a reciprocating compressor, which is very similar in design to a piston pump. The valves are made lighter; care is taken to keep the clearances small; and all parts are more carefully fitted; but otherwise their operation is like that of a piston pump.

When a liquid is moved by a pump its volume does not change appreciably with the increased pressure and therefore the term $\int PdV$ in Bernoulli's equation (page 26) is negligible. The heat equivalent of the friction losses is small and the mass of the liquid is great, so that ordinarily no appreciable temperature rise results. When a gas is compressed, however, its volume is decreased and therefore work is done upon it. This work, in addition to the frictional losses of the compressor, must appear as heat. The mass of the gas is relatively small, and consequently compressing the gas results in an appreciable rise in temperature. For the most efficient operation of a compressor this heat should be removed and the gas discharged from the compressor as nearly as possible at the temperature at which it enters. For this reason the working cylinders of most air compressors are jacketed so that they may be water cooled.

For higher pressures it is usual to operate compressors in two or more stages. A piston of large diameter is used for the first compression, and the discharge from this cylinder passes to the second cylinder of smaller diameter for further compression. In such cases, in addition to jacketing the cylinders, the gas is

usually cooled in tubular intercoolers located between the stages. The various stages may consist of cylinders in line and having their pistons on a single piston rod, or each stage may be a separate machine with its own piston rod and its own drive.

Figure 69 shows a two-stage air compressor with an intercooler between the cylinders. This is a section through the high-pressure stage of a compressor of the second type mentioned in the preceding paragraph. A cross-head *A*, a steam piston *B*, and an air piston *C* are all mounted on a single piston rod *D*. The valve gear for the steam cylinder is outside the plane of this section and does not show. The heads and walls of the air cylinder are provided with water jacket *E*. The low-pressure air cylinder is behind the one shown in section and is slightly larger, so that part of its outline appears at *F*. It discharges into the intercooler *G*, in which the air is cooled by water inside the pipes *H*. The partly compressed and cooled air passes through *J* to the intake valves. These consist of a light steel ring *K* held against a seat by springs *L*. The discharge valves *M* are similarly constructed. The discharge connection is at *N*.

Fans.—The commonest method for moving gases under moderate pressures is by means of some type of fan. These are effective for pressures from 2 or 3 in. of water up to 0.5 lb. per sq. in. They may be classified into three types: the propeller type, the plate fan, and the multiblade type.

The *propeller type* is represented by the familiar electric fan and is of no great importance for moving gases in plant practice. A typical *plate fan* is shown in Fig. 70. This consists, as its name implies, of plate-steel blades on radial arms inside a casing. These fans are satisfactory for pressures from 0 to 5 in. of water, have from 8 to 12 blades, and can be built for practically any capacity. Another variation of the steel-plate fan has blades curved like the vanes of centrifugal-pump impellers and can be used for pressures up to 27 in. of water. The *multiblade fans* are illustrated in Fig. 71 and are useful for pressures of from 0 to 5 in. of water. It is claimed that they have much higher efficiencies than the steel-plate fan. These fans will deliver much larger volumes for a given size of drum than steel-plate fans.

Cycloidal Blowers.—Any pump of the cycloidal or gear type can be used as a blower. When so used they generally have only two or three lobes on the rotating parts. Such a blower is shown

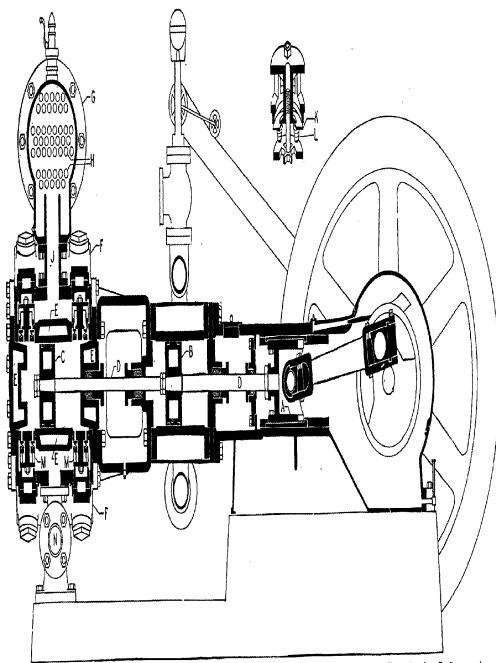



FIG. 69.—Two-stage, direct steam-driven, air compressor (section through high-pressure stage). *A*. Cross-head. *B*. Steam piston. *C*. Air piston. *D*. Piston rod. *E*. Air-cylinder water jackets. *F*. Elevation of low-pressure air cylinder. *G*. Intercooler casing. *H*. Cooling water tubes. *J*. Inlet to high-pressure stage. *K*. Valve disc. L. Valve springs. *M*. Air discharge valves. *N*. Air discharge connection

in Fig. 72. These blowers are used for pressures of from 0.5 to 10 or 12 lb. per sq. in., but their maximum efficiency is obtained below 5 lb. Their principal advantages are simplicity and large capacity. Such blowers are often used for blast furnaces and

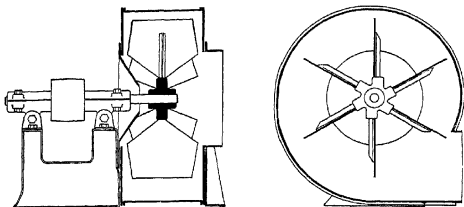



FIG. 70.—Steel-plate fan.

for other similar services where very large volumes must be delivered against pressures too high for a fan.

One type of compressor that does not fall into any of the above classifications is the *Nash hytor* (Fig. 73). A central cylindrical rotor, carrying vanes around its circumference, is

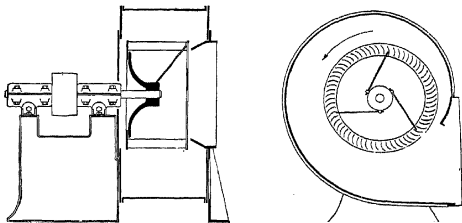



FIG. 71.—Multiblade fan.

placed inside a casing which is approximately elliptical in shape. A sufficient amount of liquid is placed inside the casing to seal the impeller at its points of least clearance from the casing, which correspond to the minor axis of the ellipse.

The rotation of the impeller causes the liquid also to rotate, but centrifugal force keeps it against the casing. The liquid there-

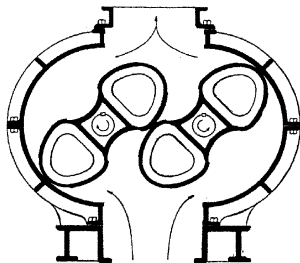



FIG. 72.—Cycloidal blower.

fore alternately advances toward, and recedes from, the center of the impeller, thus acting essentially as a series of liquid pistons. As these recede they suck in air from the inlet ports *A*, and as

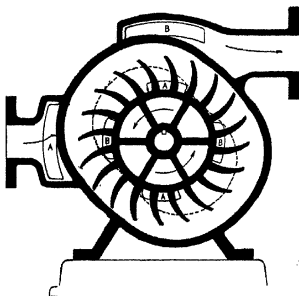



FIG. 73.—Nash hytor. *A*. Inlet ports. *B*. Discharge ports.

they advance they compress it into the outlet ports *B*. This type reaches its highest efficiency at discharge pressures from

8 to 12 lb. per sq. in. but may be operated at pressures up to 20 lb. Since the pump contains no moving parts in mechanical contact with each other, the rotor may be made of any noncorrodible material; and the actuating liquid may be any liquid that is inert to the gas being handled. For example, this pump, using concentrated sulfuric acid, is used to compress chlorine gas.

Centrifugal Blowers.—The principle of the turbine pump may be adapted to the transportation of gases. Because of the relatively low density of a gas, the centrifugal force that can be developed by the rotation of the impeller is small, and therefore the pressures that can be produced are low. Without exceedingly careful design, the efficiency is also low. The construction is quite like a turbine pump, except that the rotating parts are made as light as possible. These blowers run at high speeds (4,000 or 6,000 r.p.m.) and cannot be made to develop pressures over 8 to 10 lb. per sq. in. even when built multistage. They operate best at pressures under 5 lb. They cannot be built in small units. Within the proper pressure range they are the most efficient type of blower yet developed, but they are very expensive machines.

Nomenclature of Chapter III

C = constant
H = head in feet
V = volume in cubic feet
sub s = running (for air-lift)
sub t = total

Problems

1. Salt brine at 60°F. (sp. gr. 1.21, viscosity 1.70 centipoises) is to be pumped at the rate of 160 gal. per min. through a 3-in. line whose equivalent length (actual length of straight pipe plus equivalent length of fittings) is 350 ft. There is a vertical lift of 80 ft. between levels in feed and discharge tanks. Steam at 100 lb. gage is available. One pump company lists the following simplex pumps in the general class desired, all with a 3-in. discharge:

5 × 6 × 8	6 × 7 × 8	7 × 8 × 8
5 × 7 × 8	6 × 8 × 8	8 × 6½ × 8
5 × 8 × 8	7 × 6 × 8	8 × 7 × 8
6 × 6 × 8	7 × 7 × 8	8 × 8 × 8

(In describing a simplex or duplex pump by such expressions as the above, the first figure is the diameter of the steam cylinder, the second is the

diameter of the water cylinder, and the third is the stroke, all in inches.) If a water-end efficiency of 75 per cent and a steam-end efficiency of 60 per cent be assumed, which pump should be specified?

2. A certain centrifugal pump is rated as delivering 1,160 gal. of water per min. against a head of 110 ft. At this performance it uses 40.50 hp. What is its efficiency?

3. A reciprocating double-acting pump has a cylinder 6 in. in diameter and an average stroke of 7.95 in. The piston rod is ⅞ in. in diameter. This pump is run at the rate of 62.5 strokes per min. and discharges into a calibrated tank 4 ft. inside diameter. In a test run of 2.5 min. the water level in this tank is raised 4 ft. 3 in. What is the water-end efficiency of the pump?

4. Water is to be pumped at the rate of 100 gal. per min. from a well by means of an air-jet lift. When pumping at this rate the water level stands 50 ft. below the discharge opening. The air jet is 125 ft. below the discharge. How much free air per minute is needed?

5. What is the theoretical minimum consumption of free air to lift 100 gal. per min. of water through a height of 50 ft. (neglecting friction losses) by means of an acid egg?

CHAPTER IV

FLOW OF HEAT

Practically all the operations that are carried out by the chemical engineer involve the production or absorption of energy in the form of heat. The laws governing the transfer of heat, and the types of apparatus that have for their main object the control of heat flow, are therefore of great importance. This chapter will consider, first, the basic mechanisms of heat flow; second, the fundamental quantitative methods of calculation with especial reference to chemical engineering; and, third, the application of these principles to the design of heating and cooling equipment.

Classification of Heat-flow Processes.—Heat may flow by one or more of three basic mechanisms:

Conduction.—When heat flows through a body by the transference of the momentum of individual molecules without mixing, it is said to flow by conduction. For example, the flow of heat through the brick wall of a furnace or the metal shell of a boiler takes place by conduction as far as the solid wall or shell is concerned.

Convection.—When heat flows by actual mixing or physical turbulence, the mechanism is known as convection. Convection is restricted to the flow of heat in fluids. It is practically impossible for heat to flow through fluids by pure conduction without some convection because of the eddies set up by the changes of density with temperature. For that reason the terms "conduction" and "convection" are often used together, although in many cases the phenomena are preponderantly convection. For example, the heating of a room by means of a steam radiator and the heating of water by a hot surface are examples of heat transfer mainly by convection.

Radiation.—Radiation is a term given to the transfer of energy through space by means of electromagnetic waves. If radiation is passing through empty space it is not transformed to heat or

any other form of energy, nor is it diverted from its path. If, however, matter appears in its path, the radiation will be transmitted, reflected, or absorbed. It is only the absorbed energy that appears as heat, and this transformation is quantitative. For example, fused quartz transmits practically all of the radiation that strikes it; a polished opaque surface or mirror will reflect most of the radiation impinging on it; a black or matte surface will absorb most of the radiation received by it and will transform such absorbed energy quantitatively into heat.

CONDUCTION

The mechanism of conduction is most easily understood by the study of conduction through solids, because in this case convection is not present. The basic law of heat transfer by conduction can be written in the form of the rate Equation (1):

$$\text{Rate} = \frac{\text{driving force}}{\text{resistance}} \tag{42}$$

The driving force is the temperature drop across the solid, since it is apparent that heat can flow only when there is an inequality of temperature.

Fourier's Law.—The resistance term in Equation (42) is defined by means of Fourier's law. Consider a wall of thickness L and an area A. Let the temperature drop across the wall be Δt. Fourier's law states that the rate of heat flow through the wall is proportional to the temperature drop, to the area, and inversely proportional to the thickness of the wall. If Δt does not vary with time, the rate of heat flow is constant, and the process is one of *steady state*. If Q is the heat flow in the time θ, then Q/θ is the rate of heat flow, and Fourier's law can be written in mathematical form as Equation (43):

$$\frac{Q}{\theta} = \frac{kA\Delta t}{L} = q \tag{43}$$

where k is a proportionality constant and q replaces Q/θ. By comparing Equations (43) and (42), remembering that Δt is the driving force, it is seen that the resistance is L/kA.

Thermal Conductivity.—The constant k in Equation (43) is known as the *thermal conductivity* of the solid of which the wall

is made. If Q is measured in B.t.u., θ in hours, A in square feet, t in degrees Fahrenheit, and L in feet, k is defined as

$$k = \frac{(\text{B.t.u.})(\text{ft.})}{(\text{hr.})(\text{sq. ft.})(°\text{F.})} \tag{44}$$[1]

It is often expressed as "B.t.u. per hour per square foot per degree Fahrenheit per foot." It will be noted that on comparing this abbreviation with the definition of k given in Equation (44) it would be more strictly correct to call the thermal conductivity "B.t.u.-feet per hour per square foot per degree Fahrenheit." It must also be be thoroughly understood that the area term A is perpendicular to the direction of heat flow, and the length L is measured parallel to the heat flow.

The numerical value of the coefficient of thermal conductivity depends upon the material of which the body is made and upon its temperature. Data showing the values of this quantity for various materials are given in Appendix VI.

The thermal conductivities of liquids and of gases are very small in comparison with those of most solids. For example, at 212°F. the thermal conductivity of silver is 240 B.t.u.-ft. per hr. per sq. ft. per °F., that of building brick is about 0.8, that of water is about 0.35, and of air 0.017. In general, the variation of thermal conductivity with temperature is linear; that is,

$$k = a + bt \tag{45}$$

where a and b are constants and t is the temperature. The temperature coefficients of thermal conductivity are known with accuracy only for water among the liquids, and for silver, and possibly lead or copper, among the solids. This is because thermal conductivity is very sensitive to changes in chemical composition; and these three metals are the only ones obtainable in sufficiently pure form to permit check determinations on different samples.

Compound Resistances in Series.—Consider a flat wall constructed of a series of layers as in Fig. 74. Let the thicknesses of the layers be represented by L_1, L_2, and L_3, and the conduc-

[1] If this quantity is expressed in metric units it will be expressed as

$$k_m = \frac{(\text{Cal.})(\text{m.})}{(\text{hr.})(\text{sq.m.})(°\text{C.})}$$

tivities of the materials of which the layers are made by k_1, k_2, and k_3, respectively. Furthermore, let the area of the compound wall, at right angles to the plane of the illustration, be A. Let Δt_1 be the temperature drop across the first layer, Δt_2 that across the second, and Δt_3 that across the third. Let Δt be the temperature drop over all three layers and, therefore,

$$\Delta t = \Delta t_1 + \Delta t_2 + \Delta t_3 \tag{46}$$

It is desired, first, to derive a formula giving the rate of heat transfer through this series of resistances; and, second, to determine what expression must be used for the overall resistance if Δt is used as the overall driving force.

Equation (43) can be written for each of the layers in the following form:

$$\begin{aligned} \Delta t_1 &= q_1 \times \frac{L_1}{k_1A} \\ \Delta t_2 &= q_2 \times \frac{L_2}{k_2A} \\ \Delta t_3 &= q_3 \times \frac{L_3}{k_3A} \end{aligned} \tag{47}$$

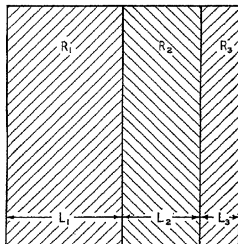



FIG. 74.—Thermal resistances in series.

If Equations (47) are added together, Equation (48) results:

$$\Delta t_1 + \Delta t_2 + \Delta t_3 = \frac{q_1L_1}{Ak_1} + \frac{q_2L_2}{Ak_2} + \frac{q_3L_3}{Ak_3} = \Delta t \tag{48}$$

Since all the heat that passes through the first resistance must pass through the second, and in turn pass through the third, q_1, q_2, and q_3 must be equal and can all be represented by q. Using this fact and solving for q, Equation (49) results:

$$\frac{\Delta t}{\frac{L_1}{k_1A} + \frac{\quad}{k_2A} + \frac{\quad}{k_3A}} \qquad \frac{\Delta t}{R_1 + R_2 + R_3} \tag{49}$$

where R_1, R_2, and R_3 are the resistances as defined on page 119. It is not necessary to memorize Equation (49), because Equation (49) is written in the form

$$\text{Rate} = \frac{\text{driving force}}{\text{resistance}}$$

and the overall resistance is equal to the sum of the individual resistances, just as is the case in the flow of electric current through a series of resistances.

Example 8.—A flat furnace wall is constructed of a 4.5-in. layer of sil-o-cel brick, with a thermal conductivity of 0.08, backed by a 9-in. layer of common brick, of conductivity 0.8. The temperature of the inner face of the wall is 1,400°F., and that of the outer face is 170°F. Calculate the heat loss through this wall in B.t.u. per square foot per hour.

Solution.—Thermal resistance has been defined as L/kA. Considering 1 sq. ft. of wall ($A = 1$), the thermal resistances are:

$$\text{For sil-o-cel brick, } R_1 = \frac{4.5/12}{0.08 \times 1} = 4.687$$

$$\text{For common brick, } R_2 = \frac{9/12}{0.8 \times 1} = 0.938$$

Since the total resistance is the sum of the individual resistances,

$$R = 4.687 + 0.938 = 5.625$$

Rate of heat flow is $\frac{\text{temperature drop}}{\text{resistance}}$. Hence,

$$\frac{q}{A} = \frac{1,400 - 170}{5.625} = 219 \text{ B.t.u. per hr. per sq. ft.}$$

It is often useful to recall the analogies between the flow of heat and the flow of electricity. The flow of heat is covered by the expression

$$\text{Rate} = \frac{\text{temperature drop}}{\text{resistance}}$$

In the flow of electricity the potential factor is the electromotive force and the rate of flow is coulombs per second, or amperes. The rate equation for electrical flow is

$$\text{Amperes} = \frac{\text{volts}}{\text{ohms}}$$

By comparing this equation with Fourier's law it is seen that rate of flow of heat in B.t.u. per hour is analogous to amperes, temperature drop to voltage, and thermal resistance to electrical

resistance. The various units for the electrical circuit have been given names such as amperes, volts, and ohms; while the corresponding units for heat flow have never been given names.

The rate of flow of heat through several resistances in series has been shown to be exactly analogous to the current flowing through several electrical resistances in series. In an electric circuit the potential drop over any one of several resistances is to the total potential drop in the circuit as the individual resistances are to the total resistance. In the same way the potential drops in a thermal circuit, which are the temperature differences,

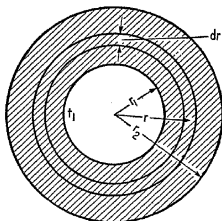



Fig. 75.—Flow of heat through thick-walled cylinder.

are to the total temperature drop as the individual thermal resistances are to the total thermal resistance. This may be expressed mathematically as

$$\Delta t:\Delta t_1:\Delta t_2:\Delta t_3::R:R_1:R_2:R_3 \qquad (50)$$

Example 9.—In example 8, what is the temperature of the interface between the refractory brick and the common brick?

Solution.—The temperature drop over any one of a series of thermal resistances is to the total drop as the individual resistance is to the total resistance, or

$$\Delta t_1: \quad \Delta t \quad :: R_1 : R$$
$$\Delta t_1:(1{,}400 - 170)::4.687:5.625$$

whence

$$\Delta t_1 = 1025°\text{F.}$$

and the temperature of the interface is $1400 - 1025 = 375°$F.

Heat Flow through a Cylinder.—Consider the hollow cylinder represented by Fig. 75. The inside radius of the cylinder is r_1, the outside radius is r_2,

and the length of the cylinder is N. The thermal conductivity of the material of which the cylinder is made is k. The temperature of the inside surface is t_1, and that of the outside is t_2. It will be assumed that t_2 is larger than t_1, and hence that heat is flowing from the outside to the inside. It is desired to calculate the rate of heat flow for this case.

Consider a very thin cylinder, concentric with the main cylinder, of radius r, where r is between r_2 and r_1. The thickness of the wall of this cylinder is dr; and if dr is small enough with respect to r so that the lines of heat flow may be considered parallel, Equation (43) can be applied and written in the form

$$q = k\frac{dt}{dr}(2\pi rN) \tag{51}$$

since the area perpendicular to the heat flow is equal to $2\pi rN$, and the L of Equation (43) is equal to dr. In order to integrate Equation (51), it is necessary only to separate the variables t and r as follows:

$$\frac{dr}{r} = \frac{2\pi Nk}{q}dt \tag{52}$$

Since all quantities except dr and dt are constant. Equation (52) can be integrated as follows:

(53)

$$\ln r_2 - \ln r_1 = \frac{2\pi Nk}{q}(t_2 - t_1) \tag{54}$$

$$q = \frac{k(2\pi N)(t_2 - t_1)}{\ln\frac{r_2}{r_1}} \tag{55}$$

Equation (55) can be used to calculate the flow of heat through a thick-walled cylinder.

Equation (55) can be put in a more convenient form. It is desirable to express the rate of flow of heat in the form of Equation (56):

$$q = \frac{k\bar{A}(t_2 - t_1)}{L} \tag{56}$$

which is of the same general form as the equation for heat flow through a flat wall [Equation (43)] with the exception of $\bar{A}$, which must be so chosen that the equation is correct. The term $\bar{A}$ can be determined by equating the right-hand sides of Equations (55) and (56) and solving for $\bar{A}$:

$$\bar{A} = \frac{2\pi N(r_2 - r_1)}{\ln\frac{r_2}{r_1}} \tag{57}$$

It will be noted from Equation (57) that $\bar{A}$ is the area of a cylinder of length N and radius r_m, where

$$r_m = \frac{r_2 - r_1}{\ln\frac{r_2}{r_1}} = \frac{r_2 - r_1}{2.303 \log\frac{r_2}{r_1}} \tag{58}$$

The form of the right-hand side of Equation (58) is important enough to repay memorizing. It is known as the *logarithmic mean,* and in the particular case of Equation (58), r_m is called *logarithmic mean radius.* It is the radius which, when applied to the integrated equation for a flat wall, will give the correct rate of heat flow through a thick-walled cylinder.

The logarithmic mean is less convenient than the arithmetic mean, and the arithmetic mean is sufficiently accurate if the tube is thin walled. The arithmetic mean gives results within 10 per cent of the logarithmic mean if the ratio r_2/r_1 is less than 3.2, and within 1 per cent if the ratio r_2/r_1 is less than 1.5. If either the inner or outer radius is used instead of the logarithmic mean, the results will be accurate to within 10 per cent if r_2/r_1 is less than 1.24, and to within 1 per cent if r_2/r_1 is less than 1.02. Consequently, for most cases in practice, the arithmetic mean is sufficiently accurate, but the use of either the inside or outside radius is usually not sufficiently accurate.

Example 10.—A tube, 2.5 in. outside diameter, is lagged with a 2-in. layer of asbestos (conductivity 0.12), which is followed with a 1.5-in. layer of cork (conductivity 0.03). If the temperature of the outer surface of the pipe is 290°F., and the temperature of the outer surface of the cork is 90°F., calculate the heat loss in B.t.u. per foot of pipe.

Solution.—These layers are too thick to use the arithmetic mean radius, and the log mean formula should be used.

For asbestos,

$$r_m = \frac{3.25 - 1.25}{2.303 \log \frac{3.25}{1.25}} = 2.09 \text{ in}$$

For cork,

$$r_m = \frac{4.75 - 3.25}{2.303 \log \frac{4.75}{3.25}} = 3.95 \text{ in.}$$

$$R_1 = \frac{L_1}{k_1 A_1} = \frac{2/12}{0.12 \times 2\pi \times \frac{2.09}{12}} = 1.268$$

$$R_2 = \frac{L_2}{k_2 A_2} = \frac{1.5/12}{0.03 \times 2\pi \times \frac{3.95}{12}} = 2.015$$

$$q = \frac{290 - 90}{1.268 + 2.015} = \frac{200}{3.283} = 60.9 \text{ B.t.u. per hr. per ft. of pipe}$$

Conduction through Fluids.—This case rarely occurs in practice except when heat flows through thin films. In these cases,

however, the thickness of the film is not known, and therefore the equations derived above cannot be applied. This difficulty is circumvented by the use of *film coefficients* which will be discussed later. Any body of fluid of appreciable size through which heat is flowing will develop convection currents of such a magnitude that heat is carried both by convection and by conduction. The difficulty of so arranging a body of fluid that heat shall flow through it by conduction only is one of the reasons for the serious lack of accurate data on the thermal conductivities of fluids.

CONVECTION

It has been pointed out in Chap. II that when fluid flow is of such a nature that the Reynolds number exceeds a certain value, the character of the flow changes from viscous to turbulent. It has also been shown that even in turbulent flow there is at the boundary a residual film that persists from viscous flow. The turbulence may be caused by a stirrer or agitator of any type, or by pumping the fluid through a pipe (*forced convection*), or by the *natural convection* currents set up when a body of fluid, otherwise stationary, is heated. If heat is passing through the retaining wall to the fluid, the comparatively stagnant film is of great importance in determining the rate of heat transfer. This follows from the facts that all the heat reaching the bulk of the fluid must pass through this film by conduction and that the thermal conductivities of fluids are low; so that although the film is thin, the resistance offered by it to the flow of heat is large. On the other hand, beyond the film the turbulence brings about a rapid equalization of temperature. In other words, an important resistance to the flow of the heat from the wall to the bulk of the fluid is that offered by the boundary film.

Temperature Gradients in Forced Convection.—The temperature distribution across a column of fluid, in forced convection, that is at the same time being heated or cooled, is intimately connected with the velocity distribution across the same column of fluid. Figure 76 represents the temperature gradients for the case where heat is flowing from a hot fluid through a metal wall into a cold fluid. The dotted lines F_1F_1 and F_2F_2 on each side of the solid wall represent the boundaries of the films in

viscous flow; and all parts of the fluids to the right of F_1F_1 and to the left of F_2F_2 are in turbulent flow. The temperature gradient from the bulk of the hot fluid to the metal wall is represented by the curved line $t_at_bt_c$. The temperature t_a is the maximum temperature in the hot fluid. The temperature t_b is the temperature at the boundary between the turbulent and viscous regimes, and the temperature t_c is the temperature at the actual interface between fluid and solid. The significance of the line $t_dt_et_f$ is similar

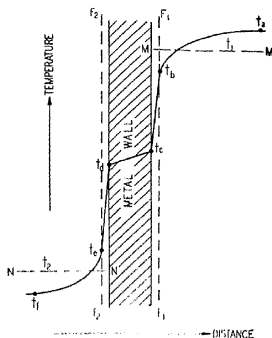



FIG. 76. Temperature gradients in forced convection.

In heat transfer calculations it is customary to use as the temperature of the fluid neither the maximum temperature, t_a, nor the temperature at the outside surface of the film, t_b, but rather the average temperature of the fluid, such as would be found by completely mixing the fluid and taking its temperature. This average temperature, t_1 will be somewhat less than t_a, and is shown as a straight line, marked *MM*. The same remarks apply to the cold fluid, whose average temperature, t_2, is that represented by the horizontal line *NN*. If the fluid is not too viscous and the pipe is not too large, these average temperatures are the ones that will be given when a thermometer is inserted

into the pipe so that its bulb is near the middle of the stream of fluid. To determine the actual course of the curve $t_a t_b t_c$, very careful measurements with very fine thermocouples are necessary.

For equal Reynolds numbers, the ratio of the temperature change across the viscous film ($t_b t_c$) to the maximum temperature change ($t_a t_c$) is roughly 99 per cent for a very viscous oil, 75 per cent for water, and 25 per cent for air.[1] The temperature gradient $t_c t_d$ is caused by the flow of heat in pure conduction, usually through a material whose thermal conductivity is known; and in most practical cases is a small fraction of the total temperature difference $t_a t_f$. It can, therefore, be calculated with a precision ample for ordinary purposes by equations already given.

Film Coefficients

An inspection of Fig. 76 will indicate that the thermal resistances in the two fluids are quite complicated. Consequently an indirect method is always used for their calculation, and this method involves the computation of *film coefficients*. In Fig. 76 suppose that q B.t.u. per hr. are flowing from the hot fluid to the cold one. Then q B.t.u. per hr. pass from the hot fluid to the metal wall and the same q B.t.u. pass from the metal wall into the cold fluid. Let the area of the metal wall on the hot side, in a plane at right angles to the flow of heat be A_1, the area on the cold side be A_2, and the average area of the metal wall be A_{av}. The film coefficient on the hot side is defined by the relation

$$h_1 = \frac{q}{A_1(t_1 - t_c)} \tag{59}$$

If Equation (59) be compared with Equation (43) it will be seen that h_1 is analogous to k/L and consequently $1/h_1A_1$ is a thermal resistance. Although the film coefficient is calculated and discussed as though it were an actual quantity in itself, reference to Fig. 76 will show that h_1 contains the effect both of the viscous film and of the thermal resistance of the turbulent core that causes the temperature difference ($t_a - t_b$).

[1] McAdams, "Heat Transmission," p. 137, McGraw-Hill Book Company, Inc., 1933. Subsequent references to McAdams refer to this book unless otherwise specified.

In the same way h_2 may be defined by

$$h_2 = \frac{}{A_2(t_d - t_2)} \tag{60}$$

Thus such a case as shown in Fig. 76 may be considered as consisting of three resistances in series; first, the resistance on the side of the hot fluid $1/(h_1A_1)$, second, the resistance of the metal wall $L/(kA_{av})$, and third, the resistance on the side of the cold fluid $1/(h_2A_2)$. If these resistances are substituted for R_1, R_2, and R_3 in Equation (49), this gives:

$$q = \frac{\Delta t}{\frac{}{h_1A_1} + \frac{}{kA} + \frac{}{h_2A_2}} \tag{61}$$

Overall Coefficients.—Assume that the numerator and denominator of the right-hand side of Equation (61) are multiplied by any one of the areas. For example, if A_1 is chosen, Equation (61) becomes

$$q = \frac{A_1\Delta t}{\frac{1}{h_1} + \frac{A_1L}{A_{av}k} + \frac{A_1}{h_2A_2}} \tag{62}$$

Define the *overall coefficient*, U_1, by the equation

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{A_1L}{A_{av}k} + \frac{A_1}{h_2A_2}} \tag{63}$$

If Equation (63) is compared with Equation (62) it is apparent that

$$q = U_1A_1\Delta t \tag{64}$$

Equation (64) states that *the rate of heat transfer is the product of three factors; overall heat-transfer coefficient, temperature drop, and area of heating surface.*

If either of the other two areas, A_{av} or A_2 had been chosen, there would have resulted coefficients based on these areas and denoted by U_{av} and U_2. Before an overall coefficient is established, therefore, a definite area must be chosen and the coefficient

so determined is automatically based on the chosen area. In general, the choice is arbitrary.

Equation (63) for the coefficient U_1 can be written in a convenient form for the case where the metal wall is tubular. In this case L is the wall thickness, the area A_1 is proportional to the corresponding diameter D_1, and the area A_2 is proportional to its corresponding diameter D_2. Equation (63) can, therefore, be written

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{D_1 L}{D_{av} k} + \frac{D_1}{D_2 h_2}} \qquad (65)$$

Analogous equations can be written for U_{av} and U_2.

Sometimes one particular area is more convenient than the others. Suppose that one film coefficient, for example h_2, is large numerically in comparison with the other coefficient h_1. In that case the term D_1/D_2h_2 becomes small in comparison with $1/h_1$. The second term in the denominator, which represents the resistance of the tube wall, is also usually small in comparison with $1/h_1$. In this case the ratios D_1/D_{av}. and D_1/D_2 have so little significance that they can be disregarded and Equation (65) becomes

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}} \qquad (66)$$

In such a case it is advantageous to base the overall coefficient on that area that corresponds to the highest resistance, or the lowest value of h.

In the case of thin-walled tubes of large diameter, flat plates, or any other case where a negligible error will be caused by using a common area A for A_1, A_{av}, and A_2, Equation (66) may be used in place of Equation (65). In such a case, U_1, U_{av}, and U_2 are all identical.

In certain cases h_1 is very small in comparison with both h_2 and L/k, and the term $1/h_1$ in Equation (65) is very large in comparison with the other terms in the denominator. In such a case it is sufficiently accurate to write $U_1 = h_1$.

The problem of predicting the rate of flow of heat from one fluid to another through a retaining wall reduces essentially

to the problem of predicting the numerical values of the film coefficients of the fluids concerned. Although this problem has not been solved for all cases, it is important that the knowledge that has been obtained be understood and used for what it is worth.

General Considerations Regarding Film Coefficients.—An equation for predicting the film coefficient in any particular case must include all the properties of the fluid and the conditions of its flow that affect the problem. In a particular case these factors might be the diameter of the pipe, the velocity of the fluid, its density, viscosity, thermal conductivity, specific heat, and possibly others. Most cases of heat transfer are so complex that it is practically hopeless to assemble these factors into an equation that shall be based on purely theoretical reasoning. In attempting to arrange these properties in an equation, one of the most useful methods yet found is that of *dimensional analysis*. This method shows in what relation to each other certain of these variables should appear, and results in arranging them into various "dimensionless" groups. Such a group, for instance, is the Reynolds number already mentioned in connection with fluid friction. For the sake of convenience the four most important groups are listed below with the names now universally given them:

Name	Formula	Symbol
Nusselt	hD/k	Nu
Reynolds	$Du\rho/\mu$	Re
Prandtl	$C\mu/k$	Pr
Grashof	$gD^3\beta\Delta t\rho^2/\mu^2$	Gr

where

h = coefficient of heat transfer.
D = diameter.
k = thermal conductivity.
u = linear velocity.
ρ = density.
μ = viscosity.
C = specific heat.
g = acceleration of gravity.
β = coefficient of thermal expansion.
Δt = temperature difference.

In the most general case it is found that an equation for a film coefficient of heat transfer to or from a moving fluid without change of state will probably be of the form

$$Nu = f(Re, Pr, Gr) \tag{67}$$

Neither dimensional analysis nor any other method known at present will give a further insight into the form of this function. It might be the sum of a few terms, it might be exponential, it might be logarithmic, an infinite series, or any type of function known to mathematics. For the sake of convenience and simplicity it has been assumed (and this assumption, though primarily one of expediency, is shown to be sufficiently correct to represent most of the experimental work done so far) that each of the four groups enters the equation only once and then as a power function. In other words; Equation (67) is arbitrarily assumed to have the form

$$Nu = KRe^aPr^bGr^c \tag{68}$$

where K, a, b, and c are constants that must be determined experimentally.

On page 39 it was stated that the Reynolds number is a measure of the degree of turbulence and, therefore, it is to be expected that in equations involving fluids in turbulent flow the Reynolds number will be of importance. The Grashof number contains the coefficient of thermal expansion and consequently measures the degree of natural convection. In natural convection the effect of turbulence is small, velocities are low, and consequently in equations for such cases the Reynolds number disappears and the Grashof number controls. The Prandtl number contains only the properties of the fluid and, therefore, changes mainly as one passes from a gas with a simple molecule to one with a complex molecule, then to water and aqueous solutions, and then to oils and organic liquids.

Certain qualitative deductions can be made from the mechanism of heat transfer in a fluid flowing in turbulent flow. It was shown on page 39 that an important factor in determining the friction loss in a pipe was the Reynolds number. It was shown that for large Reynolds numbers the viscous film becomes of less and less relative importance. Interpreting this fact in terms of Fig. 76, the result of increased Reynolds numbers, and,

therefore, increased turbulence, is to thin the viscous film and to equalize the temperature difference t_at_b. As a result of this change the gradient t_bt_c becomes much steeper and, other factors being equal, this increased gradient increases the rate of heat transfer through the fluid film to the left of F_1F_1. This in turn results in a higher rate of heat transfer from the entire fluid stream, since if one resistance in a series of resistances is reduced, the rate of heat flow through the rest is increased. It can be expected, therefore, that the coefficient h will increase with an increased Reynolds number. On inspection of the Reynolds number this indicates that increased linear velocity, increased density, and decreased viscosity all tend, in the absence of other factors, to give larger values of h. This rule does not apply to the diameter, since the diameter enters in the Nusselt number and this effect more than counterbalances the effect of diameter in the Reynolds number.

Consistent Units.—If the dimensions of the quantities occurring in any one of the four groups given above are substituted for the factors in these groups, it will be found that the dimensions will cancel and the group is therefore dimensionless; in other words it is a pure number. For example, in the Reynolds number the dimension of D is length; the dimension of u is length/time; of density, mass/length^3; and of viscosity, mass/(length $\times$ time). Substitution of these dimensions will immediately show that the Reynolds number itself is dimensionless. Analogous treatment of the other groups will also show that they are dimensionless.

One result of the use of dimensionless equations is the fact that, if in any group self-consistent units are used, the constants of the equation are independent of the system of units chosen. For example, if in a Reynolds number all length dimensions are in feet, all time dimensions are in seconds, and all mass dimensions are in pounds, the same value of the Reynolds number will be found if another entirely different but consistent set of units, such as centimeters, hours, and grams is used. Thus it was specified in Chap. II that the critical Reynolds number is 2,100. Although in Chap. II the foot-pound-second units were chosen, the critical Reynolds number would still be 2,100 if grams, centimeters, and seconds were used. It is sometimes even convenient to use one set of units in one dimensionless

group in an equation and another set of units in another group. This is entirely permissible as long as each group is kept dimensionally consistent in itself. For example, in American practice it is customary to express film coefficients in B.t.u./(hr. × sq. ft. × °F). For this reason the time unit most convenient in the Nusselt group is the hour. On the other hand, there is no necessity for changing the unit of time in the Reynolds number from seconds to hours, since velocity is customarily expressed in feet per second. In the Prandtl number either thermal conductivities can be converted to second-units by dividing the hour-unit by 3,600; or the absolute viscosity can be transformed from second-units to hour-units by multiplying the centipoise value by 2.42 instead of 0.000672.

Equations for Film Coefficients

Fluids in Turbulent Flow inside Clean Round Pipes.—For this case Equation (68) has been found to have the following form:

$$h = 0.0225\frac{k}{D}\left(\frac{Du\rho}{\mu}\right)^{0.8}\left(\frac{C\mu}{k}\right)^{0.4} \qquad (69)$$

Here the symbols have the significance given on page 131 and all the dimensions must be consistent within any one group. In a case where the temperature of the fluid varies from point to point in the apparatus, the properties of the fluid are to be evaluated at the mean of the inlet and outlet temperatures. This equation is usually known as the *Dittus-Boelter* equation.[1] It has been checked experimentally on air and other gases, water, hydrocarbon oils, and various organic liquids; for Reynolds numbers from 2,500 to 160,000 and for Prandtl numbers from 0.73 to 95. The exponent 0.4 for the Prandtl number in Equation (69) is to be used when heat is flowing from the pipe to the fluid. When the fluid is being cooled, this exponent is to be 0.3. It should be noted, however, that the heating or cooling of hydrocarbon oils does not agree with this equation below values of 7,000 for the Reynolds number. From 7,000

[1] Dittus and Boelter, *Univ. Calif. Pub. in Eng.*, **2**, 443 (1930); McAdams, p. 169.

down to the critical value of 2,100 the *Morris* and *Whitman*[1] equation should be used:

$$\frac{hD/k}{(C\mu/k)^{0.4}} \qquad Du\rho \tag{70}$$

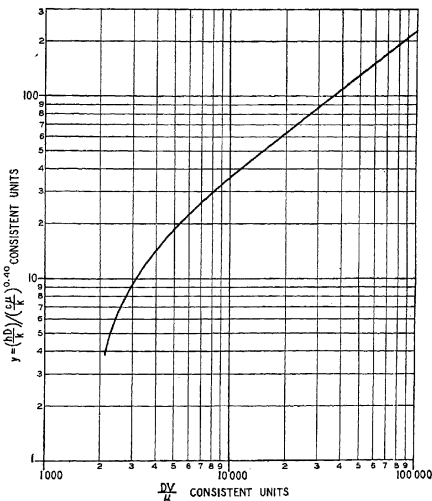



Fig. 77.—Plot of the Morris and Whitman equation.

in which ψ is a function to be taken from the curve of Fig. 77. Figure 77 is for the case where heat is flowing from the pipe into the oil. In those cases where the oil is being cooled, values of ψ 25 per cent less than those read from Fig. 77 should be used.

The product of density and velocity, which occurs in the Reynolds number, has the dimensions mass/(length^2 $\times$ time).

[1] Morris and Whitman, *Ind. Eng. Chem.*, **20**, 234–240 (1928).

This product has been given the name *mass velocity*. It is convenient to use mass velocity instead of linear velocity and density, especially in the case of gases; because this quantity is independent of pressure and temperature, whereas both linear velocity and density change with these variables. If v is mass velocity, the Reynolds number is Dv/μ. When ft.-lb.-sec. units are used, the dimensions of mass velocity are pounds per square foot per second. In this case it must be remembered that the area term means the cross-section of the fluid path and not the heating surface. So long as the cross-section of the fluid path is constant, mass velocity is constant.

Shapes Other than Circular.—In the formulas given so far for estimating film coefficients, circular pipes have been assumed and the diameter of the pipe enters into these equations. Data for developing equations that can be used in calculating film coefficients for cases wherein the cross-section of the fluid stream is not circular are very meager. In order to estimate these cases, the formulas given above are used, except that in place of the diameter there is substituted four times the so-called *shape factor*. This shape factor is defined as the cross-sectional area of the channel divided by the perimeter of the heating surface. For a circular pipe the shape factor is $D/4$, and therefore in using the shape factor instead of the diameter, four times the shape factor should be used instead of D.

Fluids in Forced Convection outside Single Cylinders and at Right Angles to Them.—For this case the best equation is that of Ulsamer:[1]

$$\frac{hD}{k_f} = K\left(\frac{Dv}{\mu_f}\right)^n\left(\frac{C\mu_f}{k_f}\right) \tag{71}$$

where the constants have the following values:

Re	K	m	n
0.1 to 50	0.91	0.31	0.385
50 to 10,000	0.60	0.31	0.50

In this equation the subscript f indicates that these properties are to be evaluated at the mean temperature of the viscous

[1] *Forschung*, **3**, 94–98 (1932).

film. This is considered to be half way between t_2 and t_a in Fig. 76.

For very low values of the Reynolds number, this equation tends to give low values. This is because at low values of the Reynolds number the turbulence is not sufficiently great to outweigh the effect of natural convection and, hence, to be strictly accurate Equation (71) should have an additional term containing the Grashof number. Experimental work has not gone far enough to develop such an equation. The change in exponents in the neighborhood of $Re = 50$ is probably connected with the complete disappearance of any appreciable effect of the Grashof number.

Gases in Forced Convection at Right Angles to Banks of Staggered Pipes.—A very common type of heater for gases involves blowing the gas across banks of pipes. For this case, where the bank is five rows deep or more, the Reiher[1] equation may be used:

$$h = 0.131\left(\frac{k_f}{D}\right)\left(\frac{Dv_{max}}{\mu_f}\right)^{0.7} \tag{72}$$

The quantity v_{max} is the mass velocity calculated on the area of the minimum opening between adjacent pipes, and the subscript f has the same significance as in Equation (71).

The indications are that for banks of pipes less than five rows deep the coefficient will be less than given by the Reiher equation, but that over five rows deep there is no further increase in the coefficient. The Reiher equation is specifically limited to gases. An extremely important case in practice involves the transfer of heat to or from liquids in forced convection outside of banks of pipes. The data for this case are so fragmentary and the results so inconclusive that no equation is justified as yet.

Gases and Liquids in Natural Convection.—This is an extremely complicated subject because the velocity attained in natural convection is a function of the size, shape, and arrangement of the heating surface, and in many cases is too complicated to be predicted. For the specific case of the transfer of heat from a single horizontal cylinder to gases or liquids in natural convection around it, the coefficient may be taken from

[1] REIHER, *Mit. Forschungsarb.*, **269**, 20 (1925); MCADAMS, p. 227.

Fig. 78.[1] This relation has been determined by experimental work covering an extremely wide range of all variables involved.

When hot bodies lose heat to their surroundings they do so both by radiation and by convection. What is ordinarily spoken of as *radiation loss* is really the sum of losses by true radiation and by convection. In the lower-temperature ranges convection is more important, in high-temperature ranges radiation is more important. True radiation cannot be cal-

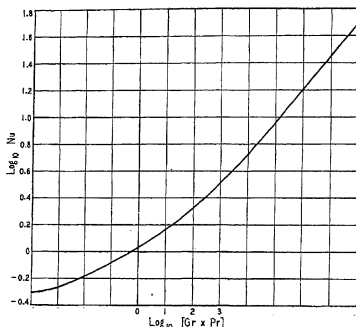



Fig. 78.—Heat transfer between single horizontal cylinders and fluids in natural convection.

culated by methods within the scope of this text. As mentioned above, convection losses vary with the shape, size, and arrangement of the hot body. For the important particular case of losses from bare horizontal iron pipe to the surrounding air at about 80°F., the sum of the convection and radiation effects may be calculated by the use of the table on page 139.[2]

The values in this table are values of h_T, which is a combined coefficient and accounts for both convection and radiation. Although radiation follows entirely different laws than does convection (especially in that heat transfer by radiation is

[1] McAdams, p. 248.
[2] McAdams, p. 252.

VALUES OF h_T FOR LOSS FROM BARE HORIZONTAL PIPE TO AIR AT 80°F.
In B.t.u. per square foot per hour per degree Fahrenheit

Nominal pipe diameter, inches	Temperature difference, degrees Fahrenheit													
	50°	100°	150°	200°	250°	300°	350°	400°	450°	500°	550°	600°	650°	700°
1	2.26	2.50	2.73	3.00	3.29	3.60	3.95	4.34	4.73	5.16	5.60	6.05	6.51	6.98
3	2.05	2.25	2.47	2.73	3.00	3.31	3.69	4.03	4.43	4.85	5.26	5.71	6.19	6.66
5	1.95	2.15	2.36	2.61	2.90	3.20	3.54	3.90	4.30					
10	1.87	2.07	2.29	2.54	2.82	3.12	3.47	3.84						

not directly proportional to temperature difference), this is compensated for by the increase in h_T with increase in temperature difference. The film coefficient between the hot fluid and the inside of the pipe is so high and the resistance of the pipe wall is so low, compared with the values in this table, that U can be considered equal to h_T (page 130).

Liquids outside Pipes.—It will be noted that in most of the formulas developed so far, the fluid under consideration must be flowing in some sort of a channel at a known velocity. The only exception to this involves the data and equation given for natural convection from single horizontal cylinders. There are many cases in practice that cannot be calculated by any of the methods given above. For example, consider a tank of liquid heated with a steam coil. The velocity of the liquid as it circulates through the tank is due only to natural convection, which in turn depends on the dimensions and proportions of the tank, the shape and area of the coil, the viscosity of the liquid, and other factors. In the present state of our knowledge, it is impossible to evaluate these, and consequently for such cases there is no method for calculating the film coefficient. In practice, values for the liquid-film coefficient may be from 10 to 200, depending on the arrangement of the apparatus and the viscosity of the liquid. Unfortunately a large number of cases of heat transfer, with which the engineer comes into contact, fall under the head of cases about which nothing is known and no calculations may be made.

Boiling Liquids.—There are no data available on the values of the film coefficient between metal and boiling liquids. It has so far been impossible to evaluate all of the factors affecting

this coefficient in such a way as to obtain any systematic results. Enough has been done, however, to show that the three principal factors involved are (1) the velocity of the liquid past the heating surface, (2) the viscosity of the liquid (although this is possibly of importance only as it affects velocity), and (3) the physical arrangement and condition of the heating surface. Since the velocity in all ordinary cases of boiling liquids is largely due to the stirring effect of the steam bubbles, it follows that the film coefficient will increase with increasing temperature drops, because larger rates of heat transfer increase the rate of steam evolution, and therefore the amount of stirring. Since the stirring is produced by the steam bubbles as they rise from the point where they originate to the surface of the liquid, it follows that the physical arrangement of the heating surface is of great importance.

Some more general statements may be made regarding *overall* coefficients involving boiling liquids, but these will be discussed in the chapter on evaporation.

Condensing Vapors.—When a saturated vapor, such as steam, transmits its heat to a metal surface and is condensed, the condensation may take place in either of two entirely distinct forms. One is *film-type condensation,* in which the condensed liquid wets the surface on which it is condensing and forms a continuous film of condensate. If the condensation is occurring on the outside surface of a horizontal metal tube (a very common case) this film of condensate drops off the under side of the tube; it runs down the whole length of the tube if the tube is vertical. The other type of condensation is *drop-wise condensation.* In this case the condensed liquid does not wet the surface, but collects in drops that may range from microscopic size up to drops easily seen with the naked eye. These drops grow for a while and then fall off the surface, leaving an apparently bare area in which new drops form.

These two types of condensation give widely different film coefficients of heat transfer. Coefficients in the case of drop-wise condensation may be double, or even more than double, those obtained on the same surface with all other conditions equal except that the condensation is film-type. The factors that cause the condensation to take one form or the other are largely unknown; and the same tube may shift erratically and

rapidly, entirely or partially, from one type of condensation to the other. In general, smooth clean surfaces seem to tend toward film-type condensation and oily or greasy surfaces seem to tend toward drop-wise condensation. The whole subject is very little understood at present.

If the condensation is drop-wise there is no method known at present by which the film coefficient of heat transfer may be predicted. For the case of a horizontal tube in true film-type condensation on which there is condensing a saturated vapor, free from any non-condensed gas and moving at low velocities, Nusselt has derived the following equation:[1]

$$h = 0.725\sqrt[4]{\frac{k^3\rho^2 g\lambda}{D\mu\Delta t}} \tag{73}$$

where

λ = latent heat of vaporization of vapor in B.t.u. per pound.
ρ = density of condensate in pounds per cubic foot.
k = thermal conductivity of condensed vapor in B.t.u.-feet per square foot per hour per degree Fahrenheit.
g = acceleration of gravity in feet per hour^2 ($= 4.18 \times 10^8$).
μ = viscosity of condensate film in foot-pound-hour units.
D = outside pipe diameter in feet.
Δt = temperature difference between vapor and metal in degrees Fahrenheit.

In using Equation (73) ρ, k, and μ are to be evaluated at mean film temperature. A limited amount of experimental work seems to check this equation moderately well.

For the case of a vertical tube with all other conditions the same as for Equation (73), Nusselt has given the equation

$$h = 0.943\sqrt[4]{\frac{k^3\rho^2 g\lambda}{L\mu\Delta t}} \tag{74}$$

where the significance of all the symbols are the same as in Equation (73) and L is the length of the tube.

Experimental work on vertical tubes has given results from 30 to 50 per cent higher than Equation (74). It should be noted that drop-wise condensation, which apparently occurs

[1] *Z. Ver. deut. Ing.*, **60**, 541, 569 (1916); McAdams, p. 263.

very often in practice, gives much higher results than film-type condensation, and, therefore, the values calculated from Equations (73) and (74) are safe.

The film coefficient between condensing vapors and metal walls increases with increasing temperature of the vapor, because of decreased viscosity of the film of condensate. It decreases with increasing temperature drop, because increasing temperature drops cause faster condensation and hence thicker liquid films. An important factor affecting film coefficients, however, (not taken into consideration in the above equations) is the presence of non-condensed gases. These accumulate near the heating surface and add their resistance to that of the liquid film.

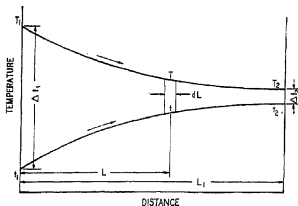



FIG. 79.—Derivation of logarithmic mean temperature difference.

VARYING TEMPERATURE DROP

Equation (64) as written applies only when the temperature drop is constant for all parts of the heating surface. When this is not so, the equation must be modified by using an average temperature drop Δt_m in place of Δt.

Derivation of Logarithmic Mean Temperature Drop.—Consider a heat interchanger. A hot fluid inside a pipe is cooled from T_1 to T_2 by transferring its heat to a cold fluid outside the pipe, entering at t_1 and heated to t_2. It is desired to calculate the length of pipe necessary for this process, assuming that the pipe has a sq. ft. of heating surface per foot of length.

The conditions in this interchanger are represented in Fig. 79, where temperatures are plotted against distance along the pipe. The temperature drop at the left-hand end of the figure is much greater than that at the right-hand end. Heat is being transferred more rapidly, therefore, at the left-hand end than at the right-hand end. Equation (64) can be applied

if the heating surface is divided into a large number of very short segments and the equation written over each segment, followed by determining the sum of the heat flowing through all these segments. Mathematically, this process is equivalent to writing Equation (64) as a differential equation over a section of infinitesimal length, and integrating the equation over the whole length of the pipe.

Consider a short segment of pipe, L ft. from the left-hand end, and let the length of this small segment be dL ft. Let the temperature of the hot fluid at this point be T, and the temperature of the cold fluid be t. There will flow through this small heating surface dq B.t.u. per hour. In passing this section the hot fluid is cooled $dT°$, and the cold fluid is heated $dt°$. Since the area of the pipe is a sq. ft. per unit of length the area of the heating surface of the small segment in question will be adL. Equation (64) can be written over this segment in the form

$$dq = U(T - t)adL \tag{75}$$

As a first approximation *it will be assumed that the value of the overall coefficient U does not vary appreciably over the whole apparatus.* This is equivalent to neglecting the effect of the temperatures of the fluids on the individual film coefficients of which U is composed. If this assumption is made, it will be seen that Equation (75) contains four variables: q, T, t, and L. If the equation is to be integrated, two of these variables must be eliminated. The variable q can be expressed in terms of either T or t. For example,

$$dq = wcdt \tag{76}$$

where

w = the pounds per hour of cold fluid passing the section.
c = the specific heat of the cold fluid.

A second assumption will be made, namely, that the specific heats of the two fluids do not vary appreciably in the apparatus. Another relationship that can be used to eliminate one of the variables of Equation (75) is a heat balance over one end of the apparatus up to the short segment considered in that equation. Thus, if the weight per hour of the hot fluid is W, and if the specific heat of the hot fluid is C, then it follows that since the heat picked up by the cold fluid must equal that lost by the hot fluid (if radiation is neglected),

$$WC(T_1 - T) = wc(t - t_1) \tag{77}$$

Solving Equation (77) for T gives

$$T = T_1 + \frac{wc}{WC}(t_1 - t) \tag{78}$$

Substituting the value for dq from Equation (76) and the value of T from Equation (78) in Equation (75) gives

$$wcdt = U\left[T_1 + \frac{wc}{WC}(t_1 - t) - t\right]adL \tag{79}$$

Equation (79) contains only two variables, t and L, and can be integrated, if the variables are separated, as follows:

$$\frac{wc\,dt}{T_1 + \frac{wct_1}{WC} - \left(\frac{wc}{WC} + 1\right)t} = Ua\,dL \tag{80}$$

Both sides of Equation (80) may be integrated

$$wc\int_{t_1}^{t_2} \frac{dt}{T_1 + \frac{wct_1}{WC} - \left(\frac{wc}{WC} + 1\right)t} = Ua\int_0^{L} dL \tag{81}$$

The limits in Equation (81) signify that when L is zero, the temperature of the cold fluid is t_1 and that when L is equal to L_1, which is the entire length of the apparatus, the temperature of the cold fluid is t_2.

The integration of Equation (81) and substitution of the limits lead to

$$-\left(\frac{wc}{WC} + 1\right)\ln\frac{T_1 + \frac{wct_1}{WC} - \left(\frac{wc}{WC} + 1\right)}{\left[T_1 + \frac{wct_1}{WC} - \left(\frac{wc}{WC} + 1\right)t_1\right]} \quad UaL_1 \tag{82}$$

Equation (82) can be used as it stands to calculate L_1 if the terminal temperatures, the amounts of the fluids and their specific heats, and the overall coefficient are all known. It is a clumsy form, however, and can be simplified, by noting that there is still available an equation representing a heat balance over the entire apparatus,

$$WC(T_1 - T_2) = wc(t_2 - t_1) \tag{83}$$

Solving Equation (83) for WC, substituting this value in Equation (82), and simplifying gives

$$\frac{wc(t_2 - t_1)}{(T_1 - t_1) - (T_2 - t_2)}\ln\frac{T_1 - t_1}{T_2 - t_2} \quad UaL_1 \tag{84}$$

Since

$$wc(t_2 - t_1) = q \tag{85}$$

and

$$T_1 - t_1 = \Delta t_1,\; T_2 - t_2 = \Delta t_2 \tag{86}$$

(where Δt_1 and Δt_2 are the terminal temperature differences), substitution in Equation (84) gives

$$q = UaL_1\frac{\Delta t_1 - \Delta t_2}{\ln\frac{\Delta t_1}{\Delta t_2}} \tag{87}$$

Let Δt_m be defined as

$$\Delta t_m = \frac{\Delta t_1 - \Delta t_2}{\ln\frac{\Delta t_1}{\Delta t_2}} \tag{88}$$

and note that the total heating surface A is

$$A = aL_1$$

then, substituting in Equation (87),

$$q = UA\Delta t_m \tag{89}$$

Equation (89) is seen to be identical with Equation (64), except that the logarithmic mean temperature difference, as defined in Equation (88), has been used instead of an arithmetic difference. The expression for the logarithmic mean temperature difference is of the same form as the expression for the logarithmic

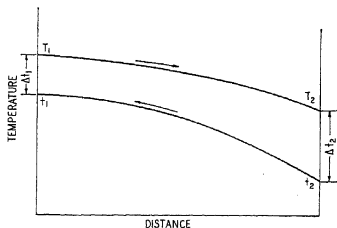



FIG. 80.—Temperature drop in countercurrent heat exchanger.

mean radius of a thick-walled tube [Equation (58)]. When Δt_1 and Δt_2 are nearly equal, their arithmetic average may be used for Δt_m within the same limits of accuracy as given for Equation (58).

Parallel-current and Countercurrent Flow.—It will be noted that, in the derivation of the formula for the logarithmic mean temperature difference, the hot fluid and the cold fluid enter at the same end of the apparatus and flow parallel to each other through it. This arrangement is known as *parallel flow.* The alternative method is to feed the hot fluid at one end of the apparatus and the cold fluid at the other, allowing the fluids to pass by each other in opposite directions. Such an arrangement is called the *countercurrent* or *counterflow method.* The temperature gradients for this case are shown diagrammatically in Fig. 80. It will be noted that in the case of countercurrent

operation the temperature drop along the length of the apparatus is much more nearly constant than is the case in parallel flow. Thus in Fig. 79, it is apparent that at that part of the apparatus near the fluid exit, a given square foot of heating surface is much less effective than a square foot at the entrance end of the apparatus, because of the much smaller temperature drop over it. In countercurrent operation on the other hand, an element of heating surface at one end may be transferring substantially the same amount of heat as an element of the same area at the other end, and therefore the heating surface has nearly constant capacity throughout the apparatus. Furthermore, it can be seen that in countercurrent operation the exit temperature of the hot fluid can be considerably less than the exit temperature of the cold fluid, and accordingly a larger proportion of the heat content of the hot fluid can be extracted for a given entrance temperature of the cold fluid.

In Figs. 79 and 80 both fluids are shown as changing in temperature. It is not necessary that either or both fluids do change in temperature. For example, consider a cold fluid being warmed by means of a hotter condensing vapor. In this case the temperature of the condensing vapor will be constant across the length of the apparatus, provided the channel containing it is large enough so that appreciable pressure differences are not produced. Also, a warm fluid may conceivably be used to vaporize a colder fluid under a constant pressure, for example, a steam boiler.

In all of these cases, the expression for Δt_m is the same, subject to the same assumptions that were used in the derivation for the case of parallel flow. In any case, it is necessary only to divide the difference between the terminal temperature drops by the natural logarithm of their ratio.

Other types of temperature gradient occur frequently, especially in multipass and baffled heat exchangers (Figs. 86 and 87). Such cases cannot be considered in this text.[1]

The simple logarithmic mean formula cannot be applied to such a case as the following, which is shown in Fig. 81. In this case a superheated vapor is the hot fluid. It is first cooled to its condensation point along the line AB. It is then condensed along the line BC, and the resulting liquid cooled along

[1] See NAGLE, *Ind. Eng. Chem.*, **25**, 604–609 (1933)

the line CD. The temperature of the cooling water rises regularly from E to H. In this case, it is best to consider the apparatus to be divided into three parts: The first section, represented by the segment AB, is considered as a vapor-cooling section, and the logarithmic mean temperature drop computed for it. This logarithmic mean temperature drop would be applied to the heat transferred in this section, and the area of the section

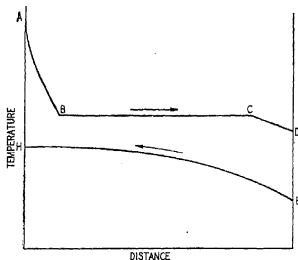



Fig. 81.—Temperature drops in condensation of superheated steam.

calculated by Equation (89). Each of the other two sections can be calculated in the same way, and the total heating surface obtained by adding the surfaces of the three sections. This method is substantially equivalent to considering that the apparatus is really three heat-transfer devices in one frame. If one were to take the average temperature difference corresponding to AH and DE, a large error might be introduced.

Varying Temperature Drop and Varying Coefficient.—In deriving Equation (87) certain assumptions were made, the most important of these being the constancy of the overall coefficient and of the specific heats. If these two quantities vary considerably over the range of the apparatus, the logarithmic mean temperature difference has no significance. In such cases it is simplest to use a method involving graphic integration.

Eliminating dq from Equations (75) and (76) gives

$$wcdt = Ua(T - t)dL \qquad (90)$$

This equation may be written

$$\frac{wc}{Ua(T - t)}dt = dL \tag{91}$$

To use Equation (91), a series of values of t are chosen. For each of these, the corresponding values of T can be calculated by Equation (78). Knowing t and T, the film coefficients (and from them, the value of U) may be calculated for each of these points. All the quantities in Equation (91), except dt and

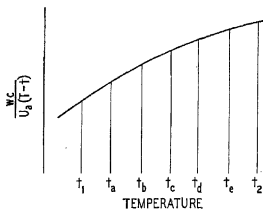



FIG. 82.—Graphic integration applied to heat-transfer problems.

dL, are then known. The quantity $\frac{wc}{Ua(T - t)}$ is then plotted against the values assumed for t as shown in Fig. 82. Graphic integration of the area under the curve between the limits t_1 and t_2 is equivalent to performing the following operation:

$$\int_{t_1}^{t_2} \frac{wc}{Ua(T - t)}dt = \int_0^{L_1} dL = L_1 \tag{92}$$

Example 11.—One thousand cubic feet per hour of dry carbon dioxide, at 15 lb. gage and 100°F., is to be cooled to 70°F. The gas is inside copper tubes, 1 in. outside diameter and 18 gage wall, and each tube is surrounded by another copper tube 1.5 in. outside diameter and 16 gage wall. Water flows through the annular space at a velocity of 1 ft. per sec. It enters at 50°F. and flows countercurrent to the gas. The inlet velocity of the gas is to be about 20 ft. per sec. How many tubes, how long, are required? The mean specific heat of carbon dioxide over the range involved here may be taken as 0.204.

Solution. *Number of Tubes.*—The inside area of the tube is 0.638 sq. in., or 0.00443 sq. ft. (Appendix V). At a velocity of 20 ft. per sec., each tube will carry, per hour,

$$0.00443 \times 3,600 \times 20 = 319 \text{ cu. ft.}$$

Hence three tubes in parallel will fulfill the conditions.

Heat to Be Transferred.—The weight of 1 cu. ft. of carbon dioxide at 15 lb. gage and 100°F. is (p. 8)

$$\frac{44}{359} \times \frac{29.7}{14.7} \times \frac{492}{560} = 0.2176 \text{ lb.}$$

(The molecular weight of carbon dioxide is 44.) The total weight of gas to be cooled is therefore 217.6 lb. per hr. The heat transferred is

$$217.6 \times 0.204 \times (100 - 70) = 1332 \text{ B.t.u.}$$

Exit Water Temperature:

	Square inches
Inside area of 1.5-in. tube =	1.473
Outside area of 1-in. tube =	0.785
Area one annulus =	0.688

But there are three tubes in parallel, hence the cross-section of the water path is 0.688 × 3 or 2.064 sq. in. At a velocity of 1 ft. per sec., this calls for

$$\frac{2.064}{144} \times 1 \times 3,600 \times 62.42 = 3,220 \text{ lb. per hr.}$$

The rise in temperature of the water will be

$$\frac{1,332}{3,220} \text{ or } 0.42°, \text{ and hence the exit temperature will be } 50.4°.$$

Mean Temperature Difference:

$$\Delta t_1 = 100 - 50.4 = 49.6°$$
$$\Delta t_2 = 70 - 50 = 20.0°$$
$$\Delta t_m = \frac{49.6 - 20.0}{2.30 \log \frac{49.6}{20.0}} = 32.6°$$

The arithmetic mean temperature difference is 34.8°, an error of 7 per cent.

Gas-film Coefficient.—For this, use Equation (69). The values used for the properties of the gas are to be taken at the mean of 100° and 70°, or 85°F., and all quantities will be expressed in lb.-ft.-hr. units. The gas is being cooled, so the exponent of the Prandtl number will be 0.3.

$$k = 0.00896$$
$$D = 0.902 \text{ in.} = 0.0752 \text{ ft.}$$

$$u\rho = v = \frac{217.6}{3 \times 0.00443} = 16{,}370$$

$\mu = 0.0149$ centipoises $= 0.0361$ lb.-ft.-hr. units.

$C = 0.204$

$$h = 0.0225\left(\frac{0.00896}{0.0752}\right)\left(\frac{0.0752 \times 16{,}370}{0.0361}\right)^{0.8}\left(\frac{0.204 \times 0.0361}{0.00896}\right)^{0.3}$$

$$= 0.002681(34{,}100)^{0.8}(0.8218)^{0.3}$$

$$= 0.002681 \times 4229 \times 0.9427 = 10.69$$

Water-film Coefficient.—Again Equation (69) will be used, the properties of water will be taken at 50°F., and ft.-lb.-hr. units will be used. The water is being heated, so the exponent of the Prandtl number will be 0.4; and since the water is in the annular space, four times the shape factor will be used for D.

Outside diameter of outside pipe	= 1.500 in.
Wall thickness = 2 × 0.065	= 0.130 in.
Inside diameter of outside pipe	= 1.370 in.
Area of circle 1.370 in. in diameter	= 1.474 sq. in.
Area of circle 1.000 in. in diameter	= 0.785 sq. in.
Area of annular space	= 0.689 sq. in.

$$\text{Shape factor} = \frac{0.689}{(3.142 \times 12)} = 0.01828 \text{ ft.}$$

$D = 4 \times 0.01828 = 0.07312$

$k = 0.333$

$u = 1.00 \times 3{,}600 = 3{,}600$

$\rho = 62.42$

$\mu = 1.310 \times 2.42 = 3.170$

$C = 1.00$

$$h = 0.0225\left(\frac{0.333}{0.0731}\right)\left(\frac{0.0731 \times 3{,}600 \times 62.42}{3.170}\right)^{0.8}\left(\frac{1.00 \times 3.170}{0.333}\right)^{0.4}$$

$$= 0.1025(5{,}180)^{0.8}(9.523)^{0.4}$$

$$= 0.1025 \times 936.5 \times 2.463 = 236.4$$

Overall Coefficient.—Since the gas-film coefficient is much lower than the water-film coefficient, the overall coefficient will approximate the gas film and hence the *inside* area will be used for calculating the heating surface. Equation (65) will be used, in which

$h_1 = 10.69$

$D_1 = 1.000 - (2 \times 0.049) = 0.902$ in. $= 0.0752$ ft.

$D_2 = 1.000$ in. $= 0.0833$ ft.

$D_{av} = 0.0792$ ft.

$L = 0.049$ in. $= 0.00401$ ft.

$k = 226$

$h_2 = 236.4$

$$U = \frac{1}{\frac{1}{10.69} + \frac{0.0752 \times 0.00401}{0.0792 \times 226} + \frac{0.0752}{0.0833 \times 236.4}}$$
$$= \frac{1}{0.09356 + 0.0000169 + 0.003817} = \frac{1}{0.09739}$$
$$= 10.17$$

Since there is so great a difference between the two film coefficients, the approximate Equation (66) might have been used. Here

$$h_1 = 10.69$$
$$L = 0.00401$$
$$k = 226$$
$$h_2 = 236.4$$
$$U = \frac{1}{\frac{1}{10.69} + \frac{0.00401}{226} + \frac{1}{236.4}}$$
$$= \frac{1}{0.09356 + 0.0000177 + 0.00423} = \frac{1}{0.09781}$$
$$= 10.23$$

Hence in this case the use of Equation (65) was unnecessary.

Heating Surface.—Substituting in Equation (89):

$$1{,}332 = 10.27 \times A \times 32.6$$
$$A = 3.98 \text{ sq. ft.}$$

From Appendix V each tube has 0.236 sq. ft. inside surface per lineal foot, and since three tubes are to be used in parallel, the length is

$$\frac{3.98}{0.236 \times 3} = 5.62 \text{ ft.} = 5 \text{ ft. } 7\tfrac{1}{2} \text{ in.}$$

Example 12.—Water, at an average temperature of 150°F., is flowing inside a horizontal 1-in. standard iron pipe at a velocity of 8 ft. per sec. Outside the pipe is saturated steam at 5 lb. per sq. in. gage. Calculate (*a*) the overall coefficient based on the inside surface, (*b*) the overall coefficient based on the outside surface, and (*c*) the per cent of the total temperature drop that takes place across each of the films and across the metal.

Solution.—The water-film coefficient may be calculated by Equation (69). Here

$$D = 1.049 \text{ in.} = 0.0873 \text{ ft.}$$
$$k = 0.381$$
$$u = 8 \times 3{,}600 = 28{,}800$$
$$\rho = 61.20$$
$$\mu = 0.433 \times 2.42 = 1.048$$
$$h = 0.0225\left(\frac{0.381}{0.0873}\right)\left(\frac{0.0873 \times 28{,}800 \times 61.20}{1.048}\right)^{0.8}\left(\frac{1.00 \times 1.048}{0.381}\right)^{0.4}$$
$$= 2{,}001$$

Steam-film Coefficient.—As a first approximation, it will be considered that the steam-film resistance will be of the same order of magnitude as the water-film resistance, and hence the temperature of the metal wall will be about halfway between steam and mean water temperatures. Steam at 5 lb. per sq. in. gage has a temperature of 227°F., hence the metal will be at about 190°F. The mean film temperature will be halfway between this and steam temperature, or about 210°F. In Equation (73)

$$k = 0.393$$
$$\rho = 59.88$$
$$g = 4.18 \times 10^8$$
$$\lambda = 960.6$$
$$D = \frac{1.315}{12} = 0.1096$$
$$\mu = 0.287 \times 2.42 = 0.694$$
$$\Delta t = 227 - 190 = 37$$
$$h = 0.725\sqrt[4]{\frac{(0.393)^3(59.88)^2(4.18 \times 10^8)(960.6)}{0.1096 \times 0.694 \times 37}}$$
$$= 1{,}711$$

To check the assumptions made in determining the approximate film temperature for use in Equation (73), the individual resistances may be calculated as follows:

$$\text{Steam-film resistance} = \frac{1}{h_1A_1}$$
$$\text{Metal resistance} = \frac{L}{kA_{av}}$$
$$\text{Water-film resistance} = \frac{1}{h_2A_2}$$

Since the areas are proportional to the diameters, D_1, D_{av}, and D_2 may be used in place of A_1, A_{av}, and A_2. The arithmetic mean diameter is sufficiently accurate, since $D_1/D_2 = 1.25$. Then

$$\text{Steam-film resistance} = \frac{1}{(1{,}711 \times 1.315)} = 0.000445$$
$$\text{Metal resistance} = \frac{(1.315 - 1.049)/(2 \times 12)}{(26 \times 1.182)} = 0.000361$$
$$\text{Water-film resistance} = \frac{1}{(2{,}001 \times 1.049)} = 0.000477$$
$$\overline{0.001283}$$

The temperature drop across the steam film is

$$(227 - 150) \times \frac{0.000445}{0.001283} \quad \text{or} \quad 27°,$$

and therefore the temperature of the steam side of the metal wall is (227 − 27) or 200°F. Mean condensate film temperature is halfway

between 227° and 200° or 213°F. Repeating the evaluation of Equation (73) gives

$$
\begin{aligned}
k &= 0.393 \\
\rho &= 59.81 \\
g &= 4.18 \times 10^8 \\
\lambda &= 960.6 \\
D &= 0.1096 \\
\Delta t &= 227 - 200 = 27° \\
\mu &= 0.284 \times 2.42 = 0.687 \\
h &= 0.725 \sqrt[4]{\frac{(0.393)^3(59.81)^2 \times (4.18 \times 10^8) \times 960.6}{0.1096 \times 0.687 \times 27}} \\
&= 1{,}855
\end{aligned}
$$

Another readjustment of film temperature and temperature drop will make changes too small to affect the steam-film coefficient by an appreciable amount.

Part a. Overall coefficient based on inside surface.

Use Equation (65). Based on the inside surface the values are

$$
\begin{aligned}
h_1 &= 2{,}000 \\
D_1 &= 1.049 \\
D_{av} &= 1.182 \\
L &= \frac{(1.315 - 1.049)}{(2 \times 12)} = 0.0111 \\
k &= 34 \\
D_2 &= 1.315 \\
U &= \frac{1}{\frac{1}{2{,}000} + \frac{1.049}{1.182} \times \frac{0.0111}{26} + \frac{1.049}{1.315 \times 1{,}855}} \\
&= \frac{1}{0.000500 + 0.000379 + 0.000429} \\
&= \frac{1}{0.001308} = 764
\end{aligned}
$$

Part b. Overall coefficient based on outside surface.

Equation (65) will now read

$$
U = \frac{1}{\frac{D_2}{D_1 h_1} + \frac{D_2 L}{D_{av} k} + \frac{1}{h_2}}
$$

Substituting the above values and solving:

$$
U = 611
$$

Part c. Temperature Drops.—The temperature drops will be directly proportional to the thermal resistances. These were calculated in part (*a*).

$$\text{Per cent drop across steam film} = \frac{0.000500}{0.001228} \times 100 = 40.7$$

$$\text{Per cent drop across metal} = \frac{0.000290}{0.001228} \times 100 = 23.6$$

$$\text{Per cent drop across water film} = \frac{0.000438}{0.001228} \times 100 = 35.7$$

$$100.0$$

Example 13.—A 2-in. standard iron pipe, carrying steam at 5 lb. gage, is immersed in a large tank of water. What is the heat-transfer coefficient when the water is at 70°F.?

Solution.—Use Fig. 78. The water-film coefficient will probably be of a much lower order of magnitude than the steam-film coefficient. All the thermal resistance may, therefore, be assumed to be on the water side. Hence there will be a negligible temperature drop through the steam film and the metal, and the pipe may be assumed to be at steam temperature, which is 227°F.

$$\begin{aligned} \text{Steam temperature} &= 227^\circ \\ \text{Water temperature} &= 70^\circ \\ \Delta t &= 157^\circ \end{aligned}$$

The mean film temperature is halfway between 227 and 70, or 148.5°F. —say, 150°.

In Fig. 78, β, the *thermal expansion* of the fluid, appears. This may be defined by the equation

$$\beta = \frac{V_1 - V_2}{V\Delta t} \tag{93}$$

where

V_1 = specific volume of the fluid at a temperature t_1.
V_2 = specific volume at a temperature t_2.
V = specific volume at the temperature at which the coefficient of thermal expansion is desired.
$\Delta t = t_2 - t_1$.

In order to obtain a reasonable accuracy, t_1 and t_2 should be chosen rather far apart on either side of the desired temperature. For this case, where the coefficient of expansion at 150° is desired, take t_1 at, say 160°F., and t_2 at 140°F. Then, noting that $V = 1/\rho$,

$$\begin{aligned} V_2 &= \frac{1}{61.38} \\ V_1 &= \frac{1}{61.00} \\ V &= \frac{1}{61.20} \\ t_1 &= 160 \\ t_2 &= 140 \end{aligned}$$

whence

$$\begin{aligned}
\beta &= 0.000337 \\
D &= 2.375 \text{ in.} = 0.198 \text{ ft.} \\
\rho &= 61.20 \\
g &= 4.18 \times 10^8 \\
\Delta t &= 157 \\
C_p &= 1.00 \\
\mu_f &= 0.433 \times 2.42 = 1.048 \\
k_f &= 0.381
\end{aligned}$$

The abscissa of Fig. 78 is, for this problem

$$\log \left[\frac{(0.198)^3 \times (61.20)^2 \times 0.000337 \times 4.18 \times 10^8 \times 157}{1.048 \times 0.381} \right] = 9.206$$

From Fig. 78, where the abscissa is 9.21, the ordinate is 1.93. Hence

$$\begin{aligned}
\log Nu &= 1.93 \\
Nu &= 85.1 \\
\frac{h \times 0.198}{0.381} &= 85.1 \\
h &= 164 \text{ B.t.u./sq. ft. °F. hr.}
\end{aligned}$$

Example 14.—15,000 cu. ft. of air per min. are to be heated from 60°F. to 120°F. by a tempering coil. One unit base has openings for pipes 4 rows deep and 22 wide. The pipes are 1-in. standard iron pipe on 2⅜ in. centers, 6 ft. 0 in. high. The casing is 54 in. wide by 6 ft. 2 in. high. Neglect horizontal pipes at top of coil. Inside pipes is steam at 5 lb. per sq. in. gage. How many units are needed?

Solution.—Gross cross-section of casing

$$54 \times 74 = 3{,}996 \text{ sq. in.}$$

1-in. pipe is 1.315 in. outside diameter.

Projection of pipe, 22 × 1.315 × 72 = 2,083 sq. in.

Net cross-section, 3,996 − 2,083 = 1,913 sq. in. = 13.28 sq. ft.

$$\rho = \frac{29}{359} \times \frac{460 + 32}{460 + 60} = 0.07644 \text{ lb. per cu. ft.}$$

$$v_{max} = \frac{15{,}000 \times 60 \times 0.07644}{13.28} = 5{,}180 \text{ lb. per sq. ft. per hr.}$$

$$\Delta t_m = \frac{167 - 107}{2.30 \log \frac{167}{107}} = \frac{60}{0.4446} = 135\text{°F.}$$

$$t_f = t_w - \tfrac{1}{2}\Delta t_m = 227 - 67.5 = 159.5, \text{ say } 160\text{°F.}$$

In Reiher's equation:

$$D = \frac{1.315}{12} = 0.1096$$

$$k_f = 0.0129 + (160 - 32)(0.000020) = 0.01546$$

$$\mu_f = 0.020 \times 2.42 = 0.0484$$

$$h = \frac{0.131 \times 0.01546}{0.1096}\left[\frac{0.1096 \times 5{,}180}{0.0484}\right]^{0.7} = 13.04$$

Specific heat of air at mean temperature (90°F.) is 0.241.

$$q = 15{,}000 \times 60 \times 0.07644 \times 0.241 \times 60 = 994{,}700 \text{ B.t.u. per hr.}$$

$$A = \frac{994{,}700}{13.04 \times 135} = 565.2$$

1-in. pipe has 1 sq. ft. outside surface for 2.904 running feet.

$$\text{Area 1 unit} = \frac{22 \times 4 \times 6}{2.904} = 181.8 \text{ sq. ft.}$$

$$\text{Units needed} = \frac{565.2}{181.8} = 3.1 \text{ or } 3 \text{ units.}$$

RADIATION

Radiation is an important method in practice for transferring heat, but unfortunately the precise calculation of most practical cases is too difficult for the purposes of this book.[1] There are certain valuable qualitative deductions that can be made from the simple theory.

Stefan's Law.—The fundamental law covering the transfer of heat by radiation is Stefan's law:

$$q = bAT^4 \tag{94}$$

where

A = the area of the radiating surface.
T = its absolute temperature in degrees Rankine.

The value of b is 1.72×10^{-9} B.t.u. per sq. ft per (°R)^4 per hr.

All bodies at all temperatures above absolute zero radiate energy according to Stefan's law. Consider a small body of area A and temperature T_2 completely surrounded by a hotter body of temperature T_1. The net amount of heat transferred from the hotter body to the colder body is therefore the algebraic sum of the radiation from the two bodies, so that Stefan's law may be written for this case as

$$q = bA(T_1^4 - T_2^4) \tag{95}$$

This assumes, however, that all of the heat radiated by the cooler body falls on the hotter and is absorbed by it.

[1] See McAdams, Chap. III; Perry, pp. 878–910; Schack, Goldschmidt, and Partridge, "Industrial Heat Transfer," pp. 173–207. John Wiley & Sons, Inc., 1933.

No actual substances fulfill this last assumption. Instead of absorbing all of the radiant energy that falls on them, they absorb only a part of it and reflect the rest. An ideal *black body* is one which reflects none of the energy falling on it. Most objects with a matte black surface approach the theoretical black body closely enough for practical purposes. Polished metal surfaces and white bodies deviate widely from the theoretical black body. It may be shown, however, that inside a closed space where the temperature is uniform throughout, all substances, no matter what their color or surface, behave as black bodies. This greatly simplifies many practical cases, and insofar as the interior of a furnace may be considered uniform in temperature, to such an extent may all the substances in it be considered black, irrespective of their nature.

Effect of Temperature.—One qualitative deduction of great practical importance that can be made from Stefan's law is that the heat transferred by radiation from one body to another depends on the difference of the fourth powers of the absolute temperatures. As the temperature of a body is raised above its surroundings, the amount of heat it can radiate to them increases at an enormous rate. If the cold surface is a water-cooled wall at 212°F. (672°R.), and the radiating body is at 1000°F. (1460°R.), the rate of heat transfer is proportional to

$$(1460^4 - 672^4).$$

If the temperature of the hot surface is raised to 1100°F., the radiation is proportional to $(1560^4 - 672^4)$. This represents an increase of 32 per cent.

Radiation from Gases.—Hot gases lose heat by radiation, but in a different manner than do hot black bodies. A black body liberates radiant energy over a continuous range of wave lengths. If the intensity of the radiation is plotted against the wave length, a smooth curve results. Hot gases, on the other hand, lose radiant energy in *bands*, *i.e.*, the plot of intensity versus wave length shows little or no radiation except in a few, fairly definitely bounded intervals of wave length. At ordinary temperatures the heat lost from a gas by radiation is small, but at higher temperatures a very large proportion of the heat lost by carbon dioxide, carbon monoxide, water vapor, or hydrocarbons is lost by radiation. Other diatomic gases do not radiate appreciable amounts

at any temperatures ordinarily reached. Such radiant heat transfer is, of course, independent of a gas film on the solid body receiving the radiation. Although methods have been developed for estimating the heat lost by gases under such conditions, these methods are beyond the scope of this book.[1]

Heat Transfer by Combined Radiation and Convection.—Consider any apparatus heated by direct fire, such as a steam

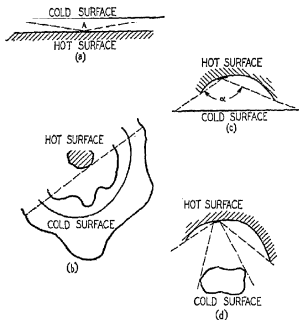



FIG. 83.—Angle of vision in radiant-heat flow.

boiler, a pipe still, or a fusion pot. Heat is transferred to the metal wall in four ways: by radiation from incandescent solids (fuel bed, brickwork, solid carbon in luminous flames), by radiation from hot gases, by conduction, and by convection. The radiation passes to the metal unhampered by any resistance in the viscous film and independent of the velocity, density, or other characteristic of the gas stream. Any quantity of heat desired may be introduced per unit area of the metal surface, dependent only on the temperature of the hot solid or gas and the area of hot surface exposed to unit area of cold surface. On the other hand, the same metal surface is also receiving heat by conduction and convection from the hot gases. This heat must pass through the viscous film and is therefore dependent on

[1] HOTTEL, *Trans. Am. Inst. Chem. Eng.*, **19**, 173 (1927).

all the properties of the gas stream involved in Equation (69). If it were not for heat received by radiation, the extent of surface necessary for steam boilers and other fire-heated apparatus would be exceedingly large. Consequently, it follows that all apparatus to be heated by direct fire should be so placed that some heat is transferred by radiation. In some cases this rule must be violated because of physical considerations such as mechanical strains which would be involved by direct exposure to high temperatures, but the general principle is that high-temperature heat may be transferred by radiation more easily than by any other method.

Angle of Vision.—Imagine a hot plane surface of indefinitely large extent (Fig. 83*a*) and consider a small element of this surface. Let the area of the element be A. This element is radiating heat to cooler surroundings according to Equation (94). Obviously this radiation extends from the point of origin in every direction. In order to intercept all of this radiation, a cooler body would need to subtend an angle of approximately 180 spherical deg. from the element A as a center.

This line of reasoning may be extended to other shapes than plane surfaces, if it is remembered that each element of the hot body is radiating heat in straight lines in every direction except back into the body. To absorb all this heat, therefore, a cold body must everywhere intercept these rays. This may be most simply expressed by saying that the hot body must be able to *see* only the cold body.

The flow of radiant heat is exactly analogous to the flow of light; qualitatively, one may follow the path of radiant heat as one would follow the path of light. If any non-transparent third body is interposed between the hot one and the cold one, it would cast a shadow on the cold body and prevent its receiving all the light leaving the hot one, and similarly it will prevent the cold body receiving all the heat from the hot one.

These considerations are illustrated in Fig. 83. In case a the unit of the hot surface can see practically nothing but the cold surface, provided both planes are of indefinitely large extent compared to their distance apart. In case c the unit of the hot surface sees the cooler surface only over the solid angle α, and through the rest of its range of vision it sees other parts of the hot surface. In d only a part of the radiation from the hot ele-

ment is received by the cold body. The balance is absorbed by the extension of the hot body or is absorbed by some distant undetermined background.

It makes no difference what the extent or distance of the cold surface may be, provided all of the cases fulfill the condition that the hot surface can see only cold surfaces. Thus in Fig. 83*b* a unit of the hot surface is radiating a certain amount of heat. This will all be received by the cold surface, no matter which one of the three positions the cold surface may occupy. The amount of heat received *per unit area* of cold surface will be different, however, in the three cases. If, instead of considering the heat radiated from a unit of hot surface, the considerations are to be based on the heat received by a unit of cold surface, the words *hot* and *cold* in Fig. 83 may be interchanged and the same qualitative conclusions hold.

From this reasoning a certain qualitative factor of design may be deduced. The total heat received by the cold body in several different positions will always be the same, provided the temperatures of both the hot and cold bodies are constant and provided the cold body can see only the hot one. The amount of heat received per unit of area of the cold body will be less as the distance from the hot body and its extent are increased. If, therefore, too great a rate of heat input would heat the cold body (say a tube in a pipe still) to too a high a temperature, by moving the tubes farther away from the source of heat and installing more of them so that they still subtend the same angle, nearly the same total amount of heat will be received, but the amount of heat per unit area (and therefore the temperature of the metal) will be greatly decreased. The cold body, such as a boiler tube, a kettle, or any other fire-heated apparatus, should be so located in the setting that the parts that are to receive heat are surrounded as nearly as possible by hot surfaces.

Projecting fins or ribs that tend to increase the cold surface do not absorb an increased amount of heat by radiation unless they increase the angle of vision of the cold surface, but they may decrease the amount of heat absorbed per square foot of cold surface.

Radiation Errors in Thermometry.—One case of heat transfer by radiation, which is often overlooked, is a source of serious error in measuring the temperatures of hot gases. Ordinary gases,

free from smoke or visible flames, are practically transparent to radiation. When such gases are flowing through a conduit of any sort, the wall of the container is usually much cooler than the average of the gas stream. If any type of temperature-measuring instrument is inserted into the gas stream, as its temperature approaches that of the gas this temperature becomes higher than that of the conduit wall. The instrument, therefore, immediately begins to radiate heat to such walls at nearly the maximum rate because it can everywhere see the colder wall. Heat is transmitted to the measuring instrument by convection, and therefore at a relatively slow rate; while heat is lost by radiation, which is not affected by any gas-film coefficient.

In such cases, the temperature-measuring device will always be colder than the gas stream. The temperature difference between it and the gas will supply heat to it by convection as fast as it loses heat to the conduit by radiation. Because of the fact that temperatures enter Stefan's equation to the fourth power, radiation is relatively more rapid at higher temperatures, and therefore the error is greater as the temperatures to be measured are higher. Various means are available to reduce this error (though it never can be entirely eliminated) such as radiation shields, high gas velocities, etc. If the shields can be made of a substance that deviates widely from black-body conditions (such as polished metals), their loss by radiation will be cut down and they will then approximate more nearly the gas temperature. If the velocity of the gas past the shield be made as rapid as possible, the rate of heat transfer from gas to shield will be increased and therefore the temperature difference between gas and shield will be decreased. The measuring instrument itself, whether it be the bulb of the thermometer, the junction of a thermocouple, or other device, should be, if possible, of bright metal to make it deviate as far as possible from a black body for the same reasons that the shield should be made bright.[1]

HEATERS

The commonest heating problem in the ordinary chemical plant is that of transferring heat from one fluid to another through a metal wall. The fluids involved may both be liquids,

[1] HASLAM and CHAPELL, *Ind. Eng. Chem.*, **17**, 402 (1925).

may both be gases, or the case may be one of heat transfer from a liquid to a gas or *vice versa*. A very common case is that of transferring heat from condensing steam to a liquid. The important special case of this method of heat transfer, namely from condensing steam to a boiling liquid, is so important that it is given a special name—evaporation—and Chap. V will be devoted to it. The other cases, such as the transferring of heat from condensing steam to non-boiling liquid, from hot liquid to cold liquid, and from steam to fixed gas, will be considered in this section.

The basic principles of heat transfer have been discussed. The types of apparatus to be considered are best understood by

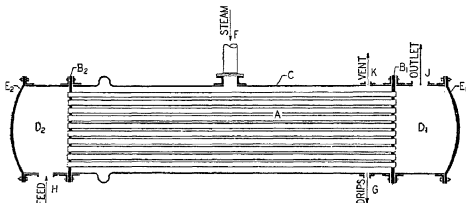



FIG. 84.—Single-pass tubular heater. *A*. Tubes. B_1, B_2. Tube sheets *C*. Shell. D_1, D_2. Liquor-distribution chambers. E_1, E_2. Covers. *F*. Steam inlet. *G*. Condensate outlet. *H*. Liquor inlet. *J*. Liquor outlet. *K*. Non-condensed gas vent.

interpreting the features of their design in terms of these theoretical principles. Three of the more important types of heat-transfer equipment are, first, tubular heaters; second, double-pipe heat exchangers; and, third, tempering coils.

Tubular Heaters.—The simplest form of tubular heater is shown in Fig. 84, which illustrates a single-pass tubular heater. It consists essentially of a bundle of parallel tubes *A*, the ends of which are expanded into two tube sheets B_1, B_2. The tube bundle is surrounded by a cylindrical casing *C* and is provided with two distribution chambers D_1, D_2, one at each end, and two covers E_1, E_2. Steam or other vapor is introduced by connection *F* into the space surrounding the tubes, condensed vapor is drained at *G*. The fluid to be heated is pumped into

one distributing head through connection H, flows through the tubes and to the other distributing head, and is discharged at J. Any non-condensed gases accompanying the vapor are removed at K. The advantage of this type of construction is that large heating surfaces can be packed into a small volume. Accordingly, by Equation (89), the capacity of the heater, which is measured by q, can be made large because of the magnitude of the A term.

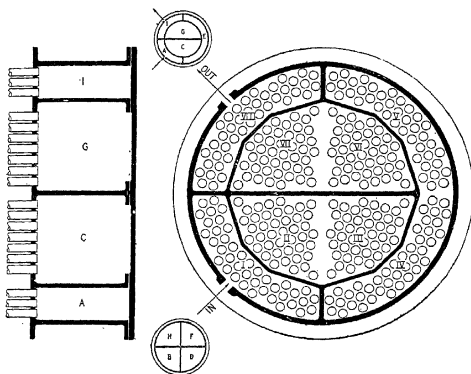



FIG. 85.—Multipass heater.

A consequence of putting a large surface into a heater of the type of Fig. 84 is that the cross-sectional area of the tubes is also large, and the velocity of the fluid through these tubes is low. This results in a low coefficient of heat transfer. This can be remedied by using a *multipass heater*, an example of which is given in Fig. 85. In this type of construction the fluid is diverted by means of baffles placed in the distributing heads so that it enters only a fraction of the tubes and thus passes back and forth through the heater several times before it leaves. Figure 85 shows in detail one distributing head with its partitions and the arrangement of the tubes. The other distributing head has a

different set of partitions, and the relation of the two is seen by a comparison of the two small plans. The liquid enters compartment *A*, flows to the left into compartment *B*, back to the right to compartment *C*, and so on till it finally leaves at *I*. This results in the tubes falling into eight groups or passes, shown by Roman numerals in the large plan. In the odd-numbered passes the liquid is flowing away from the reader, in the even-numbered ones it is flowing toward him.

Multipass construction decreases the cross-section of the fluid path and increases the fluid velocity, with a corresponding increase in the heat-transfer coefficient. The disadvantages are, first, that the heater is slightly more complicated; and, second, that the friction drop through the apparatus is increased because of the effect of velocity on friction drop and the multiplication of exit and entrance losses. The most economic design will call for such a velocity in the tubes that the increased cost of power for pumping is offset by the decreased cost of apparatus. Too low a velocity saves power for pumping but calls for an unduly large (and consequently expensive) heater. Too high a velocity saves on the first cost of the heater but more than makes up for it in the cost of power. The calculation of such an economic balance is outside the scope of this book.

Expansion.—Due to the differences in temperature that exist in heaters, expansion strains may be set up that may be severe enough to buckle the tubes or pull them loose from the tube sheets. Many heaters have a cast-iron shell but relatively thin-walled tubes. When the heater is put into or out of service, the shell heats or cools more slowly than the tubes, and the resultant strains might result in failure. The commonest method for avoiding this is the so-called *floating-head* construction, in which one of the tube sheets (and therefore one end of the tubes) is structurally independent of the shell. A two-pass floating-head heater is shown in Fig. 86. The construction is obvious, and the figure shows how the tubes may expand or contract, independent of the shell. A perforated plate is shown over the steam inlet, to prevent cutting the tubes by entrained water. Floating-head heaters may be made in multipass arrangements, but the form shown is very common. Another way of obtaining more than two passes with a floating-head arrangement, but with the two-pass construction of Fig. 86, is to make the number

of tubes per pass as small as desired and then, instead of putting more than two passes in one shell, to connect several such heaters as Fig. 86 in series. A method of allowing for expansion when the heater shell is made of sheet metal is to roll a bulge in the shell as shown in Fig. 84.

Heat Interchangers.—While the term *heater* covers many devices for transferring heat from one fluid to another, ordinarily this term is reserved for those cases where heat is transferred from condensing vapors to a liquid. The heaters of Figs. 84, 85, and 86 are primarily designed for this purpose. In such cases the film coefficient on the steam side of the heating surface is usually much larger than the film coefficient on the liquid side,

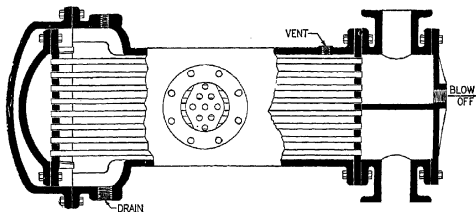



FIG. 86.—Two-pass floating-head heater.

and therefore in these heaters the emphasis has been placed on high liquid velocities. The cross-section of the space outside the tubes is large and the steam velocity is low, but because of the high values of the steam-film coefficient this is not a disadvantage.

When heat is to be transferred from one liquid to another or one gas to another the apparatus is usually known as a *heat interchanger*. In this case, the film coefficients both outside and inside the tube are of the same order of magnitude. Since the value of the overall coefficient U will be near that of the smaller of the two film coefficients, increasing one film coefficient without increasing the other is not proportionally effective in increasing U. Consequently, in heat interchangers it is desirable to increase the velocity of the fluid outside the tubes as well as that inside the tubes. Because of structural considerations it is

rarely possible to space the tubes in the tube sheet so closely that the area of the path outside the tubes will be as small as that inside the tubes, and therefore the velocity of the fluid outside the tubes will be low in such constructions as Figs. 85 and 86. To remedy this condition, baffles are placed outside the tubes to lengthen the path and decrease the cross-section of the path of the second fluid. Such a construction is shown in Fig. 87.

In this construction the baffles, *A*, consist of circular discs of sheet metal with one side cut away. These sheets are perforated to receive the tubes, *B*. The baffles are held in place by means of one or more guide rods, *C*, which are fastened between the tube sheets $D - D'$ by means of set screws. To keep the baffles properly spaced, short sections, *E*, of the same tubing

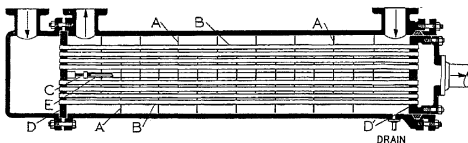



Fig. 87.—Liquid-to-liquid heat interchanger. *A*. Baffles. *B*. Tubes. *C*. Guide rod. *D*, *D'*. Tube sheets. *E*. Spacer tubes.

as is used in the rest of the heater, are slipped over the rod *C* between the baffles. In assembling such a heater, it is usual to assemble the tube sheets, spacer rods, and baffles first, and then to install the tubes. The construction of Fig. 87 shows another form of floating head in addition to that of Fig. 86. This baffling, it will be noted, not only increases the velocity of the liquid outside the tubes but also causes it to flow more or less at right angles to the tubes. This causes an added turbulence which aids in reducing the resistance to heat transfer outside the tubes. By devices such as this, the two film coefficients can both be made high, and therefore the value of the overall coefficient *U*, correspondingly increased.

Double-pipe Heat Interchangers.—When conditions are such that the relation between the volume of liquid inside the tubes, the velocity desired, and the size of the tube desired result in but a few tubes per pass, the simplest construction is the double-pipe heat interchanger shown in Fig. 88. This consists of special

fittings that are attached to standard iron pipe so that one liquid flows through the inside pipe and the second liquid flows through the annular space between the two pipes. Such a heat interchanger will usually consist of a number of passes which are almost invariably arranged in a vertical stack. If more than one pipe per pass is indicated, the proper number of such stacks are connected in parallel.

Tempering Coils.—When air is to be heated by means of steam, the device known as the tempering coil has become almost standard. The tempering coil consists essentially of a hollow cast-iron base into which steam can be introduced. Rising from this base are a number of loops of one-inch pipe, and the base is so

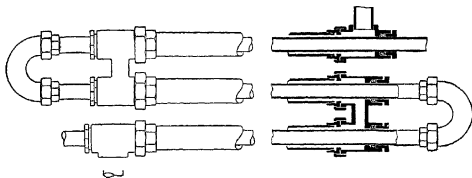



FIG. 88.—Double-pipe heat interchanger.

arranged that steam flows up through one side of each loop and down through the other. One or more such bases are arranged in parallel and enclosed in a sheet-metal housing, so that the air to be heated may be blown at right angles to the coils.

There are several well-known standard constructions, one of which is shown in Fig. 89. The cast-iron base is divided by a vertical partition *A* extending to the rear. Two rows of one-inch pipe *B* rise from one side of this partition and the loops terminate on the other side of the partition. Steam is introduced at *C* and condensate withdrawn at *D*. The proportions of these bases and the spacing of the pipes are all standardized, so that the heating surface per unit may be taken from tables in manufacturers' catalogues. If more heating surface is desired than can be obtained in one unit base, a number of these are placed side by side and connected in parallel to common steam and condensate headers.

In such a case as the tempering coil, the coefficient on the steam side may be as much as a hundred times as high as the coefficient on the air side. This means that the conditions on the air side control the rate of heat transfer. It is not possible to increase air velocities indefinitely, because of the power consumed by friction at high velocities. It is possible, however, to increase the area of the heating surface on the air side without correspondingly increasing the area on the steam side. This is done by putting fins of sheet metal, either circular or spiral, on the air side of the tubes. These fins are either pressed or sweated on

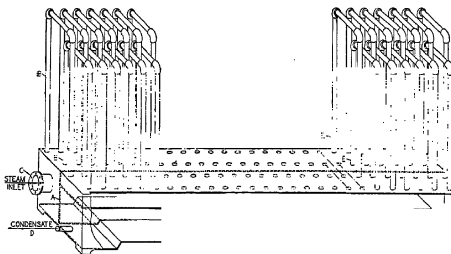



FIG. 89.—Tempering coil. *A*. Division plate. *B*. Inlet side of pipe loops.

to the tube so that there is good metallic contact. This doubles or triples the area of the air film and increases the turbulence in the air stream, thereby increasing the rate at which heat passes from the tube to the air. The increased length of path that the heat must travel by conduction through the metal of the fins offers an insignificant resistance, because of the low thermal resistance of all ordinary metals as compared with the resistance of the air film. Such fins, therefore, give rates of heat transfer several times as great as from plain pipe under the same conditions.

Nomenclature of Chapter IV

A = area in square feet
a = constant
b = constant

C or c = specific heat
D = diameter
g = acceleration of gravity
h = film coefficient
K = constant
k = thermal conductivity
L = thickness in feet
N = length in feet
Q = quantity of heat in B.t.u.
q = rate of heat flow in B.t.u. per hour
R = resistance
r = radius
T or t = temperature
Δt = temperature drop in degrees, Fahrenheit
U = overall coefficient
u = linear velocity
V = specific volume
v = mass velocity
W or w = weight
Gr = Grashof group
Nu = Nusselt group
Pr = Prandtl group
Re = Reynolds group
exponents a, b, c = constants
sub av = average
sub f = at mean temperature of viscous film
sub m = logarithmic mean
sub max = calculated on minimum cross-section
sub T = sum of radiation and convection
β = coefficient of thermal expansion
θ = time in hours
μ = viscosity
ρ = density
ψ = function from curve of Fig. 77
λ = latent heat of vaporization

Problems

1. An evaporator has condensing steam at 220°F. on one side of the heating surface and boiling liquid at 150°F. on the other side. The heating surface is iron tubing, 1.50 in. outside diameter, with a wall 0.095 in. thick. What per cent of the total temperature drop takes place across the metal when the overall coefficient (based on the outside area) is (*a*) 100, (*b*) 300, (*c*) 1,000 B.t.u. per sq. ft. per °F. per hr.?

2. A certain furnace has a wall composed of 4 in. of fire brick, 2 in. of Sil-o-cel brick, and 10 in. of common brick. The inside of the furnace is at 1400°F. and the outside surface of the wall is at 110°F. What is the loss in B.t.u. per square foot of wall?

3. A 3-in. standard iron pipe is carrying 50 lb. per min. of superheated steam at 140 lb. per sq. in. absolute and 450°F. It is covered with a layer

of 85 per cent magnesia 2 in. thick. The outer surface of the covering has a temperature of 200°F. What is the loss in B.t.u. per linear foot of pipe?

4. A tubular heater has iron tubes 1 in. outside diameter and 16 B.W.G. wall. The steam-film coefficient is 2,000 and the water film is 300. Steam is outside the tubes. What is the overall coefficient based on the inside area? What would be the effect of (*a*) substituting copper tubes 16 B.W.G. thick, (*b*) of raising the steam-film coefficient to 2,500, (*c*) of raising the liquid-film coefficient to 400?

5. Carbon dioxide at a pressure of 30 lb. gage and a temperature of 100°F. is to be cooled to 60°F. by passing it through a coil of copper pipe 1 in. outside diameter and ⅞ in. inside diameter. This coil is surrounded by cooling water that enters at 40°F. and leaves at 50°F. If 1,300 cu. ft. of CO_2 (measured at 30 in. Hg and 60°F.) are to be cooled per hour, how many feet of pipe must there be in the coil? Assume that the water-film coefficient is 200 and that the water flows countercurrent to the gas.

6. A certain tempering coil design has 148.2 sq. ft. of effective heating surface per section. This is in the form of 1-in. standard iron pipe, spaced at the vertices of equilateral triangles 2⅛ in. on a side, center to center. The minimum free area for air passage is 12.1 sq. ft. The heater is supplied with steam at 5 lb. gage. How many sections in series would be required to heat 100,000 cu. ft. of air per hr. (measured at 40°F.) from 40 to 100°F.?

7. Water is flowing through a 2-in. outside diameter horizontal copper tube with walls 1⁄16 in. thick, outside which is steam at 760 mm. average pressure. The water enters the pipe at 50°F. and leaves at 150°F. when flowing at the rate of 5 ft. per sec. inlet velocity. Calculate the length of the pipe.

8. A double-pipe heat interchanger consists of standard 1¼-in. pipe inside 2-in. pipe. The effective length per pass is 20 ft. In the 1¼-in. pipe flow 850 gal. per hr. of saturated salt brine, entering at 60°F. In the annular space flow 1,100 gal. per hr. of hot water, entering at 220°F. and leaving at 120°F. How hot will the brine be heated, and how many passes will be required? The brine may be considered to have an average viscosity of 0.95 centipoise, a density of 75 lb. per cu. ft. at 60°F., and a specific heat of 0.79.

9. Oil is flowing through 2-in. standard iron pipe at a velocity of 0.8 ft. per sec. It is being heated by steam outside the pipe, and the steam-film coefficient may be taken as 2,000. At a particular point along the pipe the oil is at 120°F., its density is 55 lb. per cu. ft., its viscosity 2.1 centipoises, its thermal conductivity is 0.078 English unit, and its specific heat is 0.52. What is the overall heat transfer coefficient at this point? If the steam temperature is 270°F., what is the amount of heat transferred per square foot of outside surface per hour at this point?

10. Oil is flowing through a 2-in. standard iron pipe at the rate of 10 ft. per sec. It is to be heated from 80 to 200°F. by means of steam at 5 lb. gage. The mean specific gravity of this oil is 0.92, its mean specific heat is 0.47, and its thermal conductivity is 0.078, over this temperature range. Its viscosity is 40 centipoises at 100°F. and 5.4 centipoises at 200°F. It may be assumed that when the logarithm of viscosity is plotted against the

logarithm of temperature, a straight line will result. The steam-film coefficient may be assumed to be 2,150. How many feet of pipe are required?

11. Twenty-five hundred pounds per hour of saturated steam at atmospheric pressure is to be condensed and the condensate cooled to 80°F. The film coefficient for condensing steam may be taken as 1,500, and for cooling the condensate, 215. The steam is inside a copper tube 1⅛ in. outside diameter, 16 B.W.G. wall. Outside this is a 3-in. standard iron pipe. Cooling water enters the annular space at 60°F. and leaves at 190°F. How long must the pipe be, and how much water is used in gallons per minute?

12. An electrically heated wire, ⅛ in. in diameter, is suspended in a current of air at 70°F. that is moving across the wire and at right angles to it at a velocity of 100 ft. per sec. It is desired to maintain the wire at 300°F. What must be the input of energy in watts per foot of wire?

13. A thermocouple is inserted in a 2.07-in. diameter steel pipe so that the leads are at right angles to the center line of the pipe and the junction is in the center. Heated air passes through the pipe at a linear velocity of 5 ft. per sec. and at a temperature of 500°F. The inside surface of the pipe has inserted in it a second thermocouple which indicates a temperature of 200°F. Estimate the temperature indicated by the first thermocouple, using the following assumptions:

The film coefficient.................................. 3
The relative blackness of the thermocouple junction bulb.. 0.9

14. A steam boiler is raising steam at such a pressure that the temperature of the boiling water is 350°F. The under side of the boiler is in sight of the fuel bed, which is at a temperature of 2500°F. (measured by an optical pyrometer). Under these conditions 80 per cent of the heat absorbed by the boiler is by radiation and the other 20 per cent by conduction and convection. If the temperature of the fuel bed is increased by 200°F., what percentage increase in the capacity of the boiler can be expected, assuming that the heat transferred into the boiler by conduction and convection in a given length of time is the same in both cases? The relative blackness of the boiler may be taken as 0.9.

CHAPTER V

EVAPORATION

One special case of heat transfer is of such importance that it warrants separate treatment. This is the transfer of heat from steam to boiling liquids. The conditions under which evaporation is carried out in practice are widely varied. The liquid to be evaporated may be more mobile than water, or it may be so viscous that it will hardly flow. It may deposit scale on the heating surface; it may precipitate crystals; it may tend to foam; or it may have a very high boiling point. This wide variety of problems has led to considerable variation in the types of mechanical construction used. Relatively little progress has been made toward a rigorous theoretical analysis of such apparatus, and consequently practical considerations and the customs of the various industries have influenced the design of evaporators to a considerable extent.

In order to carry out evaporation in the simplest possible manner, it would be necessary only to put the liquid in a container and transfer heat through the walls of the container to the boiling liquid. Such a simple device is not practical, and although the number of types of steam-heated evaporators is very great, any such evaporator must necessarily contain certain features. These features may be listed as follows:

1. A compartment for the boiling liquid.
2. A compartment for the condensing steam.
3. A heating surface through which heat flows from the steam to the boiling liquid.
4. An outlet for the vapors produced by the boiling liquid.
5. An inlet for the steam.
6. Provision for withdrawing condensate from the steam.
7. Means for withdrawing from the steam compartment gaseous impurities that are not condensed.
8. Means for charging and discharging the solution to be evaporated.
9. Suitable controls and instruments for checking the operation of the evaporator.

EVAPORATOR TYPES

Evaporator types may be classified as follows:

A. Apparatus heated by direct fire.
B. Apparatus with heating medium in jackets, double walls, etc.
C. Steam-heated evaporators with tubular heating surfaces.
- *a*. Tubes horizontal.
  - (1) Steam inside tubes.
  - (2) Steam outside tubes.
- *b*. Tubes vertical.
  - (1) Standard type.
  - (2) Basket type.
  - (3) Long-tube type.
  - (4) Forced-circulation type.
- *c*. Inclined-tube type.
- *d*. Tubes bent into special shapes such as coils, hairpin tubes, etc.

Fire-heated Apparatus.—The principal device that falls in this classification is the steam boiler. A discussion of steam-boiler design is outside the province of this book and is left to works on mechanical engineering. In those cases where the chemical engineer desires to evaporate liquids by the use of flue gases, the design has not been standardized into any definite types. The capacity of such apparatus may be calculated by methods given in the previous chapter.

Jacketed Apparatus.—When liquids are to be evaporated on a small scale, the operation is usually conducted in some form of jacketed kettle. These are available in a great variety of materials and constructions. The most common material of construction is cast iron, and for such kettles the jacket can be cast in one piece with the kettle, or it can be made of sheet metal attached to the outside. Kettles of aluminum, copper, and similar metals are widely used in the food-process industries, and in such cases the jacket is of sheet metal, riveted or welded to the body. Enameled ware also finds a wide use in the food processing, pharmaceutical, and fine chemical industries.

A typical kettle is shown in Fig. 90. The inner vessel or kettle proper is made of a single sheet of metal, in small sizes, or several sheets joined by rivets or brazing in larger sizes. The joint between jacket and kettle is usually made by riveting or brazing. To the bottom of the jacket is attached a pipe connection for condensate outlet and also a drain for the contents of the kettle.

An inlet for steam and an outlet for non-condensed gases are provided near the top of the jacket. The rate of heat transfer in such kettles may vary from 50 to 300 B.t.u. per sq. ft. per °F.

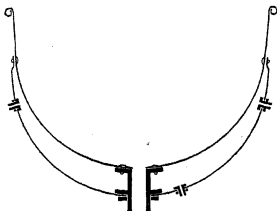



FIG. 90.—Steam-jacketed copper kettle.

per hr., depending upon the viscosity of the liquid being evaporated, the amount of agitation, and (to a lesser extent) the material of the kettle. Figure 91 illustrates some construction details for cast-iron or steel jackets.

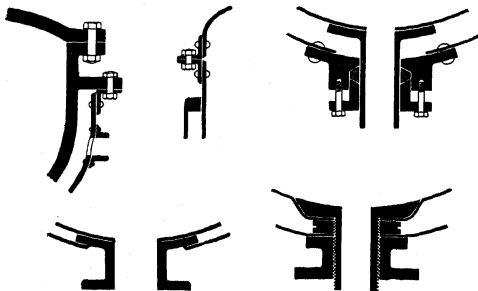



FIG. 91.—Details of kettle construction.

HORIZONTAL-TUBE EVAPORATORS

The first evaporator to receive general recognition was a design using horizontal tubes. It was built by Norbert Rillieux in Louisiana in 1843. This design did not survive long and was soon replaced by the vertical-tube types to be described in a

succeeding section. In 1879 the horizontal-tube evaporator was revived in the *Wellner-Jelinek* type. This was widely adopted, and most modern horizontal-tube evaporators are evolved from it. The name Wellner-Jelinek is still sometimes used for any horizontal-tube evaporator.

The Rillieux and Wellner-Jelinek machines had steam inside the tubes. In 1886 the Yaryan evaporator was developed, and this is the only horizontal-tube evaporator now in use with steam outside the tubes.

Steam inside the Tubes.—The usual type of horizontal-tube evaporator is shown in Fig. 92. The body of this evaporator is the liquor compartment and is in the form of a vertical cylinder. It is closed, top and bottom, with dished heads, although the bottom may be conical. The lower body ring is provided on opposite sides with steam compartments, closed on the outside by cover plates and on the inside by tube sheets. Between these tube sheets are fastened a number of horizontal tubes. The two steam chests with their connecting tubes form the steam compartment, and the tube wall is the heating surface. Steam is introduced into one steam chest at A, and as it flows through the tubes it washes non-condensed gases and condensate ahead of it, so that these are withdrawn from the opposite steam chest at B and C, respectively. It must be understood that in ordinary operation *only* condensate and non-condensed gases are removed from the exit steam chest. The connection for feeding the liquid to be evaporated may be attached to the body at any convenient point, such as D, but the discharge for thick liquor is usually in the center of the bottom E. Suitable brackets are cast on the bottom to rest on the supporting steel. Most evaporators are provided with sight glasses, such as shown at F in Fig. 92. The vapor from the boiling liquid escapes at G.

In the horizontal-tube evaporator, the tubes are almost invariably secured by a packing plate. This construction is shown in Fig. 93. A thick tube sheet is used, the holes are bored a little larger than the outside diameter of the tubes and are countersunk on the outer side. The tubes are cut long enough so that they project about an inch beyond the tube sheet at either end. Conical rubber gaskets are slipped over the tubes to fit into the countersink in the tube sheet. Each set of four tubes is secured by drawing down a packing plate which com-

presses the rubber gasket against both tube and tube sheet. This packing plate is held in place by a nut on a stud at the center of the four tubes. The main advantage of this construction is ease of tube removals.

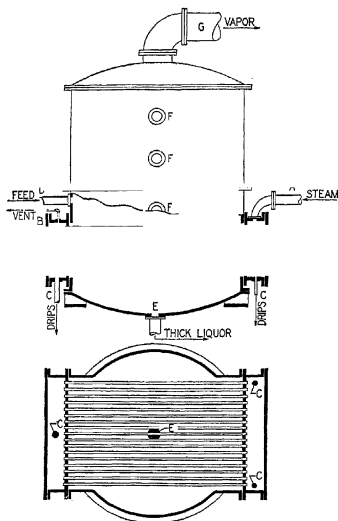



FIG. 92.—Horizontal-tube evaporator. *A*. Steam inlet. *B*. Vent for non-condensed gases. *C*. Condensate outlet. *D*. Liquor inlet. *E*. Liquor outlet. *F*. Sight glasses. *G*. Vapor outlet.

A horizontal-tube evaporator may be from 36 in. to 12 ft. in diameter, and the tubes are usually from 7/8 to 1¼ in. outside diameter. An average size for the body would be from 6 to 8 ft. in diameter and from 8 to 12 ft. high. This evaporator may be built of plate steel but is usually built of cast iron because

of the superior resistance of cast iron to moderate corrosion. The tube bank is usually shallower than it is wide, and its width is usually at least half the diameter of the body.

There are a number of minor modifications of the horizontal-tube evaporator, but these consist almost exclusively of changes in the shape of the body castings and not at all in the general arrangement or interrelationship of the parts. The horizontal-tube evaporator is best suited for non-viscous solutions, that do not deposit scale or crystals on evaporation. Its first cost per square foot of heating surface is usually less than that of the other types of evaporators.

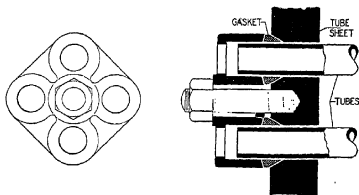



Fig. 93.—Packing plate for horizontal tubes.

Steam outside the Tubes.—The only representative of this class is the Yaryan evaporator, which is shown in Fig. 94. This evaporator consists of a cylindrical shell A closed by tube sheets B at either end. The shell may be from 8 to 20 ft. long. A number of tubes are fastened in these tube sheets by *rolling*. This term refers to a method of fastening a tube in a tube sheet by means of a tool containing rollers and inserted into the end of the tube. The rollers are so mounted that as the tool is rotated the tube is gradually expanded until it makes a tight fit with the tube sheet. Over these tube sheets are fitted cast-iron distributing plates C_1, C_2, which have return bends cast in them in such a way that the tubes are connected in series in groups of three or five. One of these distribution plates C_1 carries feed connections that register with the end of the first tube of each group. The other distribution plate C_2 has openings opposite the other end of each group. Around this end is built a large separating chamber D provided with baffles E. The shell is provided with an inlet

connection for steam F and outlets for non-condensed gases at H and condensate at G.

As the liquid is introduced into the feed end of each group of tubes, it begins to boil, and the volume of vapor so formed is

Fig. 94.—Yaryan evaporator. *A*. Shell. *B*. Tube sheets. C_1, C_2. Distributing plates. *D*. Vapor head. *E*. Baffles. *F*. Steam inlet. *G*. Condensate outlet. *H*. Non-condensed gas vent. *J*. Thick-liquor outlet. *K*. Vapor outlet. *L*. Liquor feed box.

extremely large as compared to the volume of liquid from which it is formed. As the liquid passes down the tube, therefore, the combined volume of steam and liquid becomes larger, the velocity is correspondingly greater, and a mixture of steam and spray issues from the end of each coil and strikes against the baffles that serve to separate vapor from liquid. The concentrated

liquid is drawn off at the bottom of the separating head at J, and the vapor is removed from the top at K. The liquor is not returned from J to the feed end but passes through the coils only once. In the construction shown in Fig. 94 all of the feed pipes are connected to a common feed box L, which is cast as a part of the lower edge of the feed-distributing plate C_1. The individual feed lines extending from this box to the various coils contain throttle valves or orifices to distribute the feed evenly over the various coils.

The Yaryan evaporator has never been widely used, but it has certain advantages. In the past it has been one of the most successful evaporators for handling foamy liquids, because the foam is quite completely destroyed by the high velocity with which the liquid issues from the coils and strikes against the baffles.

Vertical-tube Evaporators

Although the vertical tube was not the first evaporator to be built, it was the first type to receive wide popularity and is probably the most important type now in existence. The first one was built by Robert, director of the sugar factory at Seelowitz, Austria, about 1850, and the vertical-tube evaporator is often known as the *Robert type*. It became so common that in Europe this evaporator is known as the *standard evaporator* and is the construction always understood unless specific reference is made to some other type.

Standard Type.—A typical body is shown in Fig. 95. It is characterized by tube sheets A extending across the body, and a central downtake B. Tubes are rolled between these two tube sheets, and steam is introduced as shown, so that the liquor is inside the tubes and the steam is outside them. As the liquor boils, it spouts up through the tubes and returns through the central downtake. Condensate is removed from any convenient place on the bottom tube sheet such as C, and non-condensed gases are usually vented from somewhere near the upper tube sheet as at D. The exact position of feed E and discharge F is variable, but the positions shown in Fig. 95 are fairly typical. There may be many variations of this design; a conical bottom may be used instead of a flat bottom; and the proportions of tube length to tube diameter, ratios of body height to tube length,

and other details may be modified without altering the general principle of the evaporator shown in Fig. 95.

The first vertical-tube evaporators were built without a downtake. These were never satisfactory and the central downtake

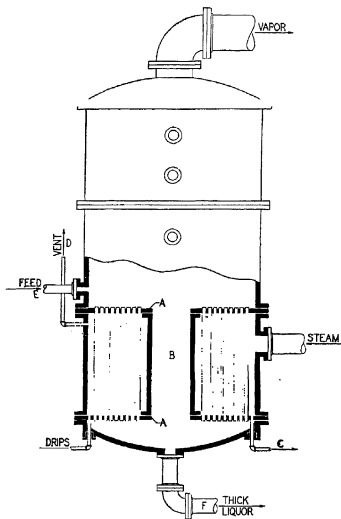



FIG. 95.—Standard vertical-tube evaporator. *A*. Tube sheets. *B*. Downtake. *C*. Condensate outlet. *D*. Non-condensed gas outlet. *E*. Liquor inlet. *F*. Thick-liquor outlet.

appeared very early. There have been many changes proposed in this arrangement, such as downtakes of different cross-section, downtakes located eccentrically, a number of scattered downtakes instead of one central one, downtake pipes entirely external to the body of the evaporator. and others. The central downtake,

however, remains a thoroughly standard construction. The *Scott evaporator* still uses scattered downtakes. The *Webre evaporator* has spiral baffles within the steam space to conduct non-condensed gases to one point. The cross-sectional area

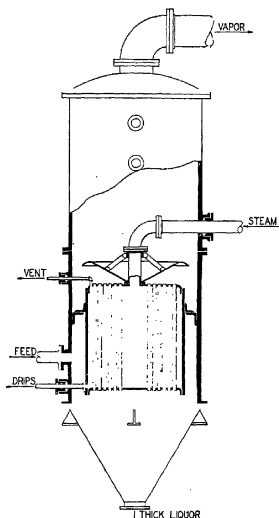



FIG. 96.—Basket-type vertical-tube evaporator.

of the downtake is usually between 75 to 150 per cent of the combined cross-section of the tubes. General practice is probably nearer the former figure than the latter one. The tubes may range from 1 to 4 in. in diameter, from 30 in. to 6 ft. long, with general practice favoring a tube about 2 in. in diameter and about 5 ft. long.

Basket Type.—The first basket-type evaporator was built in 1877. An example of this type is shown in Fig. 96. In this evaporator the liquid is inside the tubes, the steam is outside the tubes, but the downtake instead of being central is annular. In this construction the entire heating element is a single unit which may be removed bodily for repairs. The evaporator of Fig. 96 has a conical bottom, but flat bottoms are also common in this type. The tube proportions in the basket evaporator are about the same as in the standard vertical. The steam may be introduced as shown in Fig. 96, or the steam pipe may be brought straight down through the center of the top of the machine, or in rare cases it may be introduced through the side of the body in about the same location as the steam inlet of Fig. 95, but with a nipple passing through the downtake and connecting to the basket.

One important feature of the basket-type evaporator is the ease with which a deflector may be added in order to reduce entrainment from spouting. The boiling in the vertical-tube evaporator is quite violent, and this tends to cause entrainment losses. This condition is accentuated if the liquor level in the evaporator is low. A baffle such as shown in Fig. 96 largely prevents these losses and is much more easily added to the basket type than to the standard type. Other differences between the standard and the basket type are largely details of construction.

Long-tube Type.—It has long been recognized that high liquor velocity is desirable. This is obvious from the facts discussed in Chap. IV, where it was shown that the principal thermal resistance in the transfer of heat from a condensing vapor through a metal wall to a boiling liquid is in the liquid film and that this film is reduced in thickness by higher velocities. When very long tubes are used in a vertical-tube evaporator and low liquor levels are maintained, the pumping action of the bubbles formed is quite violent and the velocity of the liquid is high. The first important evaporator utilizing this effect was the *Kestner*, which was patented in 1899 and which has been very popular in Europe but has not been common in this country. The long-tube evaporator as now built in this country is shown in Fig. 97. It resembles the ordinary vertical-tube evaporator in that the liquid is inside the tubes and steam outside them.

Its characteristic feature is that the tubes are from $1\frac{1}{4}$ to $2\frac{1}{2}$ in. in diameter and 10 to 20 ft. long. The liquor level is maintained low, not more than 2 or 3 ft. above the bottom tube sheet. The mixture of steam and spray issues from the top of the tubes at a

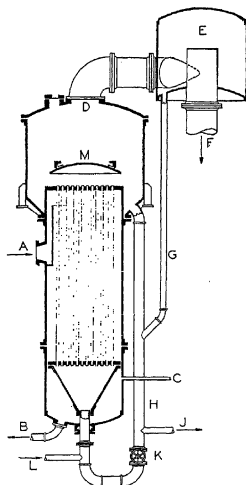



FIG. 97.—Long-tube natural-circulation evaporator. *A*. Steam inlet. *B*. Condensate outlet. *C*. Non-condensed gas vent. *D*. Vapor outlet. *E*. Entrainment separator. *F*. Vapor to condenser. *G*. Liquor return from separator. *H*. Liquor outlet pipe. *J*. Thick liquor discharge. *K*. Valve (open only when starting evaporator). *L*. Thin liquor inlet. *M*. Deflector.

high velocity and impinges on the deflector. This throws the liquid down into the lower part of the vapor head, from which it is withdrawn. This evaporator has no downtake and the liquid passes through the machine only once. The liquor level is automatically maintained, irrespective of the rate of feed.

Variations in rate of feed merely cause corresponding variations in final density.

Forced-circulation Type.—This evaporator is shown in Fig. 98. The tubes are longer and narrower than in the standard vertical (usually ¾ in. inside diameter and 8 ft. long) and are contained in a heating element A between two tube sheets. This tube bundle projects part way into the vapor head B. A return pipe C leads from the vapor head to the inlet of a pump D and this pump delivers the liquid to the tubes with a positive velocity. As the liquid rises through the tubes, it becomes heated and begins to boil, so that vapor and liquid issue from the tops of the tubes at a high velocity. This material strikes against a curved deflector E that throws the liquid downward in a sheet or curtain and makes an effective separation of liquid and vapor. The steam connection is near the bottom of the tube bundle. Just inside the shell of the tube bundle is a cylindrical baffle F that extends nearly to the top of the heating element. The steam rises between this baffle and the wall of the heating element and then flows downward around the tubes. This displaces non-condensed gases to the bottom, where they are removed at G Condensate is removed from the bottom of the heating element at H. This evaporator is especially suited for foamy liquids, for viscous liquids, and for those which tend to deposit scale or crystals on the heating surfaces. The vessel J is a salt separator and will be discussed later.

Fig. 98.—Forced-circulation evaporator. *A*. Heating-element shell. *B*. Vapor head. *C*. Downtake pipe. *D*. Circulating pump. *E*. Deflector. *F*. Steam baffle. *G*. Non-condensed gas outlet. *H*. Condensate outlet. *J*. Salt settler. *K*. Liquor-level indicator.

Inclined-tube Evaporators.—The only form of inclined-tube evaporator used in the United States was the *Buflovac,* which is now of minor importance.

Coil Evaporators.—There has been a wide variety of special evaporator constructions in which the tubes, instead of being straight, are coiled, U-shape, or deformed in some other manner. Few of these evaporators have had a wide use except the *Griscom-Russell evaporator,* which has been very popular for the preparation of distilled water for boiler feed. Another form of coil evaporator used in this country is the evaporator used for the final boiling of sugar juices and usually known either as a vacuum pan or a *strike pan.*

The sugar pan has a vertical, cylindrical shell from 8 to 12 ft. in diameter. Inside this shell are packed a number of coils of 4-in. copper pipe spaced as closely as possible. These coils are connected to a steam header outside the body of the pan and each coil is provided with its own control valve so that they may be operated individually. This form of evaporator possesses no unusual advantages and remains in the sugar industry merely because of tradition. It is not often found in Europe but is quite common in the United States.

EVAPORATOR CAPACITY

It has been shown in Chap. IV that the capacity of any heat-transfer apparatus is measured by the product of three factors: first, the area of the heating surface; second, the temperature drop across the heating surface; and, third, the overall coefficient.

The area of the heating surface is fixed by the construction of the evaporator. The actual temperature drop across the heating surface in an evaporator depends upon, first, the solution that is being evaporated; second, the difference in pressure between the steam space and the vapor space over the boiling liquid; and, third, the depth of the liquid over the heating surface. When the solution has the characteristics of pure water, its boiling point can be read from steam tables if the pressure is known, while the temperature of the condensing steam in the steam chest is also determined from the steam tables. In actual evaporators, however, two factors affect such a calculation to a greater or less extent. The first factor is boiling-point elevation and the second is hydrostatic head.

FACTORS AFFECTING TEMPERATURE DROP

Boiling-point Elevation. Dühring's Rule.—The first factor that influences the temperature drop is the property of solutions of exerting a vapor pressure lower than that of water at the same temperature. Consequently, for a given pressure, the boiling point of such a solution is higher than the boiling point of pure water. The boiling-point elevation may be as much as 150°F. for concentrated solutions of inorganic salts. The boiling-point elevation is totally lost from the available temperature drop across the heating surface. This will be best understood from a numerical example.

Suppose an evaporator is supplied with heating steam at 10 lb. gage, and a vacuum of 26 in. Hg (referred to a 30-in. barometer) is maintained in the vapor space. If the evaporator is boiling pure water, the temperature of the liquid corresponding to the vacuum is 125°F. The steam temperature is 239°F., so the available temperature drop is $239 - 125 = 114$°F.

If instead of water the evaporator is boiling saturated salt solution, the boiling point under a vacuum of 26 in. is not 125°F. but 137°F. Hence the available temperature drop is only 102°, a decrease of approximately 10 per cent as compared with water. More frequently evaporators are operated with a smaller pressure difference than in this example, so that the percentage loss is usually greater. Many solutions evaporated commercially have a greater boiling-point elevation than common salt, so that the effect of boiling-point elevation is correspondingly aggravated.

For strong solutions the boiling-point elevation cannot be calculated from any known law. A valuable empirical law is available, however, which is known as *Dühring's rule*. According to this rule, if the boiling point of a solution is plotted against the boiling point of pure water at the same pressure, a straight line results for a given concentration of solution. Different lines are obtained for different concentrations, but all of these lines are straight within the limits of precision of the available data, although they are not parallel. In Fig. 99 a set of Dühring lines is drawn for solutions of sodium chloride in water. The use of such a plot is best shown by an example. If the pressure over a 25 per cent sodium chloride solution is such that water boils at 180°F., then by reading up from the base at 180° to the

25 per cent curve and horizontally to the vertical axis it is found that the boiling point of the solution is 192°F. at this pressure, so that the boiling-point elevation for this solution is 12°F.

Thermal Effects of Boiling-point Elevation.—When a solution that has a negligible boiling-point elevation is to be evaporated,

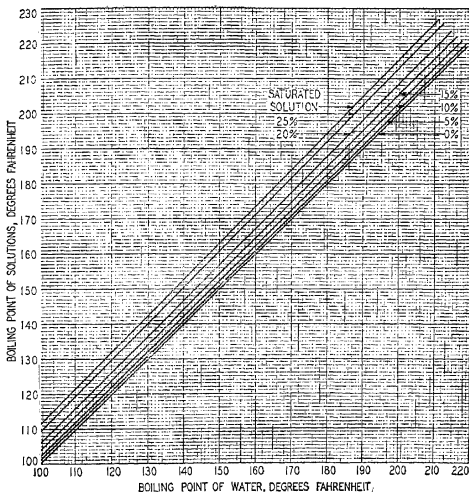



FIG. 99.—Boiling points of sodium chloride solutions.

the heat necessary to vaporize a pound of water from the solution is equal to that for water at the corresponding temperature and pressure. The vapor from such a solution is saturated. On the other hand, if the solution has an appreciable boiling-point elevation there are secondary thermal effects that tend, in general, to increase the heat required to drive a pound of water vapor from the solution as compared to what would be required for the

same evaporation from water at the same temperature. If adequate data on specific heats and heats of dilution are available, the calculation can be made rigorously.[1] Unfortunately, however, these data are not in general available and an approximation is necessary. The simplest method is to take the latent heat from the steam tables, but at the pressure under which the evaporation takes place, rather than at the temperature. Since the latent heat of water decreases with increasing temperature, this approximation gives a higher latent heat than would be the case for water at the boiling temperature of the liquid.

The Effect of Hydrostatic Head.—If there is an appreciable depth of liquid in an evaporator, the boiling point corresponding to the pressure above this liquid is the boiling point of the surface layers only. A particle of liquid at a distance X ft. below the surface will be under the pressure of the vapor space plus a head of X ft. of liquid and therefore will have a higher boiling point. In any actual evaporator, therefore, the average boiling point of the whole mass of liquid is higher than the boiling point that would correspond to the pressure in the vapor space. This increase in boiling point is at the expense of the temperature drop over the heating surface and therefore causes a decrease in capacity.

It is not possible to calculate quantitatively the effect of this elevation of boiling point on evaporator capacity. Consider a standard vertical-tube evaporator. As the liquid spouts out of the tubes it comes into equilibrium with vapor at the pressure of the vapor space and reaches the corresponding temperature. The natural circulation in the apparatus carries this liquid by way of the downtake to the bottom of the tubes. When it enters the tubes at the bottom, therefore, it is at the temperature of the vapor space and not at a temperature corresponding to the pressure at the bottom of the tubes. It is therefore below its boiling point. As it passes up through the tubes it becomes heated, but at the same time the pressure on it is diminishing. At some intermediate point in the tube it begins to boil and from this point to the top of the tube its pressure gradually drops to that of the vapor space. During this process the liquid flashes, and its temperature drops to that corresponding to the pressure. Since the temperature of the liquid in the various parts of this

[1] McCabe, *Trans. Am. Inst. Chem. Eng.* **31**, 129–164 (1935).

cycle cannot be determined or calculated, the numerical magnitude of the effect of hydrostatic head cannot be determined. The qualitative effect of hydrostatic head on temperature drop in the case of long tubes and high liquor levels should, however, be recognized.

Figure 100 shows the effect of liquor level on the capacity of a basket-type, vertical-tube evaporator like that of Fig. 96.[1] This curve was determined on an evaporator having tubes 30 in. long. It indicates how an increase in the liquor level apparently decreases the overall heat-transfer coefficient.

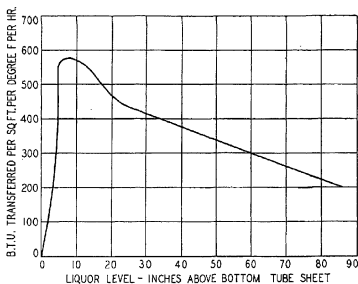



FIG. 100.—Relation between heat-transfer coefficient and liquor level in vertical-tube evaporator.

The decrease in capacity as liquor levels increase, is really the result of two factors. In the first place, higher liquor levels cause a decrease in the velocity of circulation and this, in turn, decreases the liquor-film coefficient. In the second place, higher liquor levels increase the effect of hydrostatic head and therefore decrease the true temperature drop. The rates of heat transfer represented by the curve of Fig. 100 were, however, determined by using the difference between steam temperature and the temperature of the liquid at the surface. By this method of calculation, therefore, the decrease in true mean temperature drop appears as an apparent decrease in the overall heat-transfer coefficient. It is impossible to separate these two

[1] BADGER and SHEPARD, *Trans. Am. Inst. Chem. Eng.* **13**, I, 139–149 (1920).

effects or even to tell which of these two effects has the greater influence. The above method of calculation is the usual practice in evaporator work and involves certain concepts that should be defined at this point.

Apparent Temperature Drop—Apparent Coefficients.—It is relatively easy to measure, with considerable accuracy, the pressure in the vapor space or the pressure in the steam space of an evaporator. It is not easy to measure, with a similar degree of precision, the temperatures at these points. Consequently, the temperature drop in an evaporator is usually determined, not from measured temperatures, but from temperatures calculated on the basis of pressures by means of the steam tables. If the temperatures of the liquid and of the steam are determined in this way, the difference between the two is called the *apparent temperature drop*, and the coefficient calculated by the use of this temperature drop is called the *apparent coefficient*. The ordinates of Fig. 100 are therefore apparent coefficients.

This method of calculation ignores the effect on temperature drop of the two factors of elevation in boiling point and hydrostatic head. It is based on the temperature drop that would exist if neither of these was present. Results calculated in this manner express the effect of these factors as lowered coefficients rather than as lowered temperature drops.

As pointed out above, the effect of hydrostatic head cannot be calculated and, therefore, the apparent heat-transfer coefficients cannot be corrected to true coefficients by making allowances for the effect of hydrostatic head. In many cases, however, the elevation of boiling point is known, or can be determined fairly accurately. In such cases, to the temperature calculated from the pressure in the vapor space there is added the elevation in boiling point, and this sum is then the true boiling point of the surface layer of the liquid. The difference between the steam temperature and this true surface boiling point is the *net temperature drop* or the *temperature drop corrected for elevation in boiling point.* Coefficients calculated by use of this temperature drop are known as *coefficients corrected for elevation in boiling point.* Such coefficients, rather than apparent coefficients, should be used wherever possible. This point will be discussed in detail on page 224.

The above considerations are illustrated graphically in Fig. 101. In this figure the ordinates are temperatures and the abscissas are distances along the heating surface. The steam to the evaporator may come in slightly superheated at t_s. The superheat is quickly given up and the steam drops to its saturation temperature t_1. Over the greater part of the heating surface it maintains this temperature unchanged. As the condensate issues from the evaporator it may be slightly cooled to some temperature such as t_c.

The temperature of the feed is represented by t_f. Although the diagram shows a part of the heating surface given over to heating the feed up to the boiling point, actually (in any ordinary

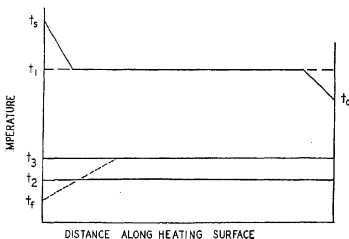



Fig. 101.—Temperature relationships in an evaporator.

evaporator that is operating properly) the amount of liquid in the evaporator is sufficiently large and the circulation is sufficiently violent so that the feed is rapidly stirred into the whole mass of liquid. At no point, except possibly in the immediate vicinity of the liquor intake, is the liquid temperature appreciably below the boiling point. For this reason the part of the curve representing the heating of feed is shown dotted.

The true boiling point of the surface layer is indicated by the temperature t_3. The temperature calculated from the pressure in the vapor space is t_2. As explained previously, the true temperature of the layers below the surface cannot be determined. The difference $t_3 - t_2$ is the elevation in boiling point. The difference $t_1 - t_2$ is the apparent temperature drop as defined above. The temperature difference $t_1 - t_3$ is the

temperature difference corrected for elevation in boiling point. The small triangle that represents the transfer of superheat is neglected, because unless the amount of superheat is very large indeed, turbulence in the steam space is sufficient to keep the tubes wet, and if the tubes are wet the effective temperature of the steam is its saturation temperature. The small triangle representing the transfer of sensible heat from the condensate is also neglected because the amount of heat involved is usually very small in comparison with the total amount of heat transferred.

It will be observed that both the apparent temperature drop and the temperature drop corrected for elevation in boiling point are rather arbitrarily defined. This is not an error, but a matter of definition. It follows, however, that coefficients calculated by the use of such arbitrarily defined temperature differences can be applied in practice only when the temperature drop to be used with them is defined in the same way. All overall heat-transfer coefficients for evaporators that have been published so far have been calculated in one of these two ways.

Factors Influencing Heat-transfer Coefficients

By referring to Equation (64) it will be seen that the total quantity of heat transferred may be affected by changes in temperature drop or by changes in the coefficient. The former has been discussed above. The various factors influencing the overall coefficient may be divided into two classes: those affecting the steam-film coefficient, and those affecting the liquid-film coefficient.

Steam-film Coefficient.—The effect on the steam-film coefficient of such factors as temperature drop, condensing temperature, and amount of non-condensed gas present has been discussed on pages 140 to 142. All these statements hold for evaporators. It will be of interest to examine the various types of evaporator in the light of these conclusions. The temperature drop and the condensing temperature are fixed by conditions of operation, not by evaporator construction. The removal of non-condensed gases, however, is definitely a function of evaporator construction. In the vertical-tube evaporator, whether it be of the central downtake type or of the basket type, the path of the steam is indefinite, steam velocities are relatively low, and the non-condensed gases are not well separated at any particular

point. If a number of thermometers are inserted in the steam space of a central downtake evaporator, it will be found that these temperatures fluctuate rapidly and erratically, often through several degrees. This is due to eddies and cross-currents sweeping pockets of non-condensed gases in an erratic path through the steam space.

In the horizontal-tube evaporator, the path of the steam is definite, its velocity is high, and therefore the non-condensed gases are positively washed into the condensate steam chest. Such evaporators as the long-tube vertical, the Yaryan, and the inclined-tube evaporator have long, cylindrical, steam spaces. If the steam is introduced at one end and the non-condensed gases removed at the other, the steam travel is fairly definite and the air removal quite complete. In the forced-circulation evaporator, the steam is given a definite path for the express purpose of washing the non-condensed gases to a definite point from which they may be withdrawn.

Liquid-film Coefficient.—The three most important factors that govern the magnitude of the liquid-film coefficient are (*a*) the velocity and (*b*) the viscosity of the liquid being evaporated and (*c*) the cleanliness of the heating surface. It has been shown in Chap. IV that the velocity of the fluid past the heating surface is of great importance in the rate of heat transfer by conduction and convection. This statement is true for the special case of a boiling liquid. In the ordinary type of evaporator, which depends upon the circulation due to natural convection, the velocity depends primarily on the shape and size and geometrical distribution of the heating surface with respect to the liquid compartment. Under such circumstances it is not possible to predict quantitatively the circulation of the boiling liquid. It is known that it depends upon such factors as viscosity, density, and the amount of heat that is passing through the heating surface per unit time, since this last factor determines the rate of bubble evolution and hence the vigor of boiling. It is not possible to separate these factors, and therefore no quantitative laws have been developed for this case.

Viscosity has its own specific effect in addition to that on the circulation or velocity. This effect is analogous to the effect of viscosity in the case of heat transfer from metal wall to non-boiling liquid. With a fixed velocity of liquid, the higher the

viscosity the thicker the resisting film and the lower the coefficient. Again, in the case of the ordinary natural-convection evaporator, it is not possible to separate the effect of viscosity from the other factors, since the viscosity cannot be changed without at the same time changing the circulation.

The cleanliness of the heating surface has been shown to have a surprising effect on the coefficient.[1] The rougher the heating surface the lower the coefficient. A very small increase in the roughness will bring about a remarkable decrease in the coefficient. It is rather hard to explain this, but it is probably due to an entangling effect on the fluid film.

The construction of the evaporator body has considerable influence on the liquid-film coefficient. For example, in the ordinary horizontal type, the circulation is mainly downward at one end of the tube nest and upward at the other. The arrangement of the heating surface in this type of evaporator is not particularly satisfactory for vigorous circulation. Under such conditions, therefore, the liquid-film coefficient is low, and the horizontal type usually gives a lower overall coefficient than does a vertical evaporator operating under the same conditions. In the standard vertical-tube evaporators, the boiling in the tubes develops a considerable circulation because of the acceleration due to the evolution of steam bubbles. The large downtakes, either central or annular, allow the circuit to be readily completed, and hence the circulation in a vertical-tube evaporator is relatively great in comparison with the horizontal type. The long-tube vertical evaporator has a still stronger pumping action, has higher velocities in the upper part of the tubes, and therefore higher coefficients. The liquid-film coefficient in the case of the forced-circulation evaporator will be much higher than in the case of natural convection. This is the basic reason for the use of forced circulation.

Overall Coefficients.—In spite of the fact that heat-transfer problems are best attacked by separating the overall resistance into the resistances of the individual films, this in general has not been done in the study of heat transfer in evaporators. The quantitative effects of the factors influencing liquid-film coefficients are practically unknown and the relative magnitudes of the film coefficients themselves have seldom been deter-

[1] Pridgeon and Badger, *Ind. Eng. Chem.* **16**, 474 (1924).

mined. The overall coefficient is the one that is always used in practice.

Figure 102 shows the effect of apparent temperature drop and boiling point of the solution on the apparent coefficient.[1] These curves were determined in a vertical-tube evaporator 30 in. in diameter, with a steam basket containing twenty-four 2-in. tubes 48 in. long. The liquid used was water. The coefficients increase with temperature drop, because the larger the temperature drop the greater is the quantity of heat passing through the heating surface and the more vigorous is the boiling. Thus,

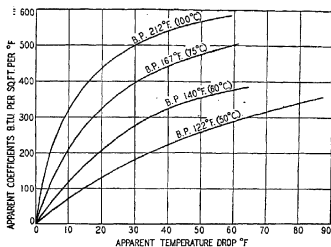



FIG. 102.—Relation between boiling point, temperature drop, and heat-transfer coefficient in a vertical-tube evaporator.

increasing the temperature drop in the evaporator increases the capacity, not only because of the increase of the Δt term in Equation (64) but also because of the increase in the coefficient. The coefficient increases with boiling point, due to the fact that the higher the boiling point the lower is the viscosity of the liquid.

The curves of Fig. 102 can be used to evaluate the heat-transfer coefficients at any boiling point and any temperature drop, for this particular evaporator, operating on distilled water or its equivalent. To extrapolate these data to other evaporators and other solutions is questionable, and for most cases the coefficients are determined as the result of experience.

[1] BADGER and SHEPARD, *Trans. Am. Inst. Chem. Eng.* **13**, I, 101–137 (1920).

EVAPORATOR CALCULATIONS—SINGLE EFFECT

There are three relationships available for the quantitative attack on evaporation problems. These relationships are

1. Material balances.
2. Heat balances.
3. The capacity equation

$$q = UA\Delta t \qquad (64)$$

where

q = the heat passing *through the heating surface* in B.t.u. per hour.
A = the area of the heating surface.
Δt = the temperature drop across the heating surface.
U = the overall coefficient based on the area A.

The applications of these quantitative tools are best demonstrated by means of typical examples.

Example 15.—It is desired to concentrate a solution of organic colloids from 10 to 50 per cent solids in a single-effect evaporator. Steam is available at 15 lb. per sq. in. gage (249°F.), and a vacuum of 26 in. Hg referred to a 30-in. barometer is to be maintained in the vapor space. This pressure corresponds to 125°F. The feed to the evaporator is 55,000 lb. per hr. The apparent overall heat-transfer coefficient can be taken as 500 B.t.u. per sq. ft. per °F. per hr. The condensate leaves the evaporator at 175°F. and the solution has a negligible elevation in boiling point.

Calculate the square feet of heating surface needed and steam consumption if the temperature of the feed is (*a*) 125°F.; (*b*) 70°F.; (*c*) 200°F.

The specific heat of the feed solution is 0.90, and the latent heat of evaporation can be taken equal to that of water. Radiation may be neglected.

Solution. *a. Feed Solution Enters at* 125°*F.*—The amount of water removed from the solution is calculated from the terminal concentrations and the weight of thin liquid. The feed solution contains $^{90}\!/_{10} = 9$ lb. of water per lb. of solid; the thick liquor contains $^{50}\!/_{50} = 1$ lb. of water per lb. of solid. The evaporation is, therefore, $9 - 1 = 8$ lb. per lb. of solid, or

$$(8)(55{,}000)(0.10) = 44{,}000 \text{ lb. per hr.}$$

and the thick liquid is $55{,}000 - 44{,}000 = 11{,}000$ lb.

A heat balance may now be written. Since the lowest temperature involved is 125°F., no attention need be paid to any heat below this temperature. In other words, 125°F. is the datum plane. Expressed in words, the heat balance is: (Heat entering in steam) + (heat entering in thin

liquor) = (heat leaving in vapor) + (heat leaving in thick liquor) + (heat leaving in condensate).

Let W be the pounds of steam used per hour. Then, from the steam tables, the latent heat of steam is 946 B.t.u. per lb. at 249° and 1,022 at 125°.

Heat entering in steam $= W[946 + (249 - 125)] = 1{,}070W$
Heat entering in feed = none (feed at datum temperature)
Heat leaving in vapor $= 44{,}000 \times 1{,}022 = 44{,}968{,}000$ B.t.u.
Heat leaving in thick liquor = none (thick liquor leaves at datum temperature)
Heat leaving in condensate $= W(175 - 125) = 50W$

The heat-balance equation is therefore

$$1{,}070W = 44{,}968{,}000 + 50W$$
$$W = 44{,}090 \text{ lb. steam per hr.}$$

Heating Surface.—This is calculated by the rate equation

$$q = UA\Delta t$$

q is the heat passing through the heating surface per hour.

The only difficulty in the use of this equation is in the value to be used for Δt. Steam condenses at 249°F. and the condensate leaves at 175°. The liquid is fed at its boiling point of 125°F. The steam gives up 946 B.t.u. in condensing at 249°F. and 74 B.t.u. in cooling to 175°F. The heat given up in condensing is so large in comparison to the heat given up in cooling condensate, that the latter operation is usually omitted *in calculating temperature drops*. It may be taken for granted that heat-transfer coefficients used in evaporator design are always calculated by using the temperature difference between condensing steam and boiling liquid.

$$1{,}070W - 50W = 44{,}968{,}000$$
$$44{,}968{,}000 = 500 \times A \times (249 - 125)$$
$$A = 725 \text{ sq. ft.}$$

b. Feed Solution Enters at 70°F.:

Material balance. This is unchanged from part *a*.

Heat balance. Only one quantity (heat entering in feed) is different from part *a*. The datum of 125°F. may be retained, and heat below it may be considered negative. The heat entering in feed is therefore

$$-[55{,}000 \times 0.9(125 - 70)] = -2{,}723{,}000 \text{ B.t.u.}$$

The heat-balance equation is

$$1{,}070W - 2{,}723{,}000 = 44{,}968{,}000 + 50W$$
$$W = 46{,}760 \text{ lb. per hr.}$$

The increase in steam consumption from 44,090 lb. in part *a* to 46,760 lb. in part *b* furnishes the heat needed to heat the feed from 70° to the boiling point.

Heating Surface.—As the cooling of condensate was neglected in calculating Δt in part a, so heating feed will also be neglected here, and Δt is still the difference between boiling liquid and condensing steam.

$$47{,}691{,}000 = 500 \times A \times 124$$
$$A = 769 \text{ sq. ft.}$$

The increase of 44 sq. ft. over part a is necessary to transmit the heat used for heating feed.

c. Feed Solution Enters at 200°F.:

Material balance. Unchanged from part a.

Heat balance. The feed is introduced at a temperature above its boiling point in the evaporator. In other words, some heat is brought in by the feed above the datum plane of 125°. The heat-balance equation is

$$1{,}070W + [55{,}000 \times 0.9 \times (200 - 125)] = 44{,}968{,}000 + 50W$$
$$1{,}020W = 41{,}255{,}000$$
$$W = 40{,}430 \text{ lb. per hr.}$$

Heating Surface:

$$41{,}255{,}000 = 500 \times A \times (249 - 125)$$
$$A = 665 \text{ sq. ft.}$$

In this case the heat above 125° brought in by the feed is available for evaporation and therefore decreases the steam needed. The vapor generated by this extra heat is formed as flash vapor when the feed is introduced into the evaporator. Consequently, the heat to produce this vapor does not need to pass through the heating surface and less heating surface is needed than in parts a and b.

Example 16.—An evaporator is fed with a 25 per cent sodium chloride solution. It is to produce 14,670 lb. of dry salt per hour. The salt removed carries 20 per cent of its weight of brine. Practically all this adhering brine is later removed from the salt and discarded. The specific heat of dry salt is 0.2. The feed is at 70°F. Steam, vacuum, and condensate temperature are as in Example 15. The heat-transfer coefficient, corrected for elevation in boiling point, is 300. The specific heat of the feed is 0.79. No solution is withdrawn from the evaporator. The latent heat of evaporation of water from this solution may be taken from the steam tables for the corresponding pressure. What are the steam consumption and heating surface required?

Solution. *Material Balance.*—Since the feed is a 25 per cent solution, it follows that 3 lb. of water must be evaporated for every pound of salt made. The evaporation would be then $3 \times 14{,}670 = 44{,}000$ lb., if no brine were withdrawn with the salt.

There are removed, with the salt, $0.2 \times 14{,}670$ or 2,930 lb. of brine. This brine is not 25 per cent solids but saturated brine of 26.9 per cent solids. To produce 2,930 lb. of this brine would require

$$2{,}930 \times \frac{26.9}{25.0} = 3{,}150 \text{ lb. of 25 per cent brine.}$$

The total feed therefore is

$$(44{,}000 + 14{,}670) + 3{,}150 = 61{,}820 \text{ lb. of brine.}$$

As 220 lb. of water were evaporated to convert 3,150 lb. of feed brine to 2,930 lb. saturated brine, the total evaporation is 44,000 + 220 = 44,220 lb.

Heat Balance.—The heat-balance equation is the same as in Example 15. It is convenient to take the boiling point as the datum. From Fig. 99 the boiling point of saturated brine, at a pressure under which water boils at 125°F., is found to be 137°F. This will be taken as the datum temperature. As before, let W be the weight of steam used. Then

Heat entering in steam $= W[946 + (249 - 137)] = 1{,}058W$
Heat entering in feed $= -61{,}820 \times 0.79 \times (137 - 70) = -3{,}272{,}000$ B.t.u.
Heat leaving in vapor $= 44{,}220 \times 1{,}022 = 45{,}190{,}000$ B.t.u.
Heat leaving in salt = none (salt leaves at datum temperature)
Heat leaving in brine = none
Heat leaving in condensate $= W(175 - 137) = 38W$

The heat-balance equation is therefore

$$1{,}058W - 3{,}272{,}000 = 45{,}190{,}000 + 38W$$
$$1{,}020W = 48{,}462{,}000$$
$$W = 47{,}500 \text{ lb. per hr.}$$

Heating Surface:

$$48{,}462{,}000 = 300 \times A \times (249 - 137)$$
$$A = 1{,}442 \text{ sq. ft.}$$

This increase in heating surface as compared to that required in Example 15 for the same evaporation is partly due to the decreased temperature drop but mainly to the decreased coefficient.

EVAPORATOR ACCESSORIES

There are various devices that must be supplied with every evaporator in addition to the evaporator body itself. These devices are useful in many other fields of chemical engineering and therefore they will be discussed in some detail.

CONDENSERS

If an evaporator is to operate under vacuum, some device must be used for condensing the vapors. Condensers may be classified into several groups as follows:

- Surface
- Contact
  - parallel-current / countercurrent
  - wet / dry
  - barometric / low level

In a *surface condenser* the vapor to be condensed and the liquid for cooling it are separated by a metal wall, while in a *contact condenser* the vapor and cooling liquid are mixed directly. A *parallel-current* condenser is one in which the non-condensed gases leave at the temperature of the exit cooling water, while a *countercurrent* condenser is one in which the non-condensed gases leave at the temperature of the entering cooling water. A *wet* condenser is one in which non-condensed gases and cooling water are removed by the same pump, whereas a *dry* condenser is one in which they are removed by separate pumps. A *barometric* condenser is one that is placed high enough so that the water escapes from it by a barometric leg, while a *low-level* condenser is one in which the water is removed by a pump. These various classifications are coordinate and theoretically are applicable to surface condensers as well as contact condensers. Also, any combination of the various subclasses is theoretically possible, but in practice parallel-current condensers are almost always wet condensers, while countercurrent condensers are always dry.

Surface Condensers.—A surface condenser does not differ from a tubular heater. The steam is usually outside the tubes, and the water inside. As is the case with any tubular heater, the heat-transfer coefficient (and therefore the capacity of the apparatus) is improved by increasing the water velocity, and this is done in surface condensers as in heaters by making them multipass. In speaking of tubular heaters it is usually assumed, though this is not necessarily true, that the steam is at pressures above atmospheric, and therefore the non-condensed gases escape into the air. In the case of a surface condenser the steam is at a pressure below atmospheric, and therefore some form of vacuum pump must be employed to remove the air.

Contact Condensers.—A countercurrent, dry-contact condenser is shown in Fig. 103. It consists of a vertical cylinder with two shelves or trays extending part way across its section. Cold water is introduced near the top and cascades from tray to tray, finally leaving from the opening at the bottom. Vapor is introduced near the bottom and must pass up through the cascades of water until condensed, so that finally only non-condensed gases leave from the top of the condenser to go to the vacuum pump. If such a condenser is set 34 ft. or more above the hot

well, it is barometric. If it is not elevated and a water pump is attached to the bottom to remove hot water, it is a low-level condenser.

Figure 104 shows a parallel-current wet condenser. Vapor to be condensed enters at the top and is met by a spray of water. Vapor and cooling water mix, and both the condensed mixture and the non-condensed gases leave together at the bottom. Such a condenser is usually mounted directly on a wet vacuum pump that removes both hot water and air. There are many modifications of both these condensers, and Figs. 103 and 104 merely show one type of each to illustrate the principles of operation.

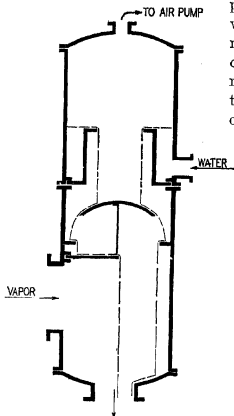



FIG. 103.—Dry countercurrent contact condenser. Dotted lines show general form of water cascades.

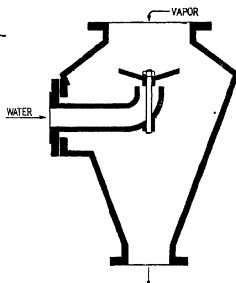



FIG. 104.—Wet parallel-current contact condenser.

If a Venturi tube is attached to the bottom of the condenser of Fig. 104, and sufficient cooling water is used so that the velocity through the throat of the Venturi is sufficiently high, the static pressure at the throat can be made low enough so that the condenser acts as its own vacuum pump and ejects the air by virtue of the water velocity. Such a condenser eliminates a vacuum pump but uses large volumes of water.

The vapor from an evaporator is usually steam and therefore has no particular value. A surface condenser is much more expensive for a given capacity than is a contact condenser. Consequently, unless the condensed steam is desired for some special purpose, or unless the vapors liberated from the solution

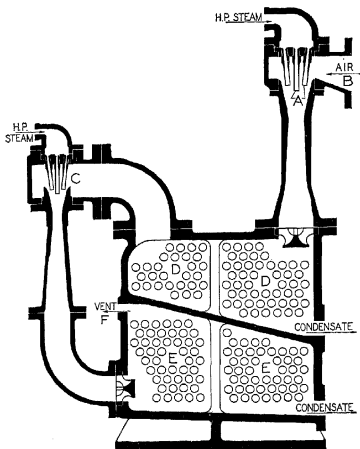



FIG. 105.—Steam-jet ejector. *A*. First-stage nozzles. *B*. Air inlet. *C*. Second-stage nozzles. *D*. First-stage condenser. *E*. Second-stage condenser. *F*. Air outlet.

must be recovered, evaporators are provided with contact condensers rather than surface condensers.

Vacuum Pumps.—The pump used to remove hot water and non-condensed gases from a parallel-current wet condenser may be exactly similar to any ordinary reciprocating pump. The displacement is made large enough so that it will suffice for both water and air. Dry vacuum pumps used on dry countercurrent barometric condensers, or in other cases where air alone is to

be removed, are constructed like ordinary air compressors except that care is taken to keep the clearances as small as possible and the valve mechanism as light as possible. Such pumps need no special description in this book.

One form of vacuum pump that is rapidly gaining in favor is the *steam-jet ejector*. An example is shown in Fig. 105. High-pressure steam is admitted to a series of nozzles *A* that send jets of very high velocity into the throat of a Venturi tube. The non-condensible gas to be removed enters at *B*. By properly proportioning the throat of this tube and the volume and velocity of the steam used, the steam can be made to entrain the non-condensed gases and produce any degree of vacuum desired. For very high vacua (low absolute pressures) the steam-air mixture from these jets goes to an auxiliary condenser *D* where the steam is largely removed. The residual air is then passed to a second series of steam jets *C*. The discharge from the second set of jets goes to another surface condenser *E*, and the non-condensible gas finally leaves the apparatus at *F* at atmospheric pressure. This two-stage ejector can be designed to produce quite high vacua, and a three-stage ejector may be designed to produce vacua of 0.5 mm. absolute. Multistage ejectors are not often applied to evaporator work. For a vacuum of 26 in. or over, the steam-jet ejector uses less steam than a vacuum pump, and for vacua over 29 in., it is practically the only device possible.

Removal of Condensate

Condensate may be removed from evaporators by two types of devices: pumps or traps. The most generally useful device is a reciprocating pump, and when this is used it is commonly made oversize so that it can handle non-condensed gases as well as condensate. Centrifugal pumps may be used, but since the centrifugal pump does not work well under vacuum, and since on evaporators it is often necessary to remove condensate from a steam space that is under vacuum, centrifugal pumps are not often used for this purpose. If the evaporator is so located that the condensate drains to the pump by gravity, centrifugal pumps may be used; or the self-priming centrifugal pump described on page 106 may also be employed.

Traps.—A device that is widely used for removing condensate, not only from evaporators but also from all types of steam-heated

equipment, such as coils, kettles, heaters, steam mains, etc., is the steam trap. The function of a steam trap is to allow condensate to drain but to prevent steam from blowing out of the space drained. There is a bewildering variety of constructions on the market, but they may be reduced to three main classes as follows: expansion traps, bucket traps, and tilt traps. Steam traps, whatever their construction, may also be divided into return traps and non-return traps. A *non-return trap* is one that will discharge condensate only to a space of lower pressure than that of the space it drains. A *return trap* will discharge condensate to a space whose pressure is as high as, or higher than, the pressure of the space being drained. Theoretically, any type

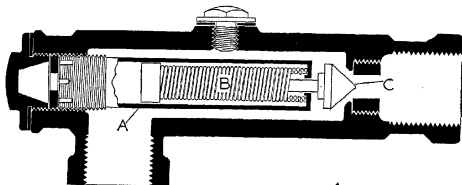



Fig. 106.—Expansion trap. *A*. Cartridge. *B*. Corrugated tube. *C*. Valve.

of trap might be either return or non-return. Actually, the first two classes are practically always non-return traps, and the return traps are found in the third group only.

Figure 106 shows an *expansion trap*. This trap contains a closed metal cartridge *A*. To one end of this metal cartridge is connected a collapsible corrugated tube *B*. The left-hand end of the tube *B* is sealed and to it is attached the stem of the valve *C*. The space between the cartridge and the corrugated tube is filled with oil. The collection of condensate against the valve, losing heat by radiation, cools the cartridge, the oil contracts, the valve opens, and the condensate is blown out. When the condensate is all discharged and steam enters the trap again, the cartridge expands and the trap closes. This device is very simple and has no moving parts. It is best suited for small capacities and requires rather delicate regulation.

A typical bucket trap is shown in Fig. 107. The condensate that enters this trap collects in the bucket A until a definite weight has been caught. The bucket then drops, pulls down on rod B, and opens the valve C at the top of the trap. This allows the condensate to be blown out. When sufficient water has been blown from the trap the bucket floats and closes the valve. These traps are intermittent and are very seldom made as return traps.

A non-return tilt trap is shown in Fig. 108. This trap consists of a drum A, which is supported on a vertical post B carried on horizontal trunnions C about which it can rotate. These trunnions are hollow but have a separating partition in the center. The left-hand side of the trunnion is connected to the vertical post B, which is hollow, and the right-hand side of the trunnion is connected to a pipe D which leads to the rear of the drum. The drum is normally held in a horizontal position by means of a counterweight E and lever F. This lever keeps the valve G in the left-hand end of the trunnion closed through the action of lever J and counterweight H. As condensate drains from the steam space into the trap, it flows into the right-hand end of the trunnion, back through pipe D, and into the tank. When sufficient water has collected to overbalance the counterweight, the drum rotates about the trunnions, the rear end of the drum drops, and this motion is transmitted by the counterweight and lever to open the valve G. The pressure in the steam space, therefore, blows the condensate out of the tank until the weight of the tank has been sufficiently reduced so that the counterweight lifts the tank and closes the discharge valve.

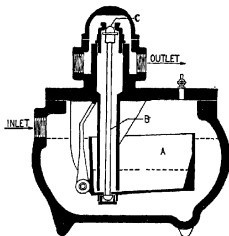



FIG. 107.—Bucket trap. *A*. Bucket. *B*. Valve rod. *C*. Discharge valve.

Figure 109 shows a *return trap*. The right-hand trunnion must be connected to a tee on either side of which there is a check valve H_1, H_2. Back of the valve G in the left-hand trunnion is another smaller valve J, which is connected to the

trunnion below the seat of the main valve G. Condensate to be removed enters through one of the check valves H_2 as indicated, high-pressure steam is connected to the left-hand trunnion, and the small vent valve J is connected either to the top of the space being drained or to a space of still lower pressure.

Let it be assumed that this trap is to remove condensate from a space that is under vacuum and discharge it to a space at

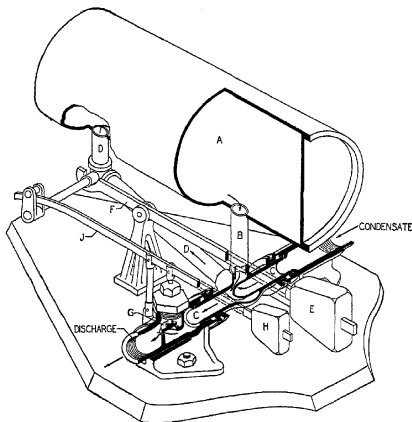



FIG. 108.—Non-return tilt trap. *A*. Drum. *B*. Outlet pipe. *C*. Trunnion. *D*. Inlet pipe. *E*. Main counterweight. *F*. Main lever *G*. Outlet valve. *H*. Outlet-valve counterweight. *J*. Outlet-valve operating lever. (*Morehead*.)

atmospheric pressure. The vacuum on the condensate line holds the check valve H_1 in the discharge line closed. Water flows to the trap through the inlet check valve H_2, through the right-hand trunnion, and back to the rear of the tank through pipe D, as shown by the arrows 1. The vent valve J is held open by the counterweight K and the lever L. The steam valve is held closed by the counterweight E and its lever F. The tank is therefore

vented to the space being drained, and condensate can flow into the tank by gravity. When the tank fills and overbalances the counterweight, the rear of the tank drops, the system of levers opens the steam valve G and closes the vent valve J, and high-pressure steam enters the tank through the left-hand side of the trunnion, as shown by the dotted arrows, and blows

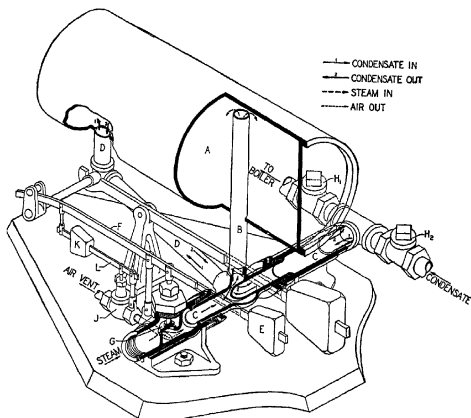



FIG. 109.—Return tilt trap. *A*. Drum. *B*. Steam inlet pipe. *C*. Trunnion. *D*. Condensate pipe. *E*. Steam-valve counterweight. *F*. Steam-valve operating lever. *G*. Steam valve. H_1, H_2. Check valves. *J*. Vent valve. *K*. Vent-valve counterweight. *L*. Vent-valve operating lever. (*Morehead*.)

condensate back through the right-hand side of the trunnion and out through the discharge check valve H_1, as shown by the arrows 2. When the tank is sufficiently empty, the counterweight drops, the tank rises, and the trap returns to the filling position.

EVAPORATOR OPERATION

In addition to the equipment of an evaporator installation, certain details of operation are important.

Operating Temperature Range.—It has been pointed out that one factor controlling the capacity of an evaporator is the temperature drop across the heating surface. Various factors influencing this temperature drop have been discussed. It should be noted that the upper limit of temperature range is fixed by such factors as the economic back pressure on the power units when the exhaust from these units is used to heat the evaporator, and by the strength of the evaporator body. The ordinary evaporator is rather bulky, is often constructed of cast iron, and cannot stand high pressures unless expensive constructions are adopted, so that the highest pressure used is seldom above 25 lb. per sq. in., and 5 lb. per sq. in. (gage) is more common. The forced-circulation and long-tube natural-circulation types offer considerable possibility for the use of higher pressures under conditions where they are economical.

Use of Vacuum.—An evaporator operated with steam at 5 lb. per sq. in., and boiling under atmospheric pressure, would have an available temperature drop of 15°F. if the liquid being boiled were water or a solution that had a negligible rise in boiling point. Many solutions handled in practice have a boiling-point rise of more than 15°F. and therefore could not be boiled under atmospheric pressure with 5-lb. steam. On the other hand, if the liquid be evaporated under a vacuum, its boiling point is lowered, the temperature drop is increased, and the capacity of the evaporator is increased. A vacuum of 26 in. (referred to a 30-in. barometer) corresponds to a boiling point of 125°F. for water. Thus, if water were boiled under a 26-in. vacuum instead of under atmospheric pressure, the temperature drop would be 102 instead of 15°. Consequently, most evaporators are operated with vacuum in order to make it possible to use exhaust steam as a heating medium on the one hand, and to get the largest possible capacity per square foot of heating surface on the other hand.

In some cases where materials like gelatin, fruit syrups, and similar sensitive liquids are boiled, the vacuum is necessary to reduce the temperature of the boiling liquid to a point where the liquid is not injured, irrespective of its effect on capacity.

At first sight it would seem that the highest possible vacuum should be maintained on an evaporator. A reference to Fig. 102 will show that as the vacuum is increased and the boiling point

lowered, the coefficient is also lowered. This effect is most pronounced at high vacua (low absolute pressures). The result may be that a very high vacuum may decrease the coefficient more than it increases the temperature drop, and thus result in a decrease, rather than an increase, in capacity. The factors determining the optimum vacuum vary too greatly to permit making a general statement, but except in unusual cases a vacuum much over 26 in. is seldom justified.

Foam and Entrainment.—These two terms are often used together. Each, however, has a special significance. *Foam,* as the name implies, represents the production of stable bubbles in the evaporating space, thereby carrying some of the liquor well above the normal level. If the vapor space is high enough, foam in itself does not necessarily result in any very great disadvantages, although if the foam gets out of control no extension of the vapor space will help.

The term *entrainment* signifies the actual loss of liquid in the form of fine mist. Entrainment may result from foaming, or it may result from a high vapor velocity that will mechanically carry over particles of liquid.

The cause of foam formation is not completely known. It apparently depends upon two factors: first, the formation of a phase that is higher or lower in surface tension than the liquid; and, second, the stabilization of this phase in the form of a film by means of a finely divided solid. To prevent or diminish the foaming in an evaporator, various practical expedients are often useful. For example, the addition of surprisingly small amounts of such materials as sulfonated castor oil or cottonseed oil will often break a foam. The reason for this effect is very obscure. A second expedient is that of dropping the liquor level below the top part of the heating surface and trusting to the transfer of heat into the bubbles to burst them through expansion. A third expedient, which is especially applicable to the forced-circulation or long-tube natural-circulation evaporators, is to give the mixture of foam and liquid a high velocity, allow it to impinge against a baffle, and break the foam through impact. This is also the reason for the success of the Yaryan evaporator on foamy liquids.

Entrainment can be cut down by reducing the velocity of the vapors. In addition to this method, *entrainment separators* have

been developed. The best type of entrainment separator seems to be one that utilizes centrifugal force to throw the drops of liquid outward from the stream of vapor and collect and remove them without allowing them to come in contact with the vapor stream. The umbrella-type baffle used in the forced-circulation evaporator accomplishes the same sort of entrainment prevention because of the scrubbing action of the downward current of liquid that leaves the periphery of the umbrella.

Air Removal.—The discussion on page 142 showed that the steam film coefficient depends to a large extent on the concentration of non-condensed gases in the steam. It is obvious that these must be removed from the steam space in order to have continuous operation. If the steam used to heat the evaporator is above atmospheric pressure, the only source of air in the steam space is air that was originally present in the steam. If the steam space is at a pressure less than atmospheric, air may be drawn in through leaks.

In those evaporator designs in which the steam has a definite path such as in the horizontal-tube evaporator, any non-condensed gases are quite completely swept to the far end of the steam path. Consequently in such evaporators air vents should be located at the end of the steam travel. In such designs as the standard vertical-tube evaporator, the path of the steam is not definite and the air is not positively swept to any one point. In such cases several connections are usually made in the top tube sheet at points a considerable distance from the steam inlet. In the basket-type evaporator the air vents are usually around the circumference of the basket. When condensate is removed by a pump, it is good practice to make the pump oversize and let it pump air with the condensate.

Where air vents are employed, it is merely necessary that they be vented to a space of lower pressure. If, therefore, the space to be vented is above atmospheric, the vent may blow into the air. If the steam space is below atmospheric pressure, the simplest way is to return the non-condensed gases to the vapor space of the same body. Where large amounts of non-condensed gases may be present, it is sometimes most convenient to vent them direct to the condenser.

Salt Removal.—In many cases during evaporation, solid material precipitates or crystallizes from the solution There

are various ways of removing this, and some of the methods are

1. Dumping the charge.
2. Salt receivers.
3. Salt elevators.
4. Continuous removal by a pump.

In the following discussion the material to be removed will be referred to as *salt*, although it is to be understood that it may be any solid material thrown out of solution.

When the solution is evaporated in batches, and especially when the batch becomes extremely viscous toward the end, if

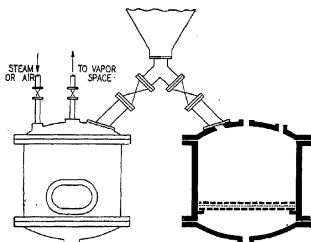



Fig. 110.—Batch salt filters.

crystals are formed they may be held in suspension and discharged with the batch. This practice is regularly followed in sugar boiling but is not general in other fields.

A more common method, where moderate amounts of salt are to be removed, is to attach to the bottom of the evaporator one or more receivers such as shown in Fig. 110. These may be merely receivers in which the salt collects and from which it is pumped at intervals to filters. More commonly, however, they are salt filters. Each filter is provided with a filtering medium, usually wire gauze, supported between two fairly heavy, perforated metal plates. A manhole is provided, with its bottom even with the filtering medium, so that the salt may be raked out. The salt filter must be provided with an equalizing line connecting it to the vapor space of the evaporator, so that a vacuum may

be put on it before it is connected to the machine. It is usually provided with connections for steam or air for displacing mother liquor from the salt as completely as possible before dumping. In cases where salt is to be separated, the evaporator is provided with a conical bottom so that salt may settle directly into the salt filter. When the amount of salt to be removed is small, one filter will be sufficient and the salt may be allowed to accumulate in the cone while the filter is being emptied. When salt accumulates at a more rapid rate, two filters should be attached to each cone so that one is always filling. A production of a ton per hour of crystallized salt is about the maximum that can be handled by this method of disposal.

In the common-salt industry, it was considered good practice for many years to extend the cone of the evaporator downward and have it discharge to a bucket elevator. This method is now obsolete.

When the quantities of salt to be removed from the evaporator are too large to permit the use of salt filters, continuous pumping is considered the best practice. In this method a centrifugal pump is attached to the cone of the evaporator and continuously pumps a stream of salt and solution out of the evaporator into a settler, the liquor overflows from the settler and is returned to the evaporator, and the salt is withdrawn from the settler to centrifuges or continuous filters. The amount of mother liquor removed by the pump is controlled by adjusting the pump capacity so that sufficient liquor is carried to prevent clogging the lines but without sending an excessive amount through the settler. Probably the thickest suspension that it is possible to pump in this manner is one part of salt to one part of brine by volume.

This method is applied to the forced-circulation evaporator by incorporating a settler in the circulating system. A small fraction of the liquid leaving the vapor head is diverted to a closed settler of relatively large cross-section where the velocity of the liquid is decreased and the heavier particles of the salt settle. The mother liquor passes from this settler back to the circulating pump, and the salt is withdrawn from the conical bottom of the settler. This method of salt removal is shown in Fig. 98.

Scale Formation.—Most solutes increase in solubility with increasing temperature. Some salts, however, such as calcium

sulfate, anhydrous sodium sulfate, and sodium carbonate monohydrate, show what are known as *inverted solubility curves;* that is, the solubility of these substances decreases with increasing temperature. When such a solution is heated or concentrated in an evaporator, the solubility of the solute is at a minimum at the tube wall where the temperature is at a maximum. Precipitation therefore takes place at the tube wall, generally with the formation of a hard, dense, strongly adhering scale. Precipitation from solutions possessing normal solubility takes place in the bulk of the fluid rather than at the tube wall. If both types of precipitation occur, however, the material having an inverted solubility curve will often include considerable quantities of the material possessing the normal solubility curve.

Various methods of removing scale are available. If the scale is soluble in water, boiling out the evaporator with water will remove it. If it is comparatively insoluble in water, it is usually removed by emptying the evaporator and drilling out the tubes with a scale-removing tool. Some scales are amenable to chemical solution in dilute acids or dilute alkalies. With reasonable care, apparatus made of iron and steel can be boiled with 0.5 per cent hydrochloric acid for a sufficient time to dissolve the scale without injury to the equipment, especially if an inhibitor be used with the acid. Sometimes an alkaline boil-out followed with an acid boil-out will remove a refractory scale such as calcium sulfate.

If a solution containing scale-forming substances is to be heated or evaporated, there is no method that will completely prevent scale formation. It has been found, however, that increasing the velocity of the liquid will greatly decrease the rate of scale formation. This is one of the important advantages of the forced-circulation evaporator.

Rate of Scale Formation.[1]—Consider an evaporator that is operating on a scale-forming liquid. The temperature drop and all factors influencing the overall coefficient (except scale) are to be considered constant. Let U be the overall coefficient θ hr. after the start of the evaporation, and let Q be the total heat that has been transferred up to the time θ. It is reasonable to assume that the formation of the scale adds a resistance in series with the resistances already present. The added resistance is proportional to the thickness of the scale. The thickness of the scale, in turn, varies directly

[1] McCabe and Robinson, *Ind. Eng. Chem.*, **16,** 478 (1924).

with the total amount of solids precipitated, and this latter is proportional to the total evaporation up to that time and hence to the total heat transferred. All this may be expressed by

$$U = \frac{1}{R_0 + R} = \frac{1}{\frac{1}{U_0} +} \tag{96}$$

where

R_0 = the initial resistance (when $\theta = 0$).
R = the added resistance due to the formation of the scale.
U_0 = the original coefficient.
α = θ constant.

The rate of heat transfer at the time θ is given by

$$\frac{dQ}{d\theta} = UA\Delta t \tag{97}$$

where

A = the area of the heating surface.
Δt = the temperature drop.

Three variables are present in Equations (96) and (97), Q, θ, and U. A relationship between θ and U is desired. Accordingly Q will be eliminated from Equations (96) and (97). Thus, from Equation (96)

$$\alpha Q = \frac{1}{U} - \frac{1}{U_0}$$

or

$$dQ \quad \frac{-dU}{\alpha U^2} \tag{98}$$

Substituting the value of dQ from Equation into Equation (97), it follows that

$$-\frac{dU}{U^3} = \alpha A \Delta t d\theta \tag{99}$$

Integrating Equation (99), it is found that

$$-\int_{U_0}^{U} \frac{dU}{U^3} = \alpha A \Delta t \int_0^{\theta} d\theta$$

$$\frac{1}{U^2} = \frac{1}{U_0^2} + 2\alpha A \Delta t \theta \tag{100}$$

Since α, A, and Δt are constants, Equation (100) can be written as

$$\frac{1}{U^2} = \frac{1}{U_0^2} + \beta\theta \tag{101}$$

where β is a constant.

Equation (101) shows that if $1/U^2$ is plotted against θ a straight line should result if the theory is correct (since U_0^2 is a constant). Figure 111 gives such a curve for sodium sulfate. The curved line is the plot of U *vs.* θ. The straight line is the plot of $1/U^2$ *vs.* θ. It will be seen that it is much easier to extrapolate or interpolate on the latter line than it is on the former. Also, two points are all that are required to determine the $1/U^2$ plot. The slope of the straight line is a measure of the rate of scale formation. The larger this slope the faster will the scale form.

There is an optimum time of operation between periods of scale removal that will give a maximum overall capacity. If the evaporator is shut down and cleaned too often, the average

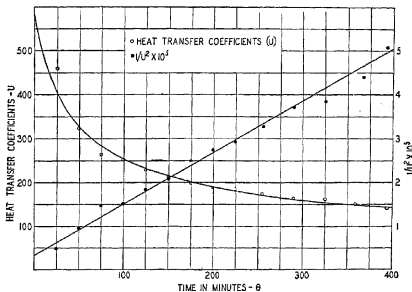



FIG. 111.—Rate of scale formation in evaporator.

production is low because too great a proportion of the total time is spent in cleaning. On the other hand, if boil-outs are too far apart, the average coefficient is low and decreased capacity results. Fortunately, however, the optimum in most cases is not sharp and is usually determined accurately enough by trial under plant conditions, although it can be calculated from curves such as Fig. 111. Such calculations, however, are beyond the scope of this book.

MULTIPLE-EFFECT EVAPORATION

All the above discussion of evaporation has been based on what are known as single-effect evaporators. A modification of this system called multiple-effect evaporation is commonly used in

large-scale operations, for the purpose of obtaining higher steam economies than it is possible to get in single effect.

Principles of Multiple-effect Evaporation.—Figure 112 shows three standard central-downtake vertical-tube evaporators connected to form a multiple-effect evaporator. The general principle is that the connections are so made that the vapor from one evaporator serves as the heating medium for the next one. It will be noted that making this evaporator into a multiple effect is merely a question of the interconnecting piping, not at all of the structure of the bodies.

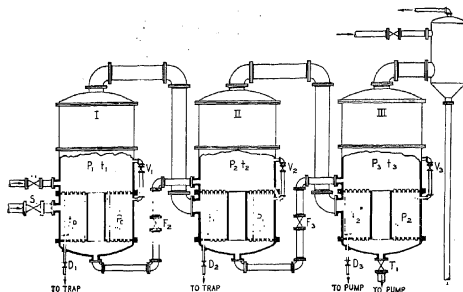



FIG. 112.—Multiple-effect evaporator. I, II, III. First, second, and third effects. D_1, D_2, D_3. Condensate valves. F_1, F_2, F_3. Feed or liquor valves. S_1. Steam valve. T. Thick-liquor valve. V_1, V_2, V_3. Vent valves. P_0, P_1, P_2, P_3. Pressures. t_0, t_1, t_2, t_3. Temperatures.

Imagine that the whole system is cold, at atmospheric pressure, and that each body is filled with the liquid to be evaporated to some predetermined level, such as the level of the top tube sheet. Now, imagine that the vacuum pump is started and that the valves V_1, V_2, and V_3 in the non-condensed gas vent lines are wide open. All of the other valves are to be imagined closed. Let it be assumed that the highest vacuum to be carried during regular operation will be 26 in. referred to a 30-in. barometer and that the vacuum pump is operating to produce such a vacuum. It follows that through the non-condensed gas lines

and through the steam lines the whole of the apparatus will be evacuated down to 26 in.

Now assume that the steam valve S_1 and the condensate valve D_1 are opened until the desired pressure P_0 is built up in the steam space of evaporator I. Let t_0 be the temperature of saturated steam at the pressure P_0. The steam will first displace any residual air in the steam space of evaporator I through the vent valve V_1; and when the air is all displaced, vent valve V_1 will be nearly closed. Since the liquid surrounding the tubes is cold, steam will condense. The trap allows the condensate to escape as fast as it collects. The liquid will become warmer until it reaches the temperature at which it boils under a vacuum of 26 in. Let it be assumed that the liquid to be evaporated has a negligible elevation in boiling point. In that case it will begin to boil at 125°F. The steam so generated will gradually displace the air in the upper part of the evaporator, in the connecting steam line, and in the steam space of evaporator II. When this vapor has completely filled such spaces, vent valve V_2 will be nearly closed.

The steam that is coming off from evaporator I will transmit its heat to the liquid in evaporator II and be condensed. Condensate valve D_2 will be opened so that this condensate will be removed as fast as it is formed. In condensing, however, it gives up its heat to the liquid in evaporator II, which becomes warmer. As the liquid becomes warmer, the temperature difference between it and the steam becomes less, the rate of condensation becomes less, and therefore the pressure in the vapor space of evaporator I will gradually build up, increasing t_1 (the boiling point of the liquid in evaporator I) and cutting down the temperature difference $t_0 - t_1$. This will continue until the liquid in evaporator II reaches a temperature of 125°F., when it will begin to boil.

The same process will be repeated in evaporator III. As the liquid in evaporator III becomes warmer and finally begins to boil, the temperature drop between it and the steam from the second effect becomes less, pressure begins to build up in the second effect and raises t_2, the boiling point there, so that the temperature difference $(t_1 - t_2)$ becomes less. This decreases the rate of condensation and builds up the pressure in the vapor space of the first effect still more, until finally the evaporator

comes to a steady equilibrium with the liquid boiling in all three bodies.

The result of boiling will be to decrease gradually the liquid levels. As soon as the level begins to come down in body I, the feed valve F_1 will be opened enough to keep the level constant. As body II begins to boil, feed valve F_2 will be adjusted and as body III begins to boil, feed valve F_3 will be adjusted. A change in any one valve obviously involves a resetting of the others, but they will be so set that the liquid levels in all three bodies are kept constant. Liquid in body III is becoming more and more concentrated, however, and when it reaches the desired finished concentration the thick-liquid valve T_1 will be opened the proper amount and the thick-liquid pump will be started. Thus, when evaporation is proceeding in a steady manner, there will be a continuous feed to the first body, from the first to the second, from the second to the third, and a continuous withdrawal of thick liquid from the third effect. The evaporator is now in continuous operation with a continuous flow of liquid through it, and all the various temperatures and pressures are in equilibrium.

The heating surface in the first effect will transmit per hour an amount of heat given by the equation

$$q_1 = A_1U_1\Delta t_1 \qquad (102)$$

If the part of this heat that goes to heat the feed to the boiling point be neglected for the moment, it follows that practically all of this heat must appear as latent heat in the vapor that leaves the first effect. The temperature of the condensate leaving through connection D_2 is very near the temperature t_1 of the vapors from the boiling liquid in the first effect. Therefore, practically all of the heat that was expended in creating vapor in the first effect must be given up when this same vapor condenses in the second effect. The heat transmitted in the second effect, however, is given by the equation

$$q_2 = A_2U_2\Delta t_2 \qquad (103)$$

but as has just been shown, q_1 and q_2 are nearly equal, and therefore

$$A_1U_1\Delta t_1 = A_2U_2\Delta t_2 \qquad (104)$$

This same reasoning may be extended to show that, roughly,

$$A_1U_1\Delta t_1 = A_2U_2\Delta t_2 = A_3U_3\Delta t_3 \tag{105}$$

It should be expressly understood that Equations (104) and (105) are only approximate equations and will need to be corrected by the addition of terms which are, however, relatively small compared to the quantities involved in the expressions above.

In ordinary practice, the heating surfaces in all the effects of a multiple-effect evaporator are equal. This is to obtain economy of construction. Therefore, from Equation (105) it follows that

$$U_1\Delta t_1 = U_2\Delta t_2 = U_3\Delta t_3 \tag{106}$$

From this it follows that the temperature drops in a multiple-effect evaporator are approximately inversely proportional to the heat-transfer coefficients.

A reference to the mechanism by which this imaginary evaporator was started up, as given in the preceding pages, will show that the temperatures t_1 and t_2 were determined as the result of the evaporator's *automatically* reaching its own equilibrium. Since t_0 and t_3 are fixed, this automatically determines Δt_1, Δt_2, and Δt_3. In other words, the equilibrium described by Equations (105) and (106) is automatically and continuously maintained, and cannot be regulated or controlled except by altering the ratios of U_1, U_2, and U_3.

Example 17.—A triple-effect evaporator is concentrating a liquid that has no appreciable elevation in boiling point. The temperature of the steam to the first effect is 227°F. (5 lb. gage). Vacuum on the last effect is 26 in. (125°F.). The overall heat-transfer coefficients are 500 in the first effect, 400 in the second effect, and 200 in the third effect. What are the approximate temperatures at which the liquid will boil in the first and second effects?

Solution.—The total temperature drop is

$$227 - 125 = 102°\text{F.}$$

The temperature drops will be approximately inversely proportional to the coefficients.

$$\Delta t_1 : \Delta t_2 : \Delta t_3 : : \frac{1}{U_1} : \frac{1}{U_2} : \frac{1}{U_3} \tag{107}$$

From this it follows that

$$\Delta t_1 = 21.5^\circ$$
$$\Delta t_2 = 27.0^\circ$$
$$\Delta t_3 = 53.5^\circ$$

consequently, the boiling point in the first effect will be 205.5° and in the second effect, 178.5°.

The above discussion has been based on triple-effect operation. It is to be understood, of course, that more or less than three effects may be used in a multiple-effect evaporator. Evaporators with any number of effects up to four are common, quintuple effects are in operation but are uncommon, and the largest number of effects that has ever been operated successfully is eleven.

Economy of Multiple-effect Evaporators.—Since the latent heat necessary to evaporate a pound of water at t_1 is very nearly the same as the latent heat liberated in condensing a pound of steam at t_o, the first effect of a multiple-effect evaporator will evaporate approximately one pound of water for every pound of steam condensed. This is almost exactly true if the feed to the first effect be nearly at the boiling point, and is roughly true unless the feed is very cold. According to the same line of reasoning, it follows that every pound of vapor evaporated in the first effect and condensed on the heating surface of the second effect will evaporate approximately a pound of water in the second effect; and in the same way for every pound of water evaporated in the second effect, there will be approximately a pound of water evaporated in the third effect. Since multiple-effect evaporators may have any number of effects (not necessarily three), it follows that in an N-effect evaporator there will be approximately N pounds of water evaporated per pound of steam supplied. This relationship is only approximate, however, and must not be used in actual calculations. Minor terms excluded in the above line of reasoning have an increasingly important effect as the number of bodies increases. However, the statement is still approximately true and, therefore, it follows that *the reason for operating an evaporator in multiple effect is to secure increased steam economy.*

Capacity of Multiple-effect Evaporators.—Although the use of the multiple-effect principle increases the steam economy,

it must not be thought that there are no compensating disadvantages. Coordinate in importance with the economy of an evaporator system is the question of its capacity. By capacity is meant the total evaporation per hour obtained in the entire system. Since latent heats are nearly constant over the ranges of pressure ordinarily involved, capacity is also measured by the total heat transferred in all effects. The heat transferred in the three effects of Fig. 112 can be represented by the following equations:

$$\begin{aligned} q_1 &= U_1A_1\Delta t_1 \\ q_2 &= U_2A_2\Delta t_2 \\ q_3 &= U_3A_3\Delta t_3 \end{aligned} \tag{108}$$

and the total capacity will be found by adding these equations, giving

$$q = q_1 + q_2 + q_3 = U_1A_1\Delta t_1 + U_2A_2\Delta t_2 + U_3A_3\Delta t_3 \tag{109}$$

Assume, now, that all effects have equal areas and that an average coefficient, U_{av}, can be applied to the system. Then Equation (109) can be written as

$$q = U_{av}A(\Delta t_1 + \Delta t_2 + \Delta t_3) \tag{110}$$

However, the sum of the individual temperature drops equals the total overall temperature drop between the temperature of the steam and the temperature in the condenser, and therefore

$$q = U_{av}A\Delta t \tag{111}$$

Suppose, now, that a single-effect evaporator of area A be operated with the same overall temperature difference, viz: with steam at 227°F. and a vapor temperature of 125°F. Assume also that the overall coefficient of the single effect is equal to the U_{av} of the triple effect. The capacity of the single effect will be

$$q = U_{av}A\Delta t$$

This is exactly the same equation as that for the triple effect. No matter how many effects one uses, provided the average overall coefficients are the same, exactly the same equation will be obtained for calculating the capacity of any evaporator.

It follows from this that if the number of effects of an evaporation system are varied, and if the total temperature difference is kept constant, the total capacity of the system remains substantially unchanged.

If the cost of one square foot of heating surface is constant, regardless of the number of effects, the investment required for an N-effect evaporator will be N times that of a single-effect evaporator of the same capacity. The choice of the proper number of effects will be dictated by an economic balance between the savings in steam obtained by multiple-effect operation and the added investment costs brought about by the added area.

Effect of Hydrostatic Head and Boiling-point Elevation.— Under the discussion of single-effect evaporators it was shown that their capacity was influenced by the effect on Δt of hydrostatic head and boiling-point elevation. These factors enter into the performance of each effect of a multiple-effect evaporator just as they did in a single effect. Their influence is even more pronounced, however, in the case of multiple-effect evaporation than in the simpler case.

As shown on page 189 the effect of hydrostatic head, though appreciable, cannot be computed. This is equally true in multiple-effect evaporators. The factors involved in any adequate hydrostatic head correction are so little understood that they cannot be subjected to even qualitative reasoning, whether as applied to single-effect or multiple-effect evaporators. The results of elevation in boiling point may be discussed more satisfactorily.

Consider an evaporator that is concentrating a solution with a high elevation of boiling point. The vapor coming from this boiling solution is at the solution temperature and is therefore superheated by the amount of the boiling-point elevation.

This superheat represents a very small fraction of the total available heat in the steam, because the specific heat of superheated steam is only about 0.5 B.t.u. per lb. Part of this superheat is dissipated by radiation before the steam reaches the next effect. Even if the steam enters the steam chest of the next effect with its superheat intact, the small amount of superheat present is quickly transmitted. Throughout most of the steam space the steam is saturated and therefore at a temperature

corresponding to the *pressure* of the previous effect rather than its temperature (see Fig. 101).

The temperature drop across any effect, therefore, is that calculated on the basis of the temperature of saturated steam at the pressure of the steam compartment, rather than the temperature of the boiling liquid in the previous effect. This means that the boiling-point elevation in any effect is lost from the total available temperature drop. This loss occurs, not in one effect, but in every effect of a multiple-effect evaporator, and the resultant loss in capacity is often important.

The effect of these losses in temperature drop on the capacity of a multiple-effect evaporator is shown in Fig. 113. The three

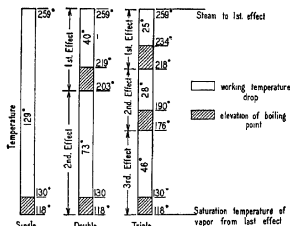



FIG. 113.—Effect of elevation in boiling point on multiple-effect evaporator capacity.

diagrams of this figure represent the temperature drops over a single, double, and triple effect, respectively. The terminal conditions are the same for all three cases; that is, the steam pressure to the first effect and the saturation temperature of the vapor to the condenser are identical in all cases. Each body contains a liquid with a boiling-point elevation. The total height of each column represents the total temperature spread from steam temperature to saturation temperature of the vapor from the last effect.

Consider the single effect. Of the total temperature drop, that part that is shaded represents the loss in temperature drop due to the boiling-point elevation. The actual driving force for the transfer of heat (working temperature drop) is represented by the unshaded portion. The diagram for the double effect

must show two such shaded portions because there is a boiling-point elevation in each of the two effects. It is apparent that the temperature drop left as a working temperature drop and represented by the unshaded portions is less than is the case in the single effect. In the triple effect, three shaded segments appear, since there is a loss of temperature drop due to boiling-point elevation in each of the three effects, and the total net working temperature drop is correspondingly less. The actual temperatures shown in Fig. 113 correspond to the evaporation of saturated sodium chloride solution.

A study of Fig. 113 will show that in the extreme case of a large number of effects or very high boiling-point elevations, it is possible that the sum of the boiling-point elevations in a multiple-effect evaporator will be equal to, or greater than, the total temperature drop available. Operation under such conditions is obviously impossible. This emphasizes the importance of correcting overall coefficients for boiling-point elevation. If the coefficients are uncorrected, the designer may be misled by the fact that he apparently has the entire temperature drop with which to work, when actually he may not have any net temperature drop available.

In the same way it will be seen that in the single-effect evaporator of Fig. 113 the apparent temperature drop is 141°F., and the temperature drop corrected for elevation in boiling point is 129°F. The ratio between the apparent and the corrected coefficients will, therefore, be 129:141 or 0.914. This is not a very large correction, considering the accuracy with which evaporator heat-transfer coefficients are known. On the other hand, in the first effect of the triple the apparent temperature drop is 41° and the corrected temperature drop is 25°; therefore in this case the ratio is 25:41 or 0.61. This is decidedly a significant difference. The error made by neglecting the elevation in boiling point varies with the working temperature drop. It is always safer to use the corrected coefficient when possible, because it does not contain such an error. The only places where the use of the apparent coefficient is justified are either (1) when working with large temperature drops, or (2) when working with solutions having a small elevation in boiling point, or (3) when working with solutions whose elevation in boiling point is not known.

The economy of a multiple effect is not influenced by boiling-point elevations if minor influences (such as the temperature of the feed) are neglected. It still must be true that a pound of steam condensing in the first effect will generate a pound of vapor, which will condense in the second effect generating another pound there, and so on. The *economy* of a multiple-effect evaporator depends on heat-balance considerations and not on rate of heat transfer. On the other hand, if the solution has an elevation of boiling point, the *capacity* of a double effect is less than half of the capacity of two single effects, each of which is operating over the same terminal temperature differences, and the capacity of a triple is less than one-third that of three singles with the same terminal temperatures.

MULTIPLE-EFFECT OPERATION

The operation of a multiple-effect evaporator has many features in common with that of a single effect. Methods of removing condensate, salt removal, scale prevention and removal, entrainment and foaming, and operating temperature ranges are governed by the same considerations for multiple effect as for single effect. There are certain features, however, that are peculiar to multiple-effect operation that are of little or no importance in single effect.

Methods of Feeding.—The usual method of feeding a multiple-effect evaporator is to pump the thin liquid into the first effect and send it in turn through the remaining effects. This is called *forward feed.* The concentration of the liquid in the various effects will increase from the first effect to the last. This method is the simplest. It requires a pump for the first effect, since this effect is often about atmospheric pressure, and a pump is required to remove the concentrated liquid from the last effect. The transfer from effect to effect, however, can be done without pumps, since the flow is in the direction of decreasing pressure, and throttle valves in the transfer line are all that is required.

Another common method of feeding is that known as *backward feed,* where the dilute liquid is pumped into the last effect and then through the successive effects to the first. This method requires a pump between each effect in addition to the thick-liquor pump, since the flow is from low pressure to high pres-

sure. This is a disadvantage, not only because of the extra pumps required but also because the stuffing boxes of the pumps leak air, since the pump casings are under a partial vacuum. If centrifugal pumps are used, this causes air binding. This can be eliminated if the evaporators are high enough so that the static head of the liquid in the transfer lines is made large enough to develop a positive pressure in the pump casing, or if pumps of the self-priming type are used.

Backward feed offers certain advantages over forward feed. For example, suppose the thick liquor is very viscous. Under conditions of forward feed the most concentrated liquor is in the last effect, where the temperature is lowest and the viscosity highest. Under such conditions the capacity of the last effect is low because of the low overall coefficient. This results in a lower capacity of the multiple-effect system as a whole. Under conditions of backward feed, however, the concentrated liquor is in the effect where the temperature is the highest and the viscosity the lowest, and the coefficient can be moderately high in spite of the viscosity.

Another reason for backward feed instead of forward feed lies in the effect of the temperature of the feed on the economy of the evaporation system. If the feed is cold and is sent to the first effect, it must be heated by live steam to the temperature prevailing in the first effect. Since the temperature in this effect is the highest in the system, and since the feed is dilute and therefore of large quantity, this means a large consumption of steam from the mains. The steam so condensed does not accomplish any evaporation in either the first or succeeding effects. Since a pound of such steam has a potential evaporation capacity of N lb. where N is the number of effects, this condensation represents a loss of economy. On the other hand, if the evaporator is fed backward, the cold feed is heated through a smaller temperature range in the last effect and is heated there by steam, each pound of which has already evaporated (N-1) lb. This is partially offset, it is true, by the fact that as the feed passes from effect to effect in backward feed, there is a heating load on each effect since its feed comes in at a temperature lower than the boiling point prevailing in it. The net effect is nevertheless an increase in economy if backward feed is used where the dilute liquid is cold. The quantitative effect of such methods

of feeding will be more apparent when heat-balance calculations are given.

If the feed to the evaporator is hot, for example at a temperature approximately that of the first effect, forward feed results in higher steam economy that does backward feed, since no steam will have to be condensed in this effect to heat the thin liquor and the transfer of solution from effect to effect will give flash evaporation, which generates extra vapors to pass on to the next succeeding effects. On the other hand, if hot feed approximating the temperature of the first effect is fed to the last effect in backward feed, as soon as it enters the last effect it flashes to the temperature of this effect, thereby performing a certain amount of evaporation, it is true, but generating vapor that goes directly to the condenser and accomplishes no evaporation in other effects. In forward feed nearly the same amount of flashing occurs, but this flashing occurs in all the effects and therefore some of this flash steam accomplishes evaporation in multiple effect before it finally reaches the condenser. In addition, when backward feed is used and the liquid has flashed to the temperature of the last effect, it then must be heated in steps as it passes from effect to effect. Consequently, for hot feed, forward feed possesses the greater economy, while for cold feed, backward feed is more economical. The dividing line between the two cases varies with the conditions of the problem and can be determined only by calculating heat balances.

It has been mentioned that one of the advantages of backward feed is attained when liquids are to be finished at high viscosity. It is sometimes possible to attain this result without incurring the inconveniences of strict backward feeding. If the difference in concentrations between the thin liquor and the thick liquor is considerable, it will be found that the effect of excessive viscosity or other similar factors that affect the heat-transfer coefficient adversely are practically confined to a single effect. In such cases it is possible to feed the thin liquor to the second effect, passing it in forward feed from the second to the last effect, pumping from the last effect to the first, and finishing in the first effect. This will eliminate one or two pumps with their suction under vacuum, will not seriously handicap the later effects, but will finish the viscous liquid at the highest temperature where its viscosity is the least and the heat-transfer coefficient the

largest. Many variations of this general idea are found, and the term *mixed feed* is applied to all such cases.

In some cases, especially in the manufacture of common salt, when the evaporator is fed with a saturated solution and no thick liquor is withdrawn, it is convenient to feed directly into each effect and transfer no material from one effect to the other. This is generally called *parallel feed* and is applicable only to those cases where no thick liquor is removed from the evaporator. Heat-balance calculations for a particular case will show whether or not this affects the economy of the evaporator favorably or adversely. Where it is used, it is ordinarily chosen on the score of convenience rather than because of its effect on economy.

Extra Steam.—An important factor in the general economy of a plant that contains a multiple-effect evaporator is one that is seldom recognized and is, in fact, fully appreciated only in the beet-sugar industry. This is the fact that a multiple-effect evaporator is not simply a device for concentrating solutions, but may be looked upon as a device for producing low-pressure steam for heating, at a very low cost.

Suppose that the second effect of a multiple-effect evaporator is boiling at 185°F., and suppose that there are places in the plant where liquids must be heated to temperatures not exceeding 175°F. If this heating is accomplished by vapor withdrawn from the second effect of the evaporator, it is done with steam that has already evaporated twice its own weight of water, and the total heat balance of the plant is thereby favorably affected, as compared with the case where this heating is done with steam from the mains. If such withdrawals of steam are small compared to the total amount of steam passing from effect to effect, no special provision need be made in the design of the evaporator, and the evaporator will ordinarily attain the proper temperature distribution to permit this larger generation of steam in the effect from which it is withdrawn. If the quantity of steam so withdrawn is large compared to the amount passing from effect to effect, it may be necessary to increase the heating surface in some of the bodies to preserve a reasonable temperature distribution. The numerical results of such steam withdrawals can be determined only by the calculation of heat balances, but it may be stated that such arrangements will in many cases prove to make possible quite astonishing economies in the steam

consumption of the plant as a whole, although it is true that such withdrawals do decrease the steam economy of the evaporator itself. This principle has not yet been properly understood or systematically applied in the process industries as a whole.

Removal of Non-condensed Gas.—The non-condensed gas from the steam space of the first effect represents only gas that was present in the original steam supply. In all effects after the first, however, air may be present that has been drawn in through leaks in the apparatus, or that was in solution in the thin liquor. In some cases, gases may be evolved by reactions during the course of evaporation.

When the amount of non-condensed gas to be removed is small, it is most convenient to vent from the steam space back into the vapor space of the same effect. In this case, if the vents are opened too wide, the steam merely by-passes one effect and is not completely lost, while the concentration of gas may not reach undesirable amounts even in the last effect. On the other hand, if the amount of non-condensed gas is large, this method of venting may result in excessive concentrations of gas in the steam in the last effect and therefore unduly low heat-transfer coefficients. In such cases it is best to vent each body directly to the condenser and thus possibly waste a little steam rather than depreciate heat-transfer coefficients. For instance, in concentrating beet-sugar juices, appreciable quantities of ammonia are evolved from the juices, and the latter method of venting is therefore always employed.

MULTIPLE-EFFECT CALCULATIONS

It should again be stated that the general relationship used on page 220, in developing the principles of multiple-effect operation, (namely, that one pound of steam evaporates one pound of water in a single effect, two pounds in a double effect, etc.), is a very rough approximation. This relationship is so affected by the temperature of the feed, the temperature range in the evaporator, the ratio between the weight of thin liquor and thick liquor, and other factors, that it should not be used for even the most approximate estimates of the performance of multiple-effect evaporators. The only way to determine in advance the performance of a proposed evaporator is to carry through the complete heat-balance calculations.

The results usually desired from the calculations for a multiple-effect evaporator are: the amount of steam to be used, the amount of heating surface needed, the approximate temperatures in the various effects, and the amount of vapor leaving the last effect and going to the condenser. As in the case of the single-effect evaporator (page 196) the important relationships available are material balances, heat balances, and the heat-transfer Equation (64). In the case of a multiple-effect evaporator, however, a trial and error method is used rather than a direct algebraic solution.

Consider, for instance, a triple-effect evaporator. There are seven equations that might be written: a heat balance for each effect, a rate equation for each effect, and the known sum of the evaporations in the three individual effects. There are seven unknowns in these equations: (1) Steam to the first effect, (2, 3, 4) evaporation in each effect, (5) temperature of boiling liquid in the first effect, (6) temperature of boiling liquid in the second effect, and (7) the heating surface in each effect. This is based on the commonest condition, where the heating surface of all the effects is to be equal. These seven equations conceivably might be solved for the seven unknowns but the method is tedious and involved. It is customary to use a method approximately as follows:

1. Assume values for the temperatures of the first and second effects.
2. By means of heat-balance equations across each effect, determine the evaporation in each effect.
3. By means of the rate equations, calculate the heating surface needed for each effect.
4. If the heating surfaces so determined are not essentially equal for the three effects, redistribute the temperature drops and repeat items 2 and 3 till the heating surfaces are equal.

The second trial can usually be made to give the desired result, so the method is not unreasonably tedious.

The arrangements possible for multiple-effect evaporators are so numerous that it is not worth while to make any formulation of specific cases. The heat-balance equations are relatively simple and can be written for each case as it arises.

The modification of the heat-balance equation, for the case where boiling-point elevation is appreciable, is simple. As suggested on page 198, the latent heat in any particular body may

be taken at the pressure in that body. Steam from that body will give up, in the next effect, not only its latent heat at the pressure in the steam space, but also the superheat equivalent to the boiling-point elevation in the effect from which the steam came. Thus, the heat given up by a pound of steam from a given effect is somewhat more than the heat necessary to generate that pound of steam in the effect from which it came.

The application of the method is best shown by some examples.

Example 18.—It is desired to concentrate a solution from 10 to 50 per cent solids in a triple-effect evaporator. Steam is available at 15 lb. per sq. in. gage (249°F.), and a vacuum of 26 in. referred to a 30-in. barometer is maintained in the vapor space of the last effect. This pressure corresponds to a boiling point of 125°F. The feed to the evaporator is 55,000 lb. per hr. and its temperature is 70°F. The condensate leaves each effect at the temperature of the steam. The solution has a negligible elevation in boiling point, and its specific heat may be taken as 1.00 at all concentrations Coefficients may be assumed as 550 in the first effect, 350 in the second, and 200 in the third, for forward feed; and 450, 350, and 275 for backward feed, all expressed as B.t.u. per sq. ft. per °F. per hr. All bodies are to have the same heating surface.

Calculate the heating surface, the steam consumption, the distribution of temperatures, and the pounds of water evaporated per pound of steam.

A. With forward feed.

B. With backward feed.

C. With forward feed and 4,000 lb. of *extra steam* withdrawn from the second effect.

Solution.—The evaporation is to be the same in all three cases and may be calculated from an overall material balance, assuming that the solids go through the evaporator without losses.

	Pounds		
	Total	Solid	Water
In feed liquor	55,000	5,500	49,500
In thick liquor	11,000	5,500	5,500
Evaporation	44,000	.....	44,000

Case A. Forward Feed.—As a guide in the first attempt to distribute the total temperature drop across the effects, certain general principles are of value. As discussed on page 219, the temperature drop across a given effect must be larger as the heat-transfer coefficient is smaller. Also, if any effect

has an extra load, that effect requires a larger proportion of the total temperature drop. In this problem, the total temperature drop is (249 − 125) or 124°F. The last effect must have the largest drop, and because of the large amount of heating to be done in the first effect, this effect may have a larger drop than the second effect. From these considerations, the first assumption will be

$$\Delta t_1 = 38°, \Delta t_2 = 33°, \Delta t_3 = 53°.$$

The conditions will then be as follows:

Steam to first effect...........	249°F., Latent heat =	946 B.t.u. per lb.
Δt_1..........................	38°F.	
Boiling point in first effect.....	211°F., Latent heat =	971
Δt_2..........................	33°F.	
Boiling point in second effect...	178°F., Latent heat =	991
Δt_3..........................	53°F.	
Boiling point in third effect....	125°F., Latent heat =	1,022

Heat-balance equations may now be written across each effect as follows:

Let

x = pounds evaporated in first effect.
y = pounds evaporated in second effect.
z = pounds evaporated in third effect.
w = pounds of steam furnished to first effect.

The datum temperature for each effect will be the boiling point in that effect. In feeding from the first to the second effect, and from the second to the third, there will be some vapor formed by flashing.

Across the first effect

(Heat in steam above 211°) − (heat to heat feed to 211°) =
(latent heat of vapor at 211°) + (heat in condensate above 211°)

It will be noted that in cases like this where condensate leaves at steam temperature, the difference of the first and last terms is the latent heat of the steam entering at 249°. Hence it follows that

$$946w - 55{,}000(211 - 70) = 971x$$

Similarly, for the second effect:

$$971x + (55{,}000 - x)(211 - 178) = 991y$$

for the third effect:

$$991y + (55{,}000 - x - y)(178 - 125) = 1{,}022z$$

and for all three effects

$$44{,}000 = x + y + z$$

Solving these four equations gives

$$x = 13{,}610 \text{ lb.}$$
$$y = 14{,}720$$
$$z = 15{,}670$$

$$\text{Total} = 44{,}000 \text{ lb.}$$

By using the rate equations for each effect, letting A_1, A_2, and A_3 be the areas of the heating surfaces in the first, second, and third effects, respectively, the areas are found as follows:

$$A_1 = \frac{971x + (55{,}000)(211 - 70)}{(38)(550)} = 1{,}003 \text{ sq. ft.}$$
$$A_2 = \frac{971x}{(33)(350)} = 1{,}144 \text{ sq. ft.}$$
$$A_3 = \frac{991y}{(53)(200)} = 1{,}375 \text{ sq. ft.}$$
$$\text{Average} = 1{,}174 \text{ sq. ft.}$$

A condition of the problem is that the heating surface is to be the same in all effects, but the above solution shows that to maintain the assumed temperature distribution requires widely different heating surfaces. Hence, the temperature distribution must be readjusted. Neglecting the small effects on latent heat, heating and flashing caused by a change in the temperature distribution, it may be assumed that the heating surfaces above determined will change inversely as the temperature drops.

The third effect is too large, and therefore must have a larger temperature drop to bring its surface down. The first two effects are too small, and must have smaller temperature drops. The tentative temperature drops may be corrected in proportion to the deviations of the surfaces from the mean surface as follows:

$$\Delta t_1 = 38 \times \frac{1003}{1174} = 32°$$
$$\Delta t_2 = 33 \times \frac{1144}{1174} = 32°$$
$$\Delta t_3 = 53 \times \frac{1375}{1174} = 62°$$

These add up to a total of 126°, but the total available is only 124°. They are therefore scaled down to $\Delta t_1 = 32°$, $\Delta t_2 = 31°$, $\Delta t_3 = 61°$. One is seldom justified in going to fractions of a degree; not only because the data are not known so accurately, but also because the terminal conditions are seldom held constant to $\pm 1°$ in operation.

The revised conditions are

Steam to first effect	249°	Latent heat = 946
Δt_1	32°	
Boiling point in first effect	217°	Latent heat = 967
Δt_2	31°	
Boiling point in second effect	186°	Latent heat = 986
Δt_3	61°	
Boiling point in third effect	125°	Latent heat = 1,022

The heat-balance equations may be rewritten as follows:

$$946w - 55{,}000(217 - 70) = 967x$$
$$967x + (55{,}000 - x)(217 - 186) = 986x$$
$$986y + (55{,}000 - x - y)(186 - 125) = 1{,}022z$$
$$x + y + z = 44{,}000$$

From these equations:

$x = 13{,}610$ lb.
$y = 14{,}660$ lb.
$z = 15{,}730$ lb.

Total $= 44{,}000$ lb.
$w = 22{,}460$ lb.
$A_1 = 1{,}207$ sq. ft.
$A_2 = 1{,}213$ sq. ft.
$A_3 = 1{,}185$ sq. ft.

Average = 1,202, or, say, 1,200 sq. ft. per effect.

The evaporation per pound of steam is $\frac{44{,}000}{22{,}460} = 1.96$ lb.

Case B. Backward Feed.—After the first trial and readjusting temperature drops by the method outlined above, the second approximation is $\Delta t_1 = 35°$, $\Delta t_2 = 42°$, $\Delta t_3 = 47°$. The conditions will be

Steam to first effect	249°	Latent heat = 946
Δt_1	35°	
Boiling point in first effect	214°	Latent heat = 969
Δt_2	42°	
Boiling point in second effect	172°	Latent heat = 995
Δt_3	47°	
Boiling point in third effect	125°	Latent heat = 1,022

Letting x, y, and z be the weight evaporated in each effect and w the steam to the first effect, and writing heat-balance equations across each effect, as before, gives

$$995y - 55{,}000(125 - 70) = 1{,}022z$$
$$969x - (55{,}000 - z)(172 - 125) = 995y$$
$$946w - (55{,}000 - y - z)(214 - 172) = 969x$$
$$x + y + z = 44{,}000$$

Solving these equations gives:

$$x = 17{,}460 \text{ lb.}$$
$$y = 14{,}950 \text{ lb.}$$
$$z = 11{,}590 \text{ lb.}$$

$$\text{Total} = 44{,}000 \text{ lb.}$$
$$w = 19{,}140 \text{ lb.}$$

The areas of the heating surfaces are

$$A_1 = 1{,}149 \text{ sq. ft.}$$
$$A_2 = 1{,}151 \text{ sq. ft.}$$
$$A_3 = 1{,}151 \text{ sq. ft.}$$

$$\text{Average} = 1{,}150 \text{ sq. ft.}$$

$$\text{Evaporation per pound of steam} = \frac{44{,}000}{19{,}140} = 2.30 \text{ lb.}$$

If these results are compared with Case A, it will be seen that backward feed effects a saving of 3,320 lb. of steam per hr., or 15 per cent of the steam used in Case A. This is the usual result of backward feed when the temperature of the feed is low. The heating surface needed for backward feed is slightly less than for forward feed, but this is largely due to the particular values assumed for the coefficients and is not a general result for all cases of backward feed. Case A gave 1.96 lb. of water evaporated per pound of steam, and Case B gave 2.30 lb. This shows how unsafe it is to base an accurate estimate on the general statement that "1 lb. of steam evaporates 1 lb. of water in a single effect, 2 lb. in a double effect, etc.," although such statements are more nearly correct if the feed temperature is between the temperatures of the first and last effects.

Case C. Withdrawal of Extra Steam.—After one trial, it is found that the temperature drops should be 35°, 36° and 53°. The latent heat of the vapors is 969 B.t.u. per lb. in the first effect and 991 in the second effect. The equations are

$$946w = 969x + (55{,}000)(214 - 70)$$
$$969x + (55{,}000 - x)(214 - 178) = 991y$$
$$991(y - 4{,}000) + (55{,}000 - x - y)(178 - 125) = 1{,}022z$$
$$x + y + z = 44{,}000$$

From which

$$x = 14{,}960 \text{ lb.}$$
$$y = 16{,}080 \text{ lb.}$$
$$z = 12{,}960 \text{ lb.}$$

$$\text{Total} = 44{,}000 \text{ lb.}$$
$$A_1 = 1{,}164 \text{ sq. ft.}$$
$$A_2 = 1{,}156 \text{ sq. ft.}$$
$$A_3 = 1{,}130 \text{ sq. ft.}$$

$$\text{Average} = 1{,}150 \text{ sq. ft.}$$
$$w = 23{,}700 \text{ lb.}$$
$$\text{Evaporation per pound steam} = 1.86 \text{ lb.}$$

If Case *C* is compared with Case *A*, it will be seen that there has been an increase of 1,240 lb. in the steam used by the evaporator. In Case *A*, however, approximately 4,000 lb. of steam must have been taken from the mains as live or exhaust steam, to do the work done by the 4,000 lb. withdrawn from the evaporator in Case *C*. Hence, the total steam consumption for the evaporator and this other operation combined was 26,460 lb. in Case *A* but only 23,700 lb. in Case *C*, a saving of 2,760 lb. of steam per hr., or 10.4 per cent of the total steam used in Case *A*. This saving has been accomplished without any changes in the evaporator except adding a connection in the vapor line between the second and third effects.

Nomenclature of Chapter V

A = area
N = number of effects
Q = total heat
q = heat flow, B.t.u. per hour
R = thermal resistance
t = temperature
Δt = temperature drop
U = overall heat-transfer coefficient
W = weight of steam
w = pounds of steam furnished to first effect
x = pounds evaporated in first effect
y = pounds evaporated in second effect
z = pounds evaporated in third effect
sub av = average
sub o = initial values
sub 1, 2, 3 = in first, second, or third effects
α, β = constants
θ = time

Problems

1. Certain experiments to determine the elevation in boiling point of NaOH solutions gave the following data:

Concentration, parts NaOH per 100 parts H_2O	Boiling point, degrees Centigrade	Pressure, millimeters or mercury
40.74	74.65	133.9
40.74	116.69	680.5
51.75	80.30	138.4
51.75	120.76	674.4

What would be the boiling point of a solution containing 45 parts NaOH per 100 parts H_2O at a pressure under which water boils at 180°F.?

2. A standard vertical-tube evaporator has 500 sq. ft. of heating surface based on outside tube area. In a certain test run 5,130 lb. of steam were condensed in 1 hr. The pressure in the vapor space was 87.8 mm., and the pressure in the steam space was 400 mm. Condensate left the evaporator at 170°F. The evaporator was boiling saturated sodium chloride solution. The evaporator tubes were 4 ft. 6 in. long and the liquor level averaged 4.5 in. above the tops of the tubes. Calculate the heat-transfer coefficients (*a*) apparent, (*b*) corrected for elevation in boiling point.

3. A certain evaporator is boiling a solution that deposits calcium sulfate scale. After 4 hr. 10 min. operation the coefficient was 545, and after 12 hr. 30 min. it was 430. What would the coefficient be after 48-hr. operation? What would be the coefficient at the start of the cycle?

4. An evaporator is to operate under the following conditions:

Feed solution: 40,000 lb. per hr. 5 per cent solids.

Thick liquor: 45 per cent solids.

Steam: 250°F. Feed temperature: 200°F.

Saturation temperature of vapor to condenser: 125°F.

Specific heats of all solutions: 1.0.

Negligible boiling-point elevation at all concentrations.

No precipitation of solids in evaporator.

Neglect radiation.

Assume all condensates leave at condensing temperature.

In multiple-effect cases, the heating surfaces of the effects are to be equal.

Calculate, for each of the following cases,

a. The square feet of heating surface per effect.

b. The steam consumption per hour.

c. The pounds of water evaporated per pound of steam.

Case I. Single effect; coefficient 700.

Case II. Double effect; forward feed; coefficients 750 in first effect, 650 in second.

Case III. Triple effect; forward feed; coefficients 700 in first effect, 650 in second, 600 in third.

5. Repeat the same problem, but assume the solution has an appreciable rise in boiling point. Use the same coefficients as in Problem 4, but assume they have been corrected for boiling-point elevation.

Case I. Single effect; boiling-point elevation 25°F.

Case II. Double effect; forward feed; boiling-point elevations 10°F. in first effect and 25° in second.

Case III. Triple effect; forward feed; boiling-point elevations 8°F. in first effect, 12° in second, 25° in third.

6. A triple-effect evaporator, each effect of which has 1,500 sq. ft. of heating surface, is to be used to concentrate a solution possessing a negligible boiling-point elevation from 5 per cent solids to 40 per cent solids. The 40 per cent liquor is quite viscous, and the 5 per cent solution is at 180°F., so a mixed feed, in the order II-III-I, is to be used. The steam available is at 250°F., and a vacuum corresponding to 110°F. is to be maintained in the last effect. It is expected that the overall coefficients of heat transfer will be:

I—300; II—500; III—400.

The specific heats of all solutions can be taken as 1.0; the condensates all leave at condensing steam temperature; and radiation is negligible. Calculate:

a. The pounds of 5 per cent liquor that can be concentrated per hour.

b. The steam economy that can be expected.

7. The evaporator of Example 18 is to be fed in parallel with 25 per cent NaCl solution at 60°F. The total evaporation is to be 44,000 lb. per hr. and no thick liquor is to be withdrawn. Use the coefficients given in Example 18 for backward feed and assume that they have been corrected for elevation in boiling point. The specific heat of brine is 0.79. Steam temperature and vacuum are to be the same as in Example 18, and all bodies are to have equal heating surfaces. Neglect heat leaving in hot salt and the brine that accompanies it. Assume that condensate leaves each effect at the saturation temperature of the steam. What are the surface required, the steam consumption, the distribution of temperatures, and the pounds water evaporated per pound of steam?

CHAPTER VI

GENERAL MECHANISM OF DIFFUSION PROCESSES

In the discussion of fluid flow it was shown that thin fluid films were set up at the interface between a solid and a moving fluid if the velocity of the fluid relative to the solid was above a definite critical magnitude. In the chapters on Flow of Heat and Evaporation it was emphasized that the rate of heat transfer from fluid to solid and *vice versa* was largely controlled by the conduction of heat through these films. The next six chapters will consider a group of unit operations that are related by the fact that their rates involve, to a greater or lesser extent, the diffusion of material as well as the transfer of heat through fluid films. These operations are (*a*) humidification (or dehumidification) of air by direct contact with liquid water (Chap. VII); (*b*) drying (Chap. VIII); (*c*) rectification (Chap. IX); (*d*) gas absorption (Chap. X); (*e*) extraction (Chap. XI); and (*f*) crystallization (Chap. XII).

Unit Operations That Depend on Diffusion.—Although it may anticipate these later chapters, it is of interest to analyze the above operations from the viewpoint of diffusion. It must be emphasized, however, that these same processes also involve the flow of heat, although this chapter will be restricted to the discussion of diffusion.

In the cases of humidification and dehumidification, water vapor diffuses through a gas film. If the gas is being humidified, the water vapor diffuses through the film from the liquid into the gas. If the gas is being dehumidified, the direction of diffusion is the reverse. In the drying of solids by hot air, moisture diffuses first (as either a liquid or a vapor) through the solid, and then as vapor through the air film. An actual drying process may, however, be characterized by the fact that the resistance to diffusion through the solid is so great that the effect of the vapor-film resistance is negligible; on the other hand, if the solid is porous, and water diffuses through it readily, the air film may offer the only important resistance to the diffusion of water

vapor, and the process becomes essentially the same as that of the humidification of air.

In the absorption of one or more components of a gaseous mixture by a liquid there are two diffusional processes that take place in series. The gas being absorbed must diffuse first through the gas film, which is made up largely of the non-absorbable gases, and second through the liquid film, since the absence of a metal wall does not affect the films themselves. Here, again, there are some cases that are simplified by the fact that one of these diffusional resistances is of controlling importance in comparison with the other. For gases of high solubility, it will be shown that the liquid-film resistance is unimportant in comparison with that of the gas film, while for very slightly soluble gases, the liquid film controls the absorption rate and the effect of the gas film can be neglected.

The extraction of soluble material from an inert solid by means of a liquid is a process that is analogous to gas absorption. The essential difference is that the solvent liquid dissolves soluble material from a solid rather than from a gas. In this case, the soluble material must, in general, diffuse through the liquid film in contact with the solid.

Crystal growth, and its reverse process, solution of soluble material, involve the diffusion of a component to or from a solid interface, through a liquid film. These processes differ from extraction in that there is no inert or insoluble component in the solid phase, and in that the solid phase has the crystalline structure, *i.e.*, its units (atoms or ions) are arranged in definite, regular array in a so-called *space lattice*.

In the above operations, one or more components diffuse in one direction through one or more films that are composed mainly of an inert material. The remaining process in the list, namely, rectification, differs from the others in that more than one component is diffusing, these components are not passing in the same direction, and there is no inert material present. For example, consider the simplest case, namely, a two-component system. This process involves the diffusion of one component from vapor to liquid, and the other component from liquid to vapor.[1] The two films are formed, not of a stationary or an inert material, but of two active components,

[1] Lewis and Chang, *Trans. Am. Inst. Chem. Eng.*, **21**, 127 (1928).

and these components are passing countercurrent to each other. This means that the rate of diffusion of one component is affected by the rate of diffusion of the other.

There are, then, certain factors that are common to all of the above operations. They all involve the diffusion of material through liquid and gaseous films. These diffusion processes are best handled by the kinetic theory, and it is to this branch of physics that one must turn for basic quantitative relationships.

Conditions at the Interface in Diffusion Processes.—In addition to the question of diffusion, however, happenings at the actual interface between the two phases (the boundary surface between the films) must be considered. It is reasonable to assume that the two phases are at equilibrium at the boundary surface. That is, if the composition of, say, the liquid phase at the interface is specified, the composition of the vapor phase at the interface is exactly the same as if the two phases had been in contact for an indefinite length of time. A corollary of this assumption is that a diffusing substance, once it gets through one of the films, passes into the second film without resistance at the interface itself; since, if there is no resistance, there is no driving force necessary, and therefore the two phases are in equilibrium at this point, in spite of the fact that there are diffusion gradients in the films at either side of the interface.[1]

In spite of the plausibility of the interfacial assumption, it can by no means be demonstrated *a priori* that it is valid. It must be experimentally proved before it can be used with safety. It is apparent that to check the assumption by direct experiment would be very difficult. It is usually possible, however, in such cases, to test such a hypothesis in an indirect manner by checking, experimentally, conclusions that are based on the hypothesis. There is enough such indirect evidence available that the assumption of interfacial equilibrium is reasonably well established, in cases involving liquids and gases.

The one exception to the hypothesis of interfacial equilibrium that has been found is that of crystal growth. In this process the evidence indicates that the solute, after diffusing through the

[1] It is important to differentiate between two phases in equilibrium at the interface and two phases in equilibrium throughout. In the latter case, there is no transfer of material, and the process has stopped. In the former case, the process is actually in operation.

liquid film, must still overcome the equivalent of a further resistance, probably involving the orientation of the molecules into the definite crystal lattice. This process will be considered further in the chapter on Crystallization.

EQUATIONS FOR RATE OF DIFFUSION

From the above analysis it is apparent that equations must be derived for diffusion through both gas and liquid films before computation methods for the diffusion process can be considered complete. At present, reasonably rigorous basic equations are available for gas diffusion, but those for the liquid side are very approximate and have been developed largely through analogy with gaseous diffusion. The equations given below are derived from those presented by Lewis and Chang,[1] as a result of their examination of earlier derivations by Maxwell and Stefan.

Gaseous Diffusion.—Consider a stream of gas, composed of two components a and b, that is flowing in turbulent flow past the surface of a liquid into which component a is diffusing. Just as a temperature gradient is necessary to make heat flow from one point to another, so a partial pressure gradient is necessary to make material diffuse from one point to another. In the discussion of flow of heat from a stream of gas in turbulent flow, it was pointed out that most, but not all, of the temperature gradient between the bulk of the gas and the interface occurred across the film. There is considerable evidence that a partial pressure gradient, similar to the temperature gradient, extends from the center of the turbulent gas stream to the interface in strict analogy with the temperature gradient of Fig. 76; and that most (but not all) of this pressure gradient is across the film.[2] In the case of diffusion, however, there are fewer experimental data available, and it is necessary to simplify the conception of the process by assuming that the entire partial-pressure gradient lies across a fictitious film, next to the interface, of such a thickness that it is equivalent to the combined resistance of both turbulent core and actual film. With this simplification, the process of diffusion of component a can be visualized as a molecular diffusion through the fictitious equivalent film.

[1] *Loc. cit.*

[2] COLBURN, *Ind. Eng. Chem.*, **22**, 967–970 (1930).

For the case of steady flow of one gas from its mixture with an inert gas to the interface, the following equation has been derived:

$$\frac{w}{A} = \frac{b_{gw}(p_{ag} - p_{ai})}{(p_b)_m B_g} \tag{112}$$ [1]

where

w = weight of component a diffusing per unit time.
A = interfacial area across which the diffusion is occurring.
b_{gw} = diffusion coefficient.
B_g = equivalent film thickness.
p_{ag} = partial pressure of component a in the bulk of the gas stream.
p_{ai} = partial pressure of component a at the interface.
p_{bi} = partial pressure of component b at the interface.
p_{bg} = partial pressure of component b in the gas stream.
$(p_b)_m$ = logarithmic mean of p_{bg} and p_{bi}.

The partial pressures are related to the total pressure by Dalton's Law, and therefore

$$P = p_{ag} + p_{bg} = p_{ai} + p_{bi} \tag{113}$$

It was pointed out on page 125 that when the ratio of the two terms of a logarithmic mean is not greater than 2:1, the arithmetic mean is amply adequate. This rule applies to the term $(p_b)_m$ of Equation (112) and in most applications of Equation (112), $(p_b)_m$ can be taken as

$$(p_b)_m = \frac{p_{bi} + p_{bg}}{2} \tag{114}$$

By comparing Equation (112) with the Fourier equation (Equation (43)) it is seen that w corresponds to q, $(p_{ag} - p_{ai})$ to Δt, b_{gw} to k, and B_g to L. Thus in this case the rate of flow (*material* in this case as compared to *heat* in Equation (43)) is proportional to a coefficient, an area, a potential difference, and inversely proportional to a distance. The principal point of difference is that in Equation (112) the term $(p_b)_m$ appears,

[1] LEWIS and CHANG, *loc. cit.;* HANKS and MCADAMS, *Ind. Eng. Chem.*, **21**, 1034 (1929).

for which there is no corresponding term in Equation (43). In strict analogy to the reasoning used in discussing the flow of heat, $\frac{(p_b)_m B_g}{b_{gw}}$ can be considered a resistance, and then Equation (112) falls into the form.

$$\text{Rate} = \frac{\text{driving force}}{\text{resistance}} \tag{42}$$

The Gas-diffusion Coefficient.—The term b_{gw} in Equation (112) is the diffusion coefficient and may be expressed as $\frac{\text{(gm.)(cm.)}}{\text{(hr.)(sq. cm.)}}$ or as $\frac{\text{(lb.)(ft.)}}{\text{(hr.)(sq. ft.)}}$. The values of the coefficient as given in the literature, however, are usually given in volume units, expressed as $\frac{\text{(cu. cm.)(cm.)}}{\text{(sec.)(sq. cm.)}}$ with the volume measured at a definite temperature and pressure. If the diffusion coefficient in c.g.s. volume units at T_0°R., and a pressure of P_0 atm., be denoted by b_{gv0} then its value (b_{gv}) at another temperature T°R. and pressure P atm. is[1]

$$b_{gv} = b_{gv0}\left(\frac{T}{T_0}\right)^{3/2}\left(\frac{P_0}{P}\right) \tag{115}$$

For engineering calculations it is more convenient to use the diffusion coefficient expressed in English units and involving weight rather than volume. If b_{gw} is the coefficient in $\frac{\text{(lb.)(ft.)}}{\text{(hr.)(sq. ft.)}}$, application of the gas laws to Equation (115) gives

$$b_{gw} = 5.32 b_{gv0} P_0 \frac{T^{1/2}}{T_0^{3/2}} M_a \tag{116}$$

where

b_{gw} = diffusion coefficient in $\frac{\text{(lb.)(ft.)}}{\text{(hr.)(sq. ft.)}}$.

b_{gv0} = diffusion coefficient, measured at P_0 and T_0, in $\frac{\text{(cu. cm.)(cm.)}}{\text{(sec.)(sq. cm.)}}$ or $\frac{\text{sq. cm.}}{\text{sec.}}$.

P = pressure in atmospheres.

T = absolute temperature in degrees Rankine.

M_a = molecular weight of diffusing component.

[1] Gilliland, *Ind. Eng. Chem.*, **26**, 681 (1934).

Example 19.—Carbon dioxide gas at 80°F. is diffusing from an air-CO_2 mixture through an air-CO_2 film in contact with a carbonate solution. The equivalent thickness of the film is 0.1 mm. The partial pressure of the CO_2 in the bulk of the gas is 150 mm. Hg, and that at the interface between the solution and gas is 10 mm. Hg. The total pressure is 760 mm. If $b_{gv0} = 0.128$ sq. cm. per sec. at 86°F. and 1 atm., how many pounds of CO_2 are being absorbed by the carbonate solution per hour per square foot of interfacial area?

Solution.—The first step is to convert b_{gv0} to weight units by Equation (116). The values to be used are:

$$b_{gv0} = 0.128$$
$$P_0 = 1.00$$
$$T = 80 + 460 = 540°\text{R.}$$
$$T_0 = 86 + 460 = 546°\text{R.}$$
$$M_a = 44$$
$$b_{gw} = 5.32 \times 0.128 \times 1.00 \times \frac{(540)^{1/2}}{(546)^{3/2}} \times 44 = 0.0546$$

The weight actually diffusing is determined by Equations (112) and (114).

$$p_{ag} = {}^{150}\!/_{760} = 0.197 \text{ atm.}$$
$$p_{ai} = {}^{10}\!/_{760} = 0.013 \text{ atm.}$$
$$(p_b)_m = \frac{(1 - 0.197) + (1 - 0.013)}{2} = 0.895 \text{ atm.}$$
$$B_g = \frac{0.1}{305} = 0.000328$$
$$\frac{w}{A} = \frac{0.0546(0.197 - 0.013)}{0.895 \times 0.000328} = 33.99 \text{ lb. per sq. ft. per hr.}$$

Liquid Diffusion.—The equations for the rate of diffusion of material through a liquid are of the same form as those representing the rate of diffusion through a gas, except that concentrations are used instead of partial pressures. In gaseous diffusion the driving force is the partial-pressure difference of the diffusing material. In liquid diffusion the driving force is the concentration difference of the component that is diffusing. The theory of liquid diffusion is not so completely worked out, however, as that of gaseous diffusion, since the kinetic theory of liquids is not so well known as that of gases because of the greater difficulty of the subject.

By analogy with the Fourier equation and Equation (112), the equation for the rate of diffusion of a solute through a liquid film may be written as

$$\frac{w}{A} = \frac{b_{Lw}(c_{ai} - c_{aL})}{B_L} \tag{117}$$

where

w = weight of solute diffusing per unit time.
A = interfacial area across which diffusion is occurring.
b_{Lw} = a diffusion coefficient.
c_{ai} = concentration of solute at the interface.
c_{aL} = concentration of solute in the bulk of the solution.
B_L = equivalent film thickness.

It will be noted that this equation is exactly analogous to the Fourier equation (Equation (43)) and differs from Equation (112) in not containing any term for the concentration of the solvent at the interface. Not enough is known about diffusion in liquids to warrant the inclusion of such a term at present.

The Liquid-diffusion Coefficient.—The constant b_{Lw}, if measured in metric units, can be expressed as

$$\frac{\text{(g.)(cm.)}}{\text{(sec.)(cm.)}^2\text{(unit difference in concentration)}}$$

or $\frac{\text{(g.)(cm.)}}{\text{(sec.)(cm.)}^2\text{(g.)/(cm.)}^3}$ This latter simplifies to $\frac{\text{(cm.)}^2}{\text{sec.}}$

The constant b_{Lw} is most conveniently used as square feet per hour. The constants obtained from the literature can be converted to this unit[1] by multiplying by $3{,}600/(30.5)^2 = 3.87$.

The constant b_{Lw} varies with both temperature and concentration.

In case b_{Lw} is used as square feet per hour, the concentration terms in Equation (117) must be expressed in pounds per cubic foot.

Nomenclature of Chapter VI

A = interfacial area across which diffusion is occurring
a = component to be absorbed
B = equivalent film thickness
b = diffusion coefficient
M = molecular weight

[1] The metric unit is sometimes given in square centimeters per day. This unit is converted to square feet per hour by multiplying by $1/(30.5)^2(24) = 0.000{,}044{,}8$.

P = total pressure
p = partial pressure
T = absolute temperature
w = weight per unit time
sub a, b refers to component a or b.
sub g refers to gas phase.
sub i refers to interface.
sub L refers to liquid phase.
sub m = mean value
sub 0 = initial or standard conditions
sub v = coefficients in volume units
sub w = coefficients in weight units

Problem

Ethyl alcohol is diffusing into an alcohol-water solution at 18°C. The concentration of the alcohol in the bulk of the solution is negligible, while that at the surface of the solution is 0.12 g. per cu. cm. The film thickness is 0.15 cm., and the coefficient b_{Lw} is 1.0×10^{-5} sq. cm. per sec. How many pounds of alcohol are diffusing through 100 sq. ft. of interface per hour?

CHAPTER VII

HUMIDITY AND AIR CONDITIONING

In many of the unit operations it is necessary to make calculations involving the properties of mixtures of air and water vapor. Such calculations may require a knowledge of the amount of water vapor carried by air under various conditions, the thermal properties of such mixtures, the changes in heat content and moisture content as air containing some moisture is brought in contact with water or wet solids, and similar problems. This chapter will discuss the fundamental properties of mixtures of air and water vapor, the mechanism of such processes as those suggested above, and the apparatus in which these processes are carried out.

Definitions.—In discussions of the physical properties of mixtures of air and water vapor, the term *humidity* has been used in the past with more than one meaning. A certain definition for this term originated with meteorologists[1] and in this form was used by engineers also. The terminology and the definitions of the meteorologist are, however, quite unsuited for engineering calculations; and a new definition of many of the terms involved was given by W. M. Grosvenor[2] in 1908. Grosvenor's work greatly simplified the calculation of air-water vapor mixtures, and his definitions will be used exclusively in this book.

Humidity is defined as the pounds of water vapor carried by 1 lb. of dry air under any given set of conditions. Humidity so defined depends only on the partial pressure of water vapor in the air and on the total pressure (*assumed throughout this chapter to be* 760 *mm.*). If the partial pressure of water vapor in the sample of air in question be p atm., the ratio of (mols water

[1] *Relative humidity* is defined in this system as $100p/p_s$, where p is the partial pressure of water vapor in the air-water mixture under discussion and p_s is the vapor pressure of liquid water at the same temperature (*i.e.*, the partial pressure of water vapor in saturated air at this temperature).

[2] *Trans. Am. Inst. Chem. Eng.*, **1**, 184–202 (1908).

vapor) to (mols dry air) is equal to the ratio of p to $(1 - p)$. Since the molecular weight of water is 18, and of air 29, the ratio of water vapor to dry air by weight is $\frac{18p}{29(1 - p)}$. Since humidity has been defined as pounds of water vapor per pound of dry air, it follows that

$$H = \frac{18p}{29(1 - p)} \tag{118}$$

where H is the humidity.

Saturated air is air in which the water vapor is in equilibrium with liquid water at the given conditions of temperature and pressure. In such a mixture, the partial pressure of water vapor in the water-air mixture is equal to the vapor pressure of pure water at that temperature.

Percentage humidity is obtained by dividing the weight of water carried by 1 lb. of dry air at any temperature and pressure by the weight of water 1 lb. of dry air could carry if saturated at that same temperature and pressure, and expressing the result on a percentage basis.

Humid heat is the number of B.t.u. necessary to raise the temperature of 1 lb. of dry air, plus whatever moisture it may carry, 1°F. If, for the temperatures ordinarily involved, the specific heat of air be assumed as 0.238, and that of water vapor as 0.48, then humid heat is defined by the equation

$$s = 0.238 + 0.48H \tag{119}$$

where

s = humid heat in B.t.u. per pound dry air.

Humid volume is the total volume in cubic feet of 1 lb. of air and its accompanying water vapor. This may be calculated from the equation

$$V = \left(\frac{359}{29}\right)\left(\frac{t + 460}{492}\right) + \left(\frac{359H}{18}\right)\left(\frac{t + 460}{492}\right) = (0.730t + 335.7)\left(\frac{1}{29} + \frac{H}{18}\right) \tag{120}$$

Where V is the humid volume of 1 lb. of dry air plus its accompanying moisture.

Saturated volume is the volume in cubic feet of 1 lb. of dry air plus that of the water vapor necessary to saturate it.

Dew point is the temperature to which a mixture of air and water vapor must be cooled (at constant humidity) in order to become saturated (*i.e.*, to be in equilibrium with liquid water at the dew point).

Wet-bulb Temperature.—If a large amount of unsaturated air is brought into contact with a small amount of water under adiabatic conditions, the water approaches a definite equilibrium temperature which is lower than that of the air but higher than the dew point. This temperature is called the wet-bulb temperature, because it is usually obtained by causing the air in question to flow over the wetted bulb of an ordinary thermometer. The mechanism of this process will be discussed later.

HUMIDITY CHART

Most of the properties of mixtures of air and water vapor needed for engineering calculations are contained in the humidity chart, Fig. 114, which will be found at the end of the book. In this chart humidities (expressed as pounds of water per pound of dry air) are plotted as ordinates against Fahrenheit temperatures as abscissas. Any point on this chart represents the temperature and humidity of a definite sample of air. The curved line marked "100%" gives the humidities of saturated air at various temperatures. Mixtures of air and water vapor represented by points above and to the left of the saturation line cannot ordinarily exist. The curved lines below the line for saturated air represent various per cent humidities. The line for humid heat is plotted with humidities from the right-hand edge of the chart as ordinates, against B.t.u. along the top of the chart as abscissas. The lines for the specific volume of dry air, and for the saturated volume, are plotted with temperatures as abscissas and cubic feet per pound of dry air along the left edge of the chart as ordinates. The humid volume of a sample of air at given temperature and humidity can be found by linear interpolation between the line for the saturated volume and the line for the specific volume of dry air.

The Wet-bulb Temperature.—Suppose that unsaturated air is brought into contact with liquid water under adiabatic conditions (*i.e.*, such that no heat is received from or given up to the sur-

roundings during the operation). Since the air is not saturated, water will evaporate into it and increase the humidity of the air. The latent heat of evaporation of this water cannot be supplied externally (since by definition the process is adiabatic) and, therefore, must be supplied by the cooling of either the air or the water, or both.

Consider first the case in which a stream of unsaturated air, at constant initial temperature and humidity, is passed over a wetted surface. If the initial temperature of the wetted surface is approximately that of the air, the evaporation of water from the wetted surface tends to lower the temperature of the liquid water. When the water becomes cooler than the air, sensible heat will be transferred from the air to the water. Ultimately an equilibrium will be reached at such a temperature that the loss of heat from the water by evaporation is exactly balanced by the heat passing from the air into the water as sensible heat. Under such conditions the temperature of the water will remain constant. This temperature is called the *wet-bulb temperature.* If the initial temperature of the wetted surface is below the wet-bulb temperature, it will rise to the wet-bulb temperature.

Wet-bulb Theory.—A more detailed discussion of the mechanism of this process is as follows: Consider a drop of water in contact with air under such conditions that the drop is at the wet-bulb temperature. Surrounding the drop of liquid will be an air film, as has been discussed in Chap. VI. Through this film there is diffusing, from the water into the air, w lb. of water vapor per hr. If the latent heat of water at the wet-bulb temperature is λ_w B.t.u. per lb., the latent heat of the diffusing vapor stream will be $\lambda_w w$ B.t.u. per hr. A further small amount of heat will be needed to superheat this vapor from the wet-bulb temperature to the air temperature, but this will be neglected in the present discussion. On the other hand, since the wet-bulb temperature is below the temperature of the bulk of the air, sensible heat equal to q B.t.u. per hr. will be flowing into the drop. Since the wet-bulb temperature is an equilibrium temperature, these two heat streams must be equal, or

$$\lambda_w w = q \tag{121}$$

It will be remembered that the transfer of sensible heat is equal to the product of three factors—the coefficient of heat transfer,

the area of the surface through which the heat is flowing, and the temperature drop. If h is the air-film heat-transfer coefficient, if A is the superficial area of the drop, if t_g is the temperature of the bulk of the air, and t_w is the temperature of the interface, then

$$q = hA(t_g - t_w) \tag{122}$$

Diffusion-film Coefficient.—It has been shown in Chap. VI that the transfer of material from a gas to a liquid takes place by diffusion through the two films, one on each side of the interface. When the liquid phase is pure water, and the vapor phase is a mixture of air and water vapor, there is no resistance to diffusion in the liquid phase, but the gas-film diffusion resistance remains. This is the same film through which the heat is flowing, as described by Equation (121). Equation (112) for gaseous diffusion can be applied to the diffusion of water vapor through this air film. There is a complication present, however, due to the fact that the rate of heat flow is great enough in such cases so that an appreciable temperature drop is set up across the film. Since the diffusion coefficient b_{gw} varies with the temperature of the film in accordance with Equation (116) it is necessary to use an average value of b_{gw} in Equation (112). Furthermore, since the percentage variation in $(p_b)_m$ is small, and since it is very difficult in practice to measure the film thickness, it is convenient to group these factors into a single coefficient, k_G, defined by Equation (123):

$$k_G = \frac{b_{gw}}{(p_b)_m B_g} \tag{123}$$

The coefficient k_G is a film coefficient for diffusion and is strictly analogous to the film coefficient for heat transfer. The rate of transfer of water vapor through the gas film away from the interface by means of diffusion is then given by Equation (124):

$$w = k_G A(p_w - p_g) \tag{124}$$

where

w = pounds of water diffusing per hour.
A = interfacial area in square feet.
p_g = partial pressure of water vapor in the bulk of the air in atmospheres.

p_w = partial pressure of water vapor at the interface in atmospheres.

k_G = film coefficient of diffusion in $\frac{\text{lb.}}{(\text{hr.})(\text{sq. ft.})(\text{atm.})}$.

Equation (124) shows that the driving force for the diffusion of water vapor through the gas film at the water-air interface is the partial-pressure difference of the water vapor over the film. For calculation purposes, however, it is convenient to express the average driving force of the reaction as a humidity difference, rather than as partial-pressure difference, because humidities can be read directly from the humidity chart, and because a water-balance equation is much simpler if written in terms of absolute humidities rather than partial pressures.

The substitution of humidity for partial pressure in Equation (124) is seen to be permissible by referring to Equation (118). Under ordinary temperature conditions, p is small in comparison with unity, and Equation (118) becomes, approximately,

$$H = {}^{18}\!/_{29}p \tag{125}$$

Within a small error, humidities are proportional to partial pressures, and humidity differences are proportional to partial-pressure differences. Equation (124) becomes, then,

$$w = (k_G)(A)({}^{29}\!/_{18})(H_w - H_g) \tag{126}$$

where H_g is the humidity of the bulk of the air, and H_w is the humidity of the air at the interface.

If k_G' is defined by the equation

$$k_G' = \frac{29k_G}{18} \tag{127}$$

Equation (126) becomes

$$w = k_G'A(H_w - H_g) \tag{128}$$

If the values of q and w from Equations (122) and (128) are substituted in Equation (121), the result is

$$H_w - H_g = \frac{h}{k_G'\lambda_w}(t_g - t_w) \tag{129}$$

Factors Influencing the Wet-bulb Temperature.—Since h and k_G' represent coefficients corresponding to the same film,

it is reasonable to expect that any influence that tends to change the thickness of this film will cause the same percentage change in each coefficient. For example, if the film thickness is halved, both h and k_G' will be doubled, and the ratio of h to k_G' will be unaffected. Therefore, the ratio $h:k_G'$ should be independent of such variables as viscosity and air velocity. This is found to be true by experiment. Such experiments also show that k_G' varies as the 0.8 power of the air velocity, just as h is shown by Equation (69) to vary as $u^{0.8}$. Mark (reported by Sherwood and Comings[1]) has shown that the ratio h/k_G' is 0.26 for water vapor, but varies from 0.40 to 0.57 for organic liquids. For any value of t_g and H_g, then, there will be a definite value of t_w and H_w that will fit Equation (129), since H_w and t_w are the coordinates on the 100 per cent humidity curve of the humidity chart (Fig. 114). *The wet-bulb temperature therefore depends only upon the temperature and humidity of the air and is independent of viscosities, air velocities, and any other factors that might influence the thickness (and hence the resistance) of the air film in contact with the water.*

Example 20.—Air at a temperature of 100°F. is found by experiment to have a wet-bulb temperature of 70°F. What is the humidity of the air? What is the percentage humidity of the air?

Solution.—From the humidity chart, the saturation humidity at 70°F. is 0.016 lb. per lb. dry air. The values for substitution in Equation (129) are:

$$H_w = 0.016 \qquad \lambda = 1053 \qquad t_g = 100 \qquad t_w = 70 \qquad h/k_G' = 0.26$$

$$0.016 - H_g = \frac{0.26}{1053}(100 - 70)$$

From which

$$H_g = 0.0086$$

From the humidity chart, the percentage humidity is 20.

Adiabatic Cooling Lines.—Equation (129) was derived on the assumption that a large amount of air was brought in contact with a small amount of water, so that the temperature and humidity of the bulk of the air were not affected. For many engineering purposes, however, it is of more interest to determine the history of a definite amount of air when brought into contact with water so that the air is changed in both temperature and humidity.

[1] *Trans. Am. Inst. Chem. Eng.*, **28**, 88–117 (1932).

Assume that a stream of air is passed through an adiabatic apparatus in which the air is brought into intimate contact with sprays of water, and that this water is recirculated so that its temperature is uniform. The air, however, as it passes through the apparatus, will be humidified and cooled. If the contact is intimate enough and the apparatus large enough, the air will leave practically saturated at the temperature of the circulating water. On the other hand, if the apparatus is smaller and the time and intimacy of contact less, the air will be discharged at a humidity less than saturation and at a temperature somewhat higher than that of the circulating water. It is desired to derive an equation, and plot a line on the humidity chart, that will represent the temperature-humidity history of air undergoing such a process.

Let the humidity of the entering air be H and its temperature t. Let the temperature of the water be t_s, and its latent heat of evaporation at t_s be λ_s. This temperature will be referred to as the *adiabatic saturation temperature*. Let H_s be the saturation humidity corresponding to t_s. If the air leaves the apparatus at H_s and t_s, a heat balance (written with t_s as a datum temperature) will state that:

Sensible heat in entering air + latent heat in entering air = latent heat in leaving air.

$$0.238(t - t_s) + 0.48H(t - t_s) + \lambda_s H = \lambda_s H_s \qquad (130)$$

from which

$$t = \frac{\lambda_s(H_s - H)}{0.238 + 0.48H} + t_s \qquad (131)$$

For any given value of t_s, this equation contains as variables only H and t and is, therefore, the equation of a line on the humidity chart. If a value be assumed for t_s, (thus fixing λ_s and H_s), Equation (131) can be plotted on the humidity chart, and this curve will intersect the 100 per cent line at the point t_s, H_s. If the temperatures and humidities are plotted in equidistant rectangular coordinates, the curve will not be a straight line, nor will the curves ending at different adiabatic saturation temperatures be parallel. In Fig. 114 the humidity coordinates have been distorted in such a manner that the adiabatic cooling

lines represented by Equation (131) are straight and parallel. This distortion has been made to facilitate interpolation. The left-hand ends of the adiabatic cooling lines are labeled with their adiabatic saturation temperature.

Relation of Adiabatic Saturation Temperature to Wet-bulb Temperature.—Lewis[1] has shown that if h/k_G' is equal to s, the humid heat, the adiabatic saturation temperature of a given sample of air is identical with its wet-bulb temperature. Fortuitously, the ratio h/k_G', for air-water vapor mixtures is very close to the humid heat. At a humidity of 0.047 the humid heat is 0.26, which is the experimental value for h/k_G'. At humidities

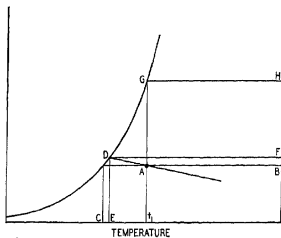



FIG. 115.—Diagram illustrating use of humidity chart.

above and below this figure there will be a divergence between the two temperatures, and for accurate work this difference should be recognized by using Equation (129) for wet-bulb problems and the adiabatic cooling line or Equation (131) for adiabatic saturation problems. With air-water mixtures under ordinary conditions, the adiabatic cooling line can be used safely for wet-bulb problems, but for extreme ranges of temperature and humidity, appreciable errors will result if this is attempted. If any liquid other than water is used, the difference between the wet-bulb temperature and the adiabatic saturation temperature is large under all conditions.

Use of the Humidity Chart.—Many of the terms that have appeared in the above discussion may be made clearer by an

[1] *Trans. A. S. M. E.*, **44**, 325 (1922).

inspection of Fig. 115. This represents a section of the humidity chart. Consider an air-water mixture of a composition and temperature represented by point A. Then by reading along the humidity coordinates to the right, point B is found, which is the humidity of this sample. By following the humidity coordinate through A to the left to its intersection with the saturation curve, the temperature C is obtained, which is the temperature at which this air would be saturated. This temperature is the dew point. By reading along an adiabatic line to the saturation curve at D and then down to the temperature axis, the point E is reached, and this represents the adiabatic saturation temperature, which is very close to the wet-bulb temperature. By reading to the right from D, the humidity F may be read, which is the humidity of saturated air at the adiabatic saturation temperature. By reading upward along the temperature coordinates to the saturation curve, the point G is reached, which represents the composition of this air if it were saturated at its initial temperature, and the corresponding humidity is read at point H. The temperature t_1 is known as the *dry-bulb temperature*, because this is the temperature that would be read by an ordinary thermometric method.

Example 21.—The air supply for a drier has a dry-bulb temperature of 70°F. and a wet-bulb temperature of 60°F. It is heated to 200°F. by tempering coils and blown into the drier. In the drier it cools adiabatically and leaves the drier fully saturated.

1. What is the dew point of the initial air?
2. What is its humidity?
3. What is its per cent humidity?
4. How much heat is needed to heat 100 cu. ft. to 200°F.?
5. How much water will be evaporated by cooling 100 cu. ft. adiabatically?
6. At what temperature does the air leave the drier?

Solution.—For the initial air, adiabatic saturation temperature and wet-bulb temperature may be considered the same. Hence, starting at the intersection of the ordinate for 60° with the saturation curve in Fig. 114 one reads along an adiabatic line to the right to the intersection of this line with the ordinate for 70°F. This intersection represents the initial condition of the air. Reading across to the right, the absolute humidity is found to be 0.0089 lb. water per lb. dry air. Reading along a constant humidity line to the left, it is found that this air, on cooling, becomes saturated at 54°, which is the dew point. The point representing the initial condition of the air falls between the curves for 50 and 60 per cent humidity. A rough interpolation gives the initial humidity as about 57 per cent.

At 70° the specific volume of dry air is 13.35 cu. ft. per lb.; and of saturated air, 13.70 cu. ft. Interpolating for 57 per cent humidity gives a humid volume of 13.55 cu. ft. per lb. The weight of 100 cu. ft. is, therefore,

$$\frac{100}{13.55} = 7.38 \text{ lb.}$$

The initial humidity of 0.0089 corresponds to a humid heat of 0.242. The heat needed to raise the temperature of this air from 70 to 200° is, therefore,

$$7.38 \times 0.242 \times (200 - 70) = 232 \text{ B.t.u.}$$

Starting at an initial humidity of 0.0089 and a temperature of 200°, and following upward and to the left parallel to an adiabatic line, it is found that at saturation the air will have a temperature of 93° and a humidity of 0.0340. The water evaporated during this process will be

$$(0.0340 - 0.0089) \times 7.38 = 1.85 \text{ lb.}$$

DETERMINATION OF HUMIDITY

From Fig. 115 it will appear that if the dew point is known the humidity may be determined by the use of Fig. 114. If the wet-bulb and dry-bulb temperatures are known the humidity may be determined by the methods described above. Most methods of determining the humidity of a given sample of air depend upon one or the other of these properties.

Dew-point Methods.—If a vessel which may be water cooled and which has a polished surface is put into the air whose humidity is to be determined, and the temperature of the metal surface is gradually lowered by lowering the temperature of the cooling water, a point will be reached where a film of moisture condenses on the polished surface. The temperature at which this mist just appears is obviously the temperature at which the air is just in equilibrium with liquid water and is therefore the dew point. From this the humidity may be read directly from Fig. 114.

Psychrometric Methods.—A more common method for determining the humidity of air is to determine simultaneously the wet-bulb and the dry-bulb temperatures. This is done by rapidly passing a stream of air over two thermometers, the bulb of one of which is dry. The bulb of the other is kept wet by means of a cloth sack either dipped in water or supplied with water. The *sling psychrometer* is regularly used in meteorological determinations. In this the two thermometers are fastened in a metal frame that may be whirled about a handle. The psychrometer

is whirled for some seconds and the reading of the wet-bulb thermometer is observed as quickly as possible. The operation is repeated until successive readings of the wet-bulb thermometer show that it has reached its minimum temperature. This locates point D in Fig. 115. By drawing an adiabatic line through point D, the intersection of this adiabatic line with the temperature t_1 fixes the point A and, therefore, determines the humidity, subject to the limitations discussed on page 256. The use of this apparatus requires considerable skill, and it also requires space enough for the operator to stand and swing the thermometer. This seriously limits its applications.

Another form of psychrometer utilizes the same principle but employs a miniature electric fan to pass the current of air over the thermometer bulbs. This fan is usually so small that its motor may be operated by a dry cell. Dry cell, motor, fan, and thermometers may be mounted in a very compact unit that can be suspended in relatively limited spaces. Since the velocity of the air is controlled by the speed of the fan, there is no chance, as is the case with the sling psychrometer, of not having long enough times of contact or high enough velocity of air fully to reach the equilibrium represented by Equation (129).

The water content of air may be determined by direct chemical methods in which a known volume of air is drawn through either sulfuric acid, phosphorus pentoxide, or other moisture-absorbing reagents, and the weight of water collected is determined. Such methods must be worked out in detail for each case to which they are applied.

AIR CONDITIONING AND WATER COOLING

It is often necessary to prepare air that shall have a known temperature and known humidity. This is most easily accomplished by bringing the air into contact with water under such conditions that a known wet-bulb temperature is nearly reached. This fixes the humidity of the air. By reheating to the desired temperature, air of any desired percentage humidity and temperature may thus be obtained.

This may be done in two ways. In the first method, the temperature of the water is so adjusted that air saturated at this temperature has the desired humidity. The air is saturated at this temperature and then reheated at constant humidity to the

desired final temperature. In the second method, the air is preheated to such an initial temperature that its adiabatic saturation temperature will correspond to the desired humidity. It is then brought into contact with water under such conditions that it is saturated adiabatically and then reheated to the final temperature. Figure 116 illustrates the two cases. Suppose that air of initial humidity H_1 and temperature t_1 (point A) is to be converted to a humidity H_2 and temperature t_2 (point B). The saturation temperature corresponding to a humidity H_2 is t_s. The first process involves maintaining the water at a temperature t_s, and the path of the air is approximately ACB. The second

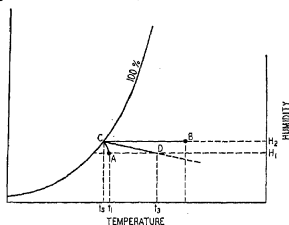



Fig. 116.—Operation of humidifiers.

process involves heating the air to such a temperature t_3 as corresponds to an adiabatic saturation temperature t_s, and the path of the air is $ADCB$.

Humidifying Equipment.—From the above considerations it follows that humidifying equipment must consist essentially of some device for heating the air, either before or after humidifying, or both, and some method of bringing air into contact with water. The heating devices are usually tempering coils or banks of finned tubes (see p. 168). The air may be brought into equilibrium with water in a variety of apparatus. Packed towers with water showered over the packing—in fact, any type of apparatus to be described later under Gas Absorption—may be employed. The usual method, however, is to spray water or steam from spray nozzles into the air. This method usually requires less space and permits a better control of the saturation temperature than the tower type of apparatus.

If the final temperature and humidity are to be widely different from the initial temperature and humidity, the second process outlined in Fig. 116 is more convenient. An apparatus for accomplishing this is shown in Fig. 117. The air is first drawn over tempering coils M and heated as indicated by the line AD in Fig. 116. It then passes through water sprays N and is adiabatically cooled and humidified (line CD, Fig. 116). A series of baffles P removes entrained water, and a second set of tempering coils Q performs the final reheating (line CB, Fig. 116). A fan R draws the air through the apparatus and discharges it to the point of use. The final temperature may be

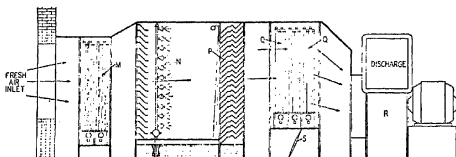



FIG. 117.—Humidifier. *M*. Tempering coil. *N*. Water-spray nozzles. *P*. Water-spray eliminators. *Q*. Reheating coils. *R*. Fan. *S*. Control damper.

regulated by controlling the steam in the second set of tempering coils or by controlling a by-pass damper S as illustrated.

The apparatus for carrying out the process corresponding to the path ACB of Fig. 116 is very similar to that shown in Fig. 117. Instead of the tempering coils M of Fig. 117, steam is injected directly into the water as it is pumped to the sprays N, maintaining it at the temperature t_s of Fig. 116. The rest of the apparatus is exactly similar to that shown in Fig. 117.

Cooling Towers.—The same operation that is used to humidify air may also be used to cool water. There are many cases in practice in which warm water is discharged from condensers or other apparatus, and where the value of this water is such that it is more economical to cool it and reuse it than to discard it. This cooling is accomplished by bringing the water into contact with unsaturated air under such conditions that the air is humidified and the water brought approximately to the wet-bulb temperature. This method is applicable only in those cases where the temperature of the water to be cooled and the temperature

and humidity of the outside air have such a relation that the wet-bulb temperature of the air is below the temperature of the incoming water. There are three types of apparatus in which this may be accomplished—first, spray ponds; second, natural-draft cooling towers; and, third, forced-draft cooling towers.

All methods for cooling water by bringing it into contact with air involve subdividing the water so as to present the largest possible surface to the air. This may be accomplished most

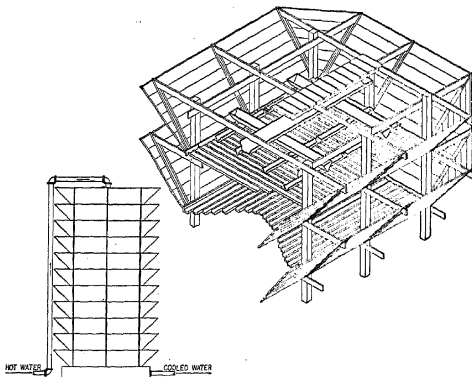



FIG. 118.—Natural-draft cooling tower.

simply by merely spraying the water from a spray nozzle. The sprays must obviously be placed over a basin to catch the water, and consequently such an arrangement is usually known as a *spray pond*. Such ponds are convenient for small capacities or where ground is not expensive but have the disadvantage that much water is lost by windage; and the power for pumping the water is appreciable, since the production of a satisfactory spray requires a certain minimum nozzle pressure.

Natural-draft cooling towers may be subdivided into two types—the chimney type and the atmospheric-circulation type. In the

atmospheric-circulation type (Fig. 118) the circulation of air through the tower is essentially across it in a horizontal direction rather than up through it in a vertical direction. Wind velocities alone are depended on for moving the air through the tower. The water is distributed by allowing it to fall over baffles of various types, and the type shown in Fig. 118 is common. This consists of flat boards 1 by 6 in. in cross-section, laid with

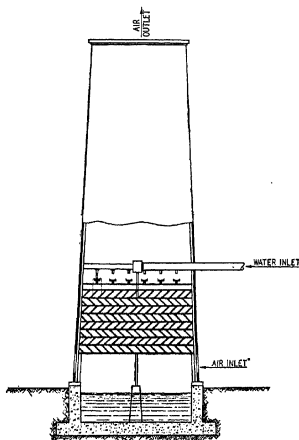



Fig. 119.—Chimney-type natural-draft cooling tower.

small gaps between the boards. All the boards in any one layer run in the same direction. Water is distributed over the tower by means of a more or less complicated system of troughs, and louvres are provided along the sides to prevent excessive amounts of water being carried away as spray by the wind. The cooling action of such a tower is not dependent on its height and therefore the question of whether it is tall and narrow or short and wide depends on the space available. The principal difficulties in the

operation of such a tower are to secure complete distribution of water over the lower surfaces and to prevent as far as possible losses of water by wind.

The chimney-type natural-draft tower depends on the fact that the air is warmed by the water and therefore may produce an upward draft. An example is shown in Fig. 119. The sides of such a tower are completely enclosed all the way to the top except for air inlets near the bottom. The grid material, which distributes the water, is confined to a relatively short section in the lower part of the tower and most of the structure is necessary for producing the draft. In towers of this type the resistance to the flow of air must be kept at a minimum, and therefore the filling of flat boards such as used in atmospheric-circulation towers is not permissible. Zigzag slats such as shown in Fig. 119 are quite common. Various other types of wood checkerwork are also used, but in all cases the boards are so arranged that they stand on edge. The disadvantage of the chimney type of tower is the height which is necessary to produce the draft and the fact that the water must be hotter than the dry-bulb temperature of the air in order to warm the air and produce a draft. The packed section cannot be so high as in atmospheric-circulation towers, or excessive friction losses will necessitate a correspondingly greater height to produce a draft.

Forced-draft towers are built like the lower part of Fig. 119 except that a fan is used at the bottom to produce the draft, and the upper part of the tower of Fig. 119, which acts as a chimney, is no longer necessary. In forced-draft cooling towers the power consumed by the fans is an appreciable item in the cost of operation. To keep this power down to the minimum it is necessary that the filling material shall offer the least possible resistance to the upward flow of air. For this reason vertical wooden rods, stoneware tile stacked vertically, or strips of galvanized-iron wire netting are used as water-distributing surfaces.

One other type of cooler for water or solutions should be mentioned in this connection. It is used where the material to be cooled cannot itself be subjected to evaporation. In such cases the liquid to be cooled is pumped through a series of pipes arranged in a vertical stack, and water is allowed to cascade from a distributing trough down over the stack of pipe. In this case

the water trickling over the pipe is cooled by the air as in a cooling tower, but the heat to be removed must be transmitted to this water through the pipe wall.

Dehumidifiers.—If moist air is to be dehumidified (for instance, dehumidifying the air discharged from a drier so that it may be reused), this can be done by bringing it into contact with a spray of water the temperature of which is lower than the dew point of the entering air. This may be accomplished by passing the air through sprays in an apparatus very similar to that of Fig. 117, except that the heater *M* is unnecessary.

Another and probably more common method is to pass the cold water inside finned tubes arranged in banks over which the air is blown. In this case, heat passes from the moist air into the water through the metal wall, and as soon as the air has been cooled to the dew point water condenses on the surface of the coils.

The recent tendency to condition air for buildings and residences has created a widespread demand for apparatus in which air must be dehumidified to a point that calls for cooling it to temperatures so low that water for this purpose is rarely available. Consequently, such a system must be supplied with artificially cooled water. This may be accomplished by ordinary refrigeration apparatus, or by circulating the cooling water into and out of a vessel in which a vacuum is maintained so high that the water is cooled to the desired temperature by flashing. For instance, if water is to be cooled to 45°F., this calls for an absolute pressure in the reservoir of about 7.6 mm. This is so low that it can ordinarily be reached only by the use of steam-jet ejectors of the type shown in Fig. 105.

Even in a case where heat is absorbed from the air through a metal wall, the collection of a layer of condensate on the heating surface results in a direct contact between cooled water and the air that is being dehumidified, and all of the above processes are equivalent from the point of view of the interaction of humid air and cold water.

THEORY AND CALCULATION OF AIR-WATER INTERACTION PROCESSES

The mechanism of the reaction of unsaturated air and water at the wet-bulb temperature of the air has been discussed under

the description of wet- and dry-bulb thermometry and the construction of the humidity chart. The process has been shown to be controlled by the flow of heat and the diffusion of water vapor through the air film at the interface between the gas and liquid. While these factors are sufficient for the discussion of

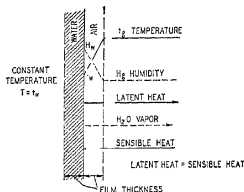



Fig. 120.—Conditions in adiabatic humidifier.

the adiabatic humidifier, when the water remains at a constant temperature, it is necessary, in the case of dehumidifiers and water coolers, where the water is changing in temperature, to consider the water film also.

In the case of adiabatic humidification, where the water remains at a constant adiabatic saturation temperature, there is no temperature gradient through the water since there is no flow of sensible heat into or from the liquid phase. In dehumidification and in water cooling, however, where the water is changing in temperature, sensible heat flows into or from the water, and a temperature gradient is thereby set up. This introduces a liquid-film resistance to the flow of heat. On the other hand, it is apparent that there can be no liquid-film diffusion resistance in any of these cases, since there can be no concentration difference in pure water.

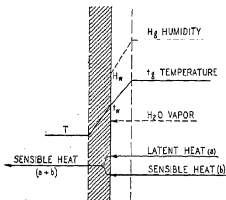



Fig. 121.—Conditions in dehumidifier.

Mechanism of Interaction between Air and Water.—It is important to obtain a correct picture of the interrelationships of the transfer of heat and of the transfer of water vapor as controlled by film resistances for all of the cases of air-water reaction processes. If such relationships are thoroughly understood, the appropriate equations can immediately be written. In Figs. 120, 121, 122, and 123 distances measured perpendicular to the

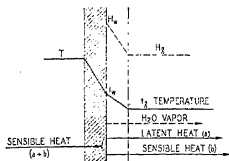



FIG. 122.—Conditions in top of cooling tower.

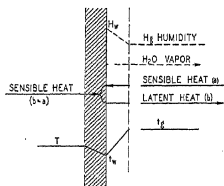



FIG. 123.—Conditions at bottom of cooling tower.

interface are plotted as abscissas, and temperatures and humidities as ordinates. In all cases,

T = temperature of the bulk of the water.
t_w = temperature at the interface.
t_g = temperature of the bulk of the air.
H_w = humidity at the interface.
H_g = humidity of the bulk of the air.

Broken arrows represent the diffusion of water vapor through the gas film, and full arrows represent the flow of heat (latent and sensible) through both air and water films. In all cases also, t_w and H_w represent equilibrium conditions and are therefore coordinates of points lying on the 100 per cent saturation line of the humidity chart of Fig. 114.

The simplest case, that of adiabatic humidification with the water at a constant temperature, is shown diagrammatically in Fig. 120. In this case the latent heat flow from water to air just balances the sensible heat flow from air to water, and there is no temperature gradient in the water. The air temperature t_g must be greater than the interface temperature t_w in order that sensible heat may flow to the interface; and H_w must be greater than H_g in order that the air be humidified.

Conditions at some particular point in a dehumidifier are shown in Fig. 121. In this case H_g is greater than H_w, and therefore water vapor must diffuse to the interface. Since t_w and H_w represent saturated air, t_g must be greater than t_w or the bulk of the air would be supersaturated with water vapor.

The consequence of this reasoning is the conclusion that moisture can be removed from unsaturated air by direct contact with sufficiently cold water without first bringing the bulk of the air to saturation.

This operation is shown in Fig. 124. A is the composition of the air that is to be dehumidified. Suppose that water is available at such a temperature that saturation conditions at

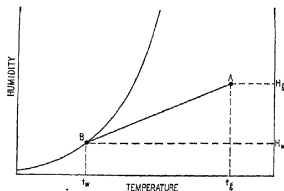



FIG. 124.—Dehumidification by cold water.

the interface are represented by point B. It has been shown experimentally[1] that in such a process the path of the air on the humidity chart is practically a straight line between A and B and not, as might be thought, cooling at a constant humidity, H_g, until the air is saturated and then condensing along the saturation line to point B.

As a result of the humidity and temperature gradients, the interface is receiving both sensible heat and water vapor from the air. The condensation of the water liberates latent heat, and both latent heat and sensible heat are transferred through the water film. This requires a temperature difference $t_w - T$, through the water film.

The conditions in a counterflow cooling tower will depend upon whether the temperature of the water is above the dry-bulb temperature of the air or between the dry-bulb and wet-bulb

[1] KEEVIL and LEWIS, *Ind. Eng. Chem.*, **20**, 1058 (1928).

temperatures of the air. In the first case, as, for example, in the upper part of the cooling tower, the conditions may be shown diagrammatically as in Fig. 122. In this case the flow of heat and of material (and hence the direction of temperature and humidity gradients) are exactly the reverse of those shown in Fig. 121. The water is being cooled both by evaporation and by transfer of sensible heat, the humidity and temperature gradients of the air film decrease in the direction of interface to air, and the temperature gradient ($T - t_w$) through the water film must result in a heat-transfer rate high enough to account for both of these heat items.

In the lower part of the cooling tower, where the temperature of the water is higher than that of the wet-bulb temperature of the air but below the dry-bulb temperature, the conditions shown in Fig. 123 prevail. In this case the water is being cooled, hence the interface must be cooler than the bulk of the water, and the temperature gradient through the water film is toward the interface (t_w is less than T). On the other hand, since the air is being humidified adiabatically, there must be a flow of sensible heat from the bulk of the air to the interface (t_G is greater than t_w). The sum of the heat flowing from the bulk of the water to the interface, and from the bulk of the air to the interface, results in evaporation at the interface, and the resulting water vapor diffuses through the air film (H_w is greater than H_G). This flow of water vapor carries away from the interface as latent heat all the heat supplied to the interface, from both sides, as sensible heat. The resulting temperature gradient, $T - t_w - T_G$, has a striking V-shape, as shown in Fig. 123. It will be understood that the latent heat b approaches the sensible heat a more and more closely as the bottom of the tower is approached, and at the bottom of an infinitely tall tower T becomes equal to t_w at the wet-bulb temperature of the entering air.

Film Coefficients of Heat Transfer and Diffusion.—The rates of diffusion and of heat transfer through the gas film depend on the coefficients h and k_G'. The rate of heat transfer through the water film depends, of course, on the film coefficient for heat transfer through this film. The experimental determination of these film coefficients is complicated by two factors. In the first place, since a very common method of obtaining a large surface for the reaction is the use of sprays, it is impractical to measure

the A term in Equations (126) and (128). This difficulty is best handled by expressing the product hA in the form

$$hA = haV \tag{132}$$

where V is the active volume of the apparatus and a is the interfacial area per unit volume. In this case the two constants h and a are kept together, and their product ha used as a volumetric rate-of-reaction coefficient. In the same way, the product $k_G'A$ can be written as

$$k_G'A = k_G'aV \tag{133}$$

where $k_G'a$ is used as a volumetric coefficient.

The second difficulty in the experimental determination of the heat-transfer coefficients is the direct measuring of the temperature of the interface t_w. This temperature must be known if the individual heat-transfer coefficients are to be determined. Also, since H_w depends on t_w, the lack of knowledge of this temperature prevents the determination of k_G', since the driving force $H_w - H_g$ is not known. For these reasons, one uses an overall heat-transfer coefficient, based on the *sensible* heat transferred per degree Fahrenheit overall temperature difference $(T - t_g)$, and a diffusion coefficient, based on a humidity difference of $H - H_g$, where H is the saturation humidity corresponding to the water temperature T.

It will be noted that in case of the adiabatic humidifier, where $T = t_w$ and $H = H_w$, the true gas-film coefficient can be obtained by experiment.

Although the coefficients can be evaluated only experimentally, it is possible to predict the qualitative effects of operating variables on the individual film coefficients, and hence on the overall coefficients.

Factors Influencing the Heat-transfer and Diffusion Coefficients.—It will be remembered that both h and k_G' are film coefficients, *i.e.*, each depends largely on two factors: first, a constant peculiar to the material of the film; and, second, the thickness of the film. The first factor is the thermal conductivity, in the case of the heat-transfer coefficient, and is the diffusion coefficient in the case of the material-transfer coefficient. In each case, these factors are functions of the composition of the film and its temperature. In the case of heat transfer it has been shown in Chap.

IV that the film thickness, which is the second factor determining a film coefficient, is primarily a function of the mass velocity of the fluid past the film, the viscosity of the fluid in the film, and the direction of flow of the fluid with respect to the film. These same variables determine the film thickness in the case of the diffusion coefficient. A high mass velocity, high temperature (and therefore low viscosity), and a flow perpendicular to the film will result in a higher diffusion film coefficient than will a low mass velocity, low temperature, and flow parallel to the film just as is the case with the heat-transfer film coefficient.

Humidifier Calculations.—In that method of humidifying in which the incoming air is blown through a spray chamber, and the process is allowed to follow an adiabatic cooling line, the water from the sprays is recirculated, and just enough make-up water is added to replace that carried away because of the increased humidity of the air. Under these conditions the amount of make-up water is small in comparison with the amount of water recirculated, and the water can be considered to be at a constant temperature equal to the adiabatic saturation temperature of the air. The conditions shown in Fig. 120 apply, and the latent heat of evaporation is balanced by the flow of sensible heat from the air to the water. Equation (122) can be written

$$q_s = hA(t - T)_m \tag{134}$$

where

q_s = sensible heat transferred in B.t.u. per hour.
h = film coefficient of heat transfer through the air film.
A = area of contact between the water and air.
$(t - T)_m$ = average temperature difference from air to water computed by the usual logarithmic mean formula.

This equation is entirely analogous to Equation (89). Since

$$q_s = Gs_1(t_1 - t_2) \tag{135}$$

and

$$hA = haV \tag{132}$$

where

G = pounds of dry air per hour.
s_1 = initial humid heat of the air.
t_1 = temperature of air entering humidifier.
t_2 = temperature of air leaving humidifier.
ha = B.t.u. of sensible heat transferred per hour per cubic foot of active volume of humidifier per degree Fahrenheit.
V = volume of humidifier in cubic feet.

Then

$$Gs_1(t_1 - t_2) = haV(t - T)_m \tag{136}$$

Equation (136) is used to calculate the volume of the humidifier if ha is known and also serves the purpose of calculating ha from experimental data.

Equation (128) can be written:

$$w = k_G'aV(H_w - H_g)_m \tag{137}$$

where

w = pounds of water evaporated per hour.
$k_G'a$ = pounds of water evaporated per hour per cubic foot of actual volume per unit humidity difference.
$(H_w - H_g)_m$ = average humidity difference, calculated by the logarithmic mean formula.

Since

$$w = G(H_2 - H_1) \tag{138}$$

where

H_2 = humidity of leaving air,
H_1 = humidity of entering air,

it follows that

$$G(H_2 - H_1) = k_G'aV(H_w - H_g)_m \tag{139}$$

Equation (139) can be used instead of Equation (136) for calculating the volume of the apparatus.

The water evaporated in the humidifier may be calculated from either a heat balance or a material balance:

$$w = \frac{q_s}{\lambda_s} \tag{140}$$

or

$$w = G(H_2 - H_1) \tag{138}$$

where

λ_s = latent heat of evaporation of water at the adiabatic saturation temperature.

Example 22.—For a certain drier there are needed 12,300 lb. dry air per hr., at 30 per cent humidity and 110°F. This is to be prepared from air at 10 per cent humidity and 70°F. by first heating, then saturating adiabatically, and then reheating to 110°. The saturation is to be carried out in a spray chamber of the type for which coefficients are given on page 276. To what temperature must the air be preheated, at what temperature will it leave the spray chamber, and what is the total heat used per hour? What must be the volume of the spray chamber if the exit air is brought to within 1° of the saturation temperature? 0.1°? 2°? 5°? What would the volume be for complete saturation?

Solution.—From the chart it is found that air at 30 per cent humidity and 110°F. has a humidity of 0.0178 lb. water per lb. air. This air is saturated at 73°, and hence this is the temperature at which it leaves the spray chamber. The initial air has a humidity of 0.0016. Starting from the saturation curve at 73° and following parallel to the adiabatic lines, it is found that the adiabatic process must start at 143°.

The initial air has a humid heat of 0.2385. Hence, the heat used in preheating is

$$12{,}300 \times 0.2385 \times (143 - 70) = 214{,}200 \text{ B.t.u.}$$

Saturated air at 73° has a humid heat of 0.246. Hence the reheating will use

$$12{,}300 \times 0.246 \times (110 - 73) = 111{,}900 \text{ B.t.u.}$$

The total heat used per hour is therefore 214,200 + 111,900 or 326,100 B.t.u.

For calculating the spray chamber, use Equation (136):

$$Gs_1(t_1 - t_2) = haV(t - T)_m$$
$$G = 12{,}300$$
$$s_1 = 0.2385$$
$$t_1 = 143$$
$$t_2 = 74$$
$$ha = 85 \text{ (see p. 276)}$$
$$(t - T)_m = \frac{(143 - 73) - (74 - 73)}{\ln\frac{(143 - 73)}{(74 - 73)}} = \frac{69}{\ln\frac{70}{1}} = 16°$$
$$12{,}300 \times 0.2385 \times 69 = 85 \times V \times 16$$
$$V = 147 \text{ cu. ft.}$$

If the final temperature difference is 0.1°, the mean temperature difference is 10.7° and the volume needed is 226 cu. ft. Similarly,

$t_2 - T_2$, degrees	$(t - T)_m$	Volume, cubic feet
2	19.1	123
3	21.3	109
5	24.6	91
0	0	∞

Non-adiabatic Humidifiers.—In the process of humidification that is not adiabatic and in which the water is kept hotter than the wet-bulb temperature of the inlet air (p. 259), the above equations do not apply. The external heat supplied by steam may be calculated by a heat-balance equation. If the datum temperature be taken at the temperature of the incoming air, then

$$q = G\{(H_2 - H_1)[\lambda_T + 0.48(t_2 - T)] + s_1(t_2 - t_1)\} \quad (141)$$

where

t_1 = temperature of entering air in degrees Fahrenheit.
t_2 = temperature of leaving air in degrees Fahrenheit.
H_1 = humidity of entering air in pounds of water per pounds of dry air.
H_2 = humidity of leaving air in pounds of water per pounds of dry air.
T = temperature of water in degrees Fahrenheit.
G = dry air entering in pounds per hour.
λ_T = latent heat of water at T°F. in B.t.u. per pound.
s_1 = humid heat of entering air in B.t.u. per pound of dry air.
q = heat supplied in steam above t_1°F. in B.t.u. per hour.

The rate equations are the same as those for cooling towers.

Calculations for Water Coolers.—Consider unsaturated air in contact with water at a temperature higher than the dry-bulb temperature of the air. It has been shown that the water will be cooled by two processes: first, by transfer of sensible heat from the warm water to the cooler air; and, second, through the loss of latent heat carried away by the water vapor diffusing from the water into the air. In order to write a heat-balance

equation, it is most convenient to take the exit water temperature as the datum. The cycle to be imagined is that all the water is first cooled to this final temperature, and that the heat so liberated is used to (1) vaporize some of the water at the exit (datum) temperature, (2) superheat the water vapor so produced to the exit air temperature, and (3) heat the incoming air to its exit temperature. Hence

$$Gs_1(t_2 - t_1) + G(H_2 - H_1)[\lambda_T + 0.48(t_2 - T_1)] = W(T_2 - T_1) \tag{142}$$

where

G = dry air entering in pounds per hour.
s_1 = initial humid heat of air in B.t.u. per pound.
λ_T = latent heat of vaporization of water at T_1, B.t.u. per pound.
T_1 = exit temperature of water in degrees Fahrenheit.
T_2 = inlet temperature of water in degrees Fahrenheit.
t_1 = inlet temperature of air in degrees Fahrenheit.
t_2 = exit temperature of air in degrees Fahrenheit.

To determine the capacity of a cooling tower a rate equation is necessary. This equation is to be written in terms of sensible heat transferred, because all determinations of the coefficient ha have been so calculated. In such a case,

$$q_s = Gs_m(t_2 - t_1) = haV(T - t)_m \tag{143}$$

The corresponding rate equation, involving the diffusion coefficient, is

$$G(H_2 - H_1) = k_G'aV(H_w - H_g)_m \tag{144}$$

where s_m is the mean humid heat of air between the humidities H_2 and H_1, and all other symbols have the same significance as in Equations (136) and (137). Counterflow will be the rule for the forced-draft cooling towers, and the natural-draft chimney type of tower. For this case the ratio of the initial and final values of $T - t$ is usually almost unity, and the arithmetic mean temperature difference is sufficiently accurate.

It will be remembered that a cooling tower can cool the water to a temperature below that of the air by the mechanism shown in Fig. 123. The process is terminated when the temperature of

the water drops to the wet-bulb temperature of the entering air. For the last part of the process, then, the water is being cooled through the loss of latent heat faster than it is being warmed by the inflow of sensible heat. Equations (142), (143), and (144) apply here, however, if due attention is paid to signs.

NUMERICAL VALUES OF COEFFICIENTS

The only available values for the coefficients needed to design air-water reaction equipment are those given by Walker, Lewis, and McAdams.[1] The data given below are from this source.

Horizontal-spray Chambers.—The coefficient ha varies from 84 to 104 B.t.u. per cu. ft. active volume per °F. per hr. These values are affected by water pressure, air velocity, water temperature, and nozzle spacing. Possibly a fair average is 90 to 95, though the coefficient should be especially determined for each set of conditions involved. The coefficient for dehumidification is about 25 per cent of the coefficient for humidification.

Coke-packed Towers.—For towers packed with about 3-in. coke, and for water velocities of 20 lb. or over per min. per sq. ft. of cross-section,

$$ha = 72 + 0.42g \tag{145}$$

where g = lb. dry air per min. per sq. ft. of total cross-section.

Cooling Towers.—Robinson[2] has published tests of forced-draft cooling towers in which he develops the following equation:

$$ha = 4.17u \tag{146}$$

where u is the velocity of air in feet per second calculated on the cross-section of the empty tower. The same equation probably applies to chimney-type towers, but measurements of air velocities through these towers have not been published. Further data of this type are very badly needed.

Spray Ponds.—As a rough approximation it may be assumed that spray ponds will cool 15 to 20 gal. water per sq. ft. superficial pond area per hr. when cooling water from 110 or 120 to 70°F. In this case the sprays must be so arranged that the entire pond

[1] "Principles of Chemical Engineering," 2d ed., pp. 472–474.

[2] *Trans. A.S.M.E.*, **44**, 669 (1922).

area is covered with spray. Cooling towers will cool 1 to 2 gal. per min. per sq. ft. ground area for atmospheric towers or 6 gal. for natural- or forced-draft towers, when cooling through the same temperature range as above. This assumes a packed section about 30 ft. high and a total height of 60 to 80 ft. for natural-draft towers. The loss by windage will be from 10 to 25 per cent for spray ponds, 5 to 10 per cent for forced-draft towers, and 2 to 5 per cent for atmospheric towers.

Nomenclature of Chapter VII

A = area
a = interfacial area per unit volume
G = pounds of dry air per hour
g = pounds of dry air per minute per square foot
H = humidity
h = heat-transfer coefficient
k = modified film coefficient for diffusion
k' = film coefficient for diffusion in terms of humidity difference
p = partial pressure
q = heat transferred, B.t.u. per hour.
s = humid heat, B.t.u. per pound dry air
T or t = temperature
u = velocity of air in feet per second
V = volume
w = weight
sub G = for air film
sub g = of bulk of air
sub s = at saturation temperature; sensible heat
sub w = at interface, or at wet-bulb temperature
sub 1 = initial conditions
sub 2 = final conditions
λ = latent heat of vaporization

Problems

1. Construct the following parts of a humidity chart for air and alcohol on a basis of 760 mm. total pressure.

1. Percentage humidity lines for 100, 50, and 10 per cent.
2. Saturated volume *vs.* temperature.
3. Latent heat of alcohol *vs.* temperature.
4. Humid heat *vs.* humidity.
5. Adiabatic cooling lines for adiabatic saturation temperatures of 40°, 80°, and 100°F.
6. Wet- and dry-bulb temperature lines (Equation (129)) for 40°, 80°, and 100°F.

Data:

Specific heat of alcohol vapor = 0.45 B.t.u. per lb. For alcohol vapor, h/k_G' is 0.40. Heat of vaporization of alcohol:

Temperature, degrees Centigrade	Latent heat, B.t.u. per pound
20	392
40	387
60	379
78.3	368
100	349
120	327

Vapor Pressure of Alcohol:

Temperature, degrees Fahrenheit	Pressure, millimeters mercury	Temperature, degrees Fahrenheit	Pressure, millimeters mercury
32	12.2	104	135.3
50	23.6	113	174.0
68	43.9	122	222.2
77	59.0	131	280.6
86	78.8	140	352.7
95	103.7	158	542.5

2. Air, dry bulb 80°F., wet bulb 60°F., is passed over wet yarn until its absolute humidity is 90 per cent.

a. How many pounds of water will 100 lb. of dry air remove from the yarn?

b. The humid air of part *a* is heated to 150°F. and again passed over the yarn, so that the exit temperature of the air is 92°F. What is the resulting percentage humidity of the air?

c. How many pounds of water are removed from the yarn by this second operation, per 100 lb. of dry air?

d. What is the volume of the 100 lb. of dry air after passing it over the yarn the second time?

3. The air in a room 35 × 60 × 12 ft. is to be changed every 2 min. and is to be maintained at 70°F. and 40 per cent humidity. The worst winter conditions outside will be 0°F. and a humidity so low that it may be neglected. To what temperature must the air be preheated if the operation is to consist only of heating and then humidifying to the desired conditions? What will be the volume of the spray chamber? What will be the cycle of operations and the volume of the spray chamber if the air is preheated, humidified to 95 per cent, and then reheated to give the desired conditions? How much steam at 5 lb. gage will be condensed per hour in each heater?

4. The following data were obtained from a test on a forced-draft cooling tower:[1]

Water entering per minute	= 640 gal.
Temperature of entering water	= 109.9°F.
Temperature of leaving water	= 90.5°F.
Humidity of entering air	= 0.012
Humidity of leaving air	= 0.031
Temperature of entering air	= 83°F.
Temperature of leaving air	= 95°F.
Volume of tower	= 2,200 cu. ft.

From these data calculate

a. The cubic feet of air entering the tower per minute.
b. The value of the coefficient *ha*.

[1] ROBINSON, *loc. cit.*

CHAPTER VIII

DRYING

A rigid definition of drying that shall sharply differentiate it from evaporation is difficult to formulate. The term *drying* usually infers the removal of relatively small amounts of water from solid or nearly solid material, and the term *evaporation* is usually limited to the removal of relatively large amounts of water from solutions. In most cases drying involves the removal of water at temperatures below its boiling point, whereas evaporation means the removal of water by boiling a solution. Another distinction is that in evaporation the water is removed from the material as practically pure water vapor, mixed with other gases only because of unavoidable leaks. In drying, on the other hand, water is removed by circulating air or some other gas over the material in order to carry away the water vapor. The above definitions hold in many cases, but there are also notable exceptions to every one of them. In the last analysis, the question of whether a given operation is called evaporation or drying is largely a question of common usage. Thus the removal of water from a solution by spraying it into a current of superheated steam fulfills most of the definitions of evaporation; but because this is done in an apparatus exactly like the apparatus in which true drying operations are carried out, it is customarily considered a drying operation.

There are methods of drying other than those suggested in the above definitions. For example, air is sometimes dried for iron blast furnaces by passing it over refrigerating coils which remove water as either liquid water or ice. Transformer oils are dried by adding calcium carbide which decomposes the water present. Gases may be dried by passing them through solutions such as calcium chloride or by passing them over absorbent materials such as silica gel. All of the above are spoken of as drying operations but are rather too specialized to be considered at length in this book.

DRYING APPARATUS

A wide variety of types of equipment has grown up in the course of many years of empirical experience. There are often several devices of widely different construction used for quite similar operations, merely because it has become customary in one industry to use a certain type and in another industry to use an entirely different type for the same purpose. This variety of constructions leads to difficulty in classification. A number of classifications are available but the following one, based on Cronshaw,[1] is useful. It is based on the form in which the material is handled through the drying process.

I. Materials in sheets or masses carried through on conveyors or trays.
  - *a*. Batch driers.
    - (1) Atmospheric-compartment driers.
    - (2) Vacuum tray driers.
  - *b*. Continuous driers.
    - (1) Tunnel driers.

II. Granular or loose materials.
  - *a*. Rotary driers.

III. Material in continuous sheets.
  - *a*. Cylinder driers.

IV. Pastes and sludges or caking crystals.
  - *a*. Agitator driers.
    - (1) Atmospheric.
    - (2) Vacuum.

V. Materials in solution.
  - *a*. Drum driers.
    - (1) Atmospheric.
    - (2) Vacuum.
  - *b*. Spray driers.
    - (1) Air.
    - (2) Superheated steam.

Compartment Driers.—Whenever the consistency of the raw material or of the dried product is such that it is most easily handled on trays, some form of compartment drier is used. This includes many sticky or plastic substances, granular masses such as crystalline material, pastes, and precipitates. Yarn and other textile products, and many similar materials not necessarily handled on trays, are also dried in this type of equipment. When the material is on trays it is easy to handle

[1] "Modern Drying Machinery," Ernest Benn, Ltd., London, 1926.

it in both loading and unloading without losses, and therefore valuable products or small batches are handled by this method.

The apparatus consists essentially of a rectangular chamber whose walls contain suitable heat-insulating materials. Inside the chamber are either racks made of light angles upon which trays may slide or tracks for cars so that a car loaded with trays may be run into the drier and doors closed behind it. There is

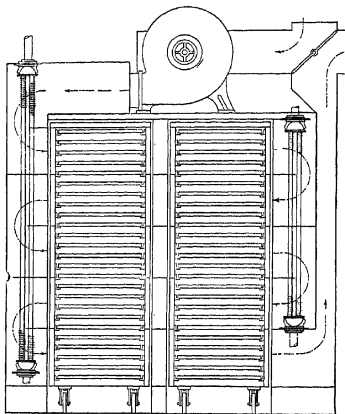



FIG. 125.—Compartment tray drier.

provision for circulating air over and between trays, and most driers of this type also have provision for heating the air inside the drier rather than outside. Such a drier, considerably conventionalized, is shown in Fig. 125. Air is taken in at the connection at the upper right-hand corner, passes through the fan, and then over a stack of heating tubes as shown by the arrows. A partition is so placed that the air after being heated passes over only a few trays. As it issues from these trays on the right-hand side of the drier, it again passes over heating coils and

is returned over the next set of trays, until it finally issues at the lower right-hand corner and passes upward through a duct along the right side. The damper shown in the upper right-hand corner may be so set as to feed only fresh air to the fan and discharge all the moist air from the drier. It may also take up intermediate positions, in which some of the moist air coming from the bottom of the drier is returned with the fresh air and only a portion is discharged.

It will be noted that the air passing through the drier is not heated all at once but is reheated in several steps. The advantage of this may be seen by an inspection of Fig. 126, which is a section

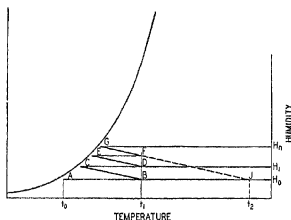



FIG. 126.—Effect of air reheating on drier operation.

of the humidity chart of Fig. 114. Suppose that the outside air is at a temperature and humidity indicated by point A on the diagram. Suppose that the properties of the material being dried are such that it should not be heated above some temperature such as t_1. The initial heating of the air is represented in Fig. 126 by the line AB. If the air were then allowed to pass over the moist material and if its time of contact with the material were such that it left nearly saturated, it would leave at a condition approximated by the point C, if BC is an adiabatic cooling curve. The air which entered would have a humidity of H_0, and by cooling to the point C it would have acquired a humidity of H_1. Every pound of air put through the drier would therefore carry out $H_1 - H_0$ lb. of water. On the other hand, if this air (represented by point C) were again heated to t_1 along the line CD, it could again be passed over more material and

pick up moisture by adiabatic cooling as represented by line DE. This process may be repeated an indefinite number of times by repeated reheatings, and the air that finally leaves, represented in Fig. 126 by point G, removes $H_n - H_0$ lb. of water per lb. of air instead of only $H_1 - H_0$ lb.

The above procedure has two advantages. In the first place, by repeated reheating, fewer pounds of air are necessary to remove a given weight of water. Although the heat necessary to evaporate the water is an appreciable part of the heat consumption of the drier, the large volumes of air required carry off so much heat as sensible heat that this is an important part of the total heat consumption. By reducing the amount of air discarded per pound of water evaporated, this heat loss may be considerably reduced. A second advantage of reheating is that more moderate temperatures may be employed. If air entering under conditions represented by the point A were to be discharged with the humidity of H_n (represented by point G) and if the heating were all done in one step, it is obvious that it would be necessary to heat the air to the point J at a temperature of t_2. In many cases materials would be injured by contact with air at such high temperatures; and in any case, the employment of high initial temperatures increases radiation losses and demands higher-pressure steam for heating.

Most materials entering the drier will carry considerable surface moisture. This can be evaporated rapidly without affecting the properties of the material. Consequently, at the beginning of the operation, outside air is used, because after heating this has a low percentage humidity and therefore results in a rapid rate of drying. After surface moisture is removed and the problem becomes one of causing moisture to diffuse from the interior of the material to the surface, the drying rate should be less in order to prevent surface hardening. The drying rate may be decreased either by using lower temperatures or by using a higher humidity. The latter is conveniently accomplished by the use of the damper mentioned in the description of the drier. If the damper be so set that some of the air that has passed through the drier is returned, the air entering the drier will contain more moisture and therefore drying will be slower. Further, the exit air from the drier will be warmer than the inlet air, and consequently a return of a portion of this exit

air increases the average temperature of the entering air and therefore decreases the steam consumption of the drier.

Vacuum-compartment Driers.—In many cases it may be desirable to dry materials on trays more rapidly than can be done by passing a stream of air over them and yet maintain the temperature lower than would correspond to the evaporation of water at atmospheric pressure. In such cases a vacuum

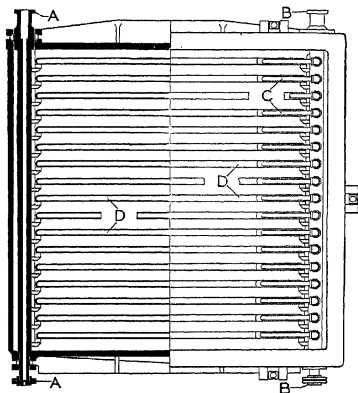



FIG. 127.—Vacuum shelf drier. *A*. Steam-inlet manifold. *B*. Condensate manifold. *C*. Connections from manifold to shelves. *D*. Shelves.

shelf drier is used. Such a drier is shown in Fig. 127. It consists of a cast-iron shell, usually rectangular in cross-section, and this shell contains a number of shelves *D*. These shelves are hollow and during operation are filled with steam or hot water. In the front of the drier at either side are vertical manifolds *A* and *B*, and a short connection *C* extends from either manifold to each shelf. One of these manifolds *A* is for the introduction of steam, and the other *B* is for the removal of condensate and non-condensed gases. The material to be dried is spread on trays which are placed on these shelves. The door is closed and the

interior of the drier placed under a vacuum by means of a vacuum pump. The steam in the shelves gradually heats the material in the trays to a temperature such that the water will evaporate under the pressure existing in the drier. This water is condensed in a condenser placed between the drier and the vacuum pump.

Such driers are extensively used for drying precipitates of all sorts in the form of filter press cakes and in general for drying any solid where the rate of diffusion of the solvent through the cake is not an important factor. Because of the amount of labor involved in filling and emptying the trays, this type of drier is confined to relatively valuable materials.

Tunnel Driers.—The compartment driers that have just been described are necessarily intermittent, and each unit has a

Fig. 128.—Tunnel drier.

relatively small capacity. If large amounts of material are to be dried and if the material is uniform in moisture content and general properties, a continuous system is desirable. This can be accomplished by building the drier in the form of a long tunnel as shown in Fig. 128 and conveying the material through this tunnel on cars, either continuously or by having the tunnel so arranged that a car leaves the discharge end when a fresh car is put in at the entrance. Such driers are usually provided with countercurrent circulation of air. This prevents the use of internal reheating, but the substances handled in tunnel driers are usually such that the material can stand the high temperatures at the discharge end. The countercurrent system can be so arranged that the air is discharged very nearly saturated and a relatively good heat economy is obtained. The principal advantage of the tunnel drier over the compartment drier is in the convenience of a continuous operation as compared to an intermittent one. The tunnel drier is generally used on brick, ceramic products, lumber, and other materials that must be dried rather slowly but in relatively large quantities. In drying

lumber by this system it may be necessary to humidify the air in order to prevent too rapid drying at the hot end.

Rotary Driers

The material handled in a rotary drier must be granular or crystalline, must be handled in bulk, must be dry enough at the beginning of the operation to be handled by ordinary conveying methods, and must not be sticky enough to build up on the walls of the drier.

Rotary driers all consist of a cylindrical shell, set with its axis at a slight angle to the horizontal, and mounted on rollers so that it can be rotated. The material to be dried is fed to the high end of the drier and, by the rotation of the drier, usually assisted by internal shelves or flights, is gradually advanced to the lower end where it is discharged. These driers may be classified into several types according to the methods by which they are heated. A possible classification would be as follows:

Direct heat	parallel current	steam heated
Indirect heat	counter current	fire heated
	reversed current	

In a *direct-heat drier* the heat is applied within the shell, and in an *indirect-heat drier* it is applied outside the shell. In parallel-current operation, the air and the material to be dried flow in the same direction; in countercurrent operation they flow in opposite directions; and in a *reverse-current drier* the air travels through the drier twice, once in either direction.

Examples of Rotary Driers.—Figure 129 illustrates a *direct-heated, countercurrent, steam-heated drier.* A rotating shell *A* made of sheet steel is supported on two sets of rollers *B* and driven by a gear and pinion *C*. At the upper end there is a hood *D* which connects through a fan *E* to a stack and a spout *F* which brings in wet material from the feed hopper. Flights *G*, which serve to lift the material and shower it down through the current of hot air, are riveted along the inside of the shell. At the lower end, the material discharges from the drum into a screw conveyor *H*, which removes it from the drier casing. Just beyond this screw conveyor is a stack of tempering coils *J* (see p. 167) which preheat the air. The air is moved through the drier by the fan, and this may, if desired, be placed to the right of the tempering

coils, so that the whole drier is under a plus pressure. The fan may be placed in the stack as shown, especially when the material

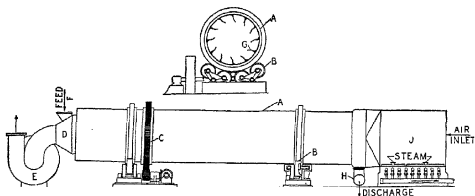



FIG. 129.—Direct steam-heated countercurrent rotary drier. *A*. Drier shell. *B*. Shell-supporting rolls. *C*. Drive gear. *D*. Discharge hood. *E*. Discharge fan. *F*. Feed chute. *G*. Lifting flights. *H*. Product discharge. *J*. Air heater.

carries dust, and thus the whole drier may be kept under a slight diminished pressure. Such driers are widely used for salt, sugar, and all kinds of granular and crystalline material which must be

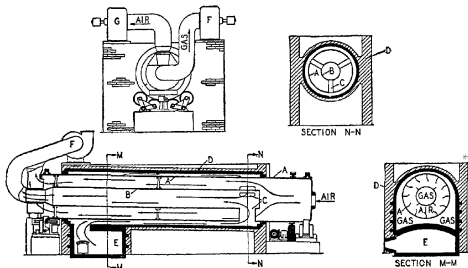



FIG. 130.—Indirect fire-heated, reversed-current rotary drier. *A*. Drier shell. *B*. Flue-gas duct. *C*. Connecting ports. *D*. Brick setting. *E*. Fire box. *F*. Flue-gas discharge fan. *G*. Air-discharge fan.

kept clean, and which may not be submitted to the temperature of gases direct from a fire.

It will be noted that in this drier the air must be heated to such a temperature before it enters that it will correspond to the

point J on Fig. 126 if the air is to leave at a humidity H_n. In many cases a steam drum or steam coils are introduced inside the shell toward the discharge end. This drum serves to reheat the air and therefore the temperature of the inlet air may be less than t_2 in Fig. 126. The surface of the drum also assists the drying and increases the capacity of the drier, as the material falls on the drum during rotation.

Figure 130 represents an *indirect-heated, reversed-current, fire-heated drier*. The drier consists of an outer shell A and an inner tube B concentric with the shell. At the right end of the shell there are nozzles C connecting the interior of the inner tube with the outside of the shell. The whole shell is mounted in a brick chamber D, which contains a fire box E under the left end. Flue gases from the fire box pass around the outside shell, traveling from left to right, pass through the nozzles C into the inner tube B, through the inner tube from right to left, and then to a fan F or stack. These gases do not come in contact with the material at any point in their travel. Material is fed at the left end, passes down the annular space between the shell and the central tube, and is discharged at the right end through a spout not shown in the figure. Air is drawn in at the right-hand end, passes through the drier in the annular space in countercurrent to the material, and is removed by a separate fan G. This drier would be used for materials which could stand the high temperature of the flue gases, but which should be protected from dirt, such as high-grade clays and similar inert materials.

Figure 131 shows a *direct-heated, reversed-current, fire-heated drier*. Here the drier consists of a shell A and a central tube B, with the material in the annular space between the shell and the central tube. In the particular drier shown, the central tube is given a special cross-section to increase the area of contact with the flue gases. A fire box C is built at one end, and the hot gases pass first from left to right through the central tube in parallel current with the material. They enter the central tube through ports D. At the right end the material is discharged by spout E, but the gases pass back through the annular space, then to a fan F, and out through a stack. In this way the stack gases themselves, instead of air, carry the moisture removed from the material. The actual drying in the annular space is done by gases moving in countercurrent to the material, and the hotter

gases in the central tube have a very similar function to the steam drum inside a steam-heated drier, since they effect some intermediate reheating during the drying process. This type of drier is quite common and is used on such a wide variety of materials that they cannot be mentioned specifically. By using coke as a fuel and firing very carefully, it is even possible to dry table salt in such an apparatus without contaminating it with dust.

In general, the fire-heated driers have a higher capacity than steam-heated driers because of the higher temperatures of the

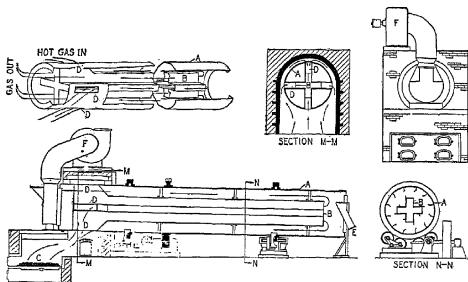



FIG. 131.—Direct fire-heated, reversed-current drier. *A*. Drier shell. *B*. Flue-gas tube. *C*. Fire box. *D*. Flue-gas inlet ports. *E*. Discharge spout. *F*. Discharge fan.

gases used. They are usually limited to inorganic materials, while the steam-heated driers are used either where lower temperatures must be maintained or where cleanliness is essential. Organic materials may be dried in direct-heated, parallel-current driers under careful control. In this way the hottest gases strike the wet material, and before the material becomes dry enough to burn or scorch, the gases have been cooled to such a temperature that this cannot take place. The parallel-current drier has a somewhat lower fuel efficiency than the countercurrent drier, because the air or fire gases leave at a higher temperature than in the case of countercurrent operation.

Cylinder Driers.—In the manufacture of paper, textiles, and other materials that are produced in continuous sheets,

it is desirable to dry these sheets continuously at the same rate at which they are made. The drier on a paper machine is an excellent example of this and is illustrated diagrammatically in Fig. 132. After the sheet of paper has been formed, it is carried in a zigzag manner over the surface of a number of hollow steam-heated rolls. These rolls are driven mechanically at the same speed at which the sheet of paper is produced. The number of these rolls, their temperature, and the speed of the sheet

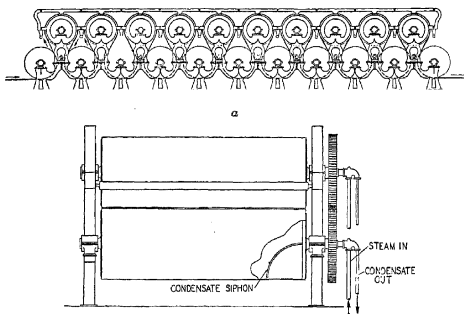



FIG. 132.—Paper-machine drier. *a*. General view. *b*. Detail of roll construction.

are so adjusted that the sheet is dry when it leaves the last roll. A modification of this drier is one in which a pasty material such as a heavy solution, a sticky filter press cake, or similar material is carried on an endless belt of wire netting or fabric and passed over rolls in a similar way until it is dried. This may be further modified by enclosing the rolls in a drying chamber and supplying the heat in the form of hot air, rather than in the form of steam inside the rolls.

Mechanically Agitated Driers.—Many materials that are too sticky to handle in continuous rotary driers but which are not valuable enough to dry in tray or compartment driers are handled in mechanically agitated driers, of which there are many varia-

tions. One type is shown in Fig. 133. In this a horizontal, steam-jacketed, cylindrical shell is provided with an inlet at one end and an outlet at the other. A central rotating shaft carries a number of paddles or scrapers. The material is fed in at the left end and the vapor is driven off from the connection at the right end. The machine operates as a batch drier and is discharged by opening the door at the lower right end. It will be noted that in this drier no current of air is employed to sweep out the vapors and that the process is essentially one of evaporation. This type of apparatus, however, is universally known as a drier. Driers of this type may be operated at atmospheric

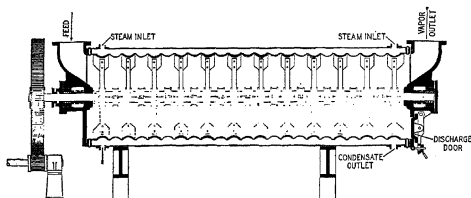



FIG. 133.—Agitated batch drier. (*Hamler.*)

pressure, but it is much more common to connect the vapor outlet to condenser and vacuum pump and operate the drier under vacuum. This permits operation at a lower temperature, which both decreases the risk of injuring the material and increases the temperature difference between the boiling point of the moisture in the material and the temperature of the steam in the jackets.

Small batch driers for materials that must be agitated during operation are built as shown in Fig. 134. This drier consists essentially of a shallow cast-iron pan A with a steam jacket B, and a central shaft C which carries rotating scraper blades D. Material is shoveled into the pan, and at the end of the drying operation a door E in the side of the pan is opened and the action of the blades works the material out through this door. These driers may be left open and operated at atmospheric pressure, or by putting on a cover they may be operated under vacuum as

shown in Fig. 134. They are usually used where small batches of granular or sticky material are to be handled.

Drum Driers

When one step in a process delivers a solution from which the product is to be obtained, the next step is often evaporation,

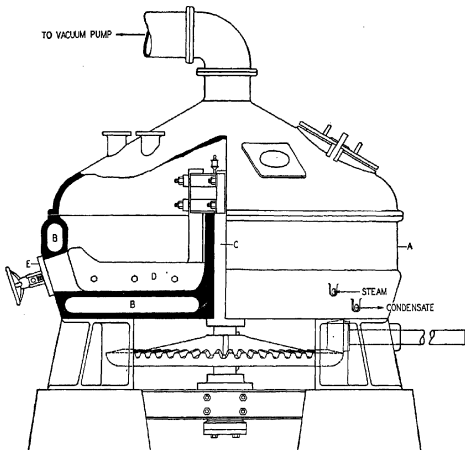



Fig. 134.—Vacuum drier for batch operation. *A*. Drier pan. *B*. Steam jackets. *C*. Shaft. *D*. Stirrer arms. *E*. Discharge door. (*Buflovac.*)

crystallizing out the desired material either in the evaporator or in a subsequent operation. On the other hand, many materials, especially colloids, cannot be crystallized from solution, and the evaporator can remove water only as long as the material is still fluid. As the solution becomes more concentrated and more viscous the operation of the evaporator becomes less satisfactory, until finally a point is reached where evaporation as such is no longer commercially feasible, and the apparatus that removes

the rest of the water is known as a drier. It is obvious that none of the types so far mentioned, except possibly the vacuum shelf drier, is suitable for this type of work. The final removal of moisture from a concentrated solution is usually done on drum driers of one sort or another. The general characteristic of this group is that a thin layer of a viscous solution is applied to a slowly rotating, internally heated metal roll. The speed of the roll and its temperature are so regulated that by the time the

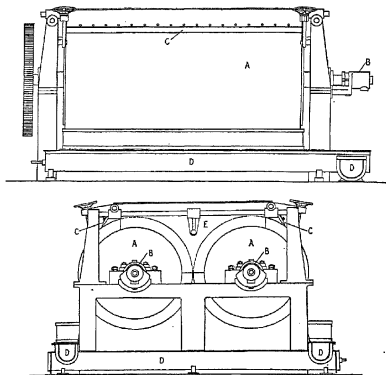



FIG. 135.—Double-roll atmospheric drum drier. *A*. Drying rolls. *B*. Trunnions. *C*. Doctor knife. *D*. Product conveyors. *E*. End plate.

material has made less than one complete revolution it has been dried and can be removed by a doctor knife.

Atmospheric Drum Driers.—A double-roll drum drier is shown in Fig. 135. This drier consists essentially of two large cast-iron rolls *A* with a smooth external surface. Pipes for introducing steam and removing condensate from the interior of the drum pass through trunnions *B* on which the rolls are supported. The rolls rotate toward each other and the liquid to be dried is fed directly into the V-shaped space between the rolls. Loss of material is prevented by closing the ends of this space with

cover plates *E*. No special feeding device is ordinarily used. The thickness of the coating applied to the rolls is determined by the space between them. Doctor knives *C* are placed near the top of the rolls on the outside and the product falls into conveyors *D*. For smaller capacities the drier may consist of a single roll. This is usually fed by allowing the bottom of the roll

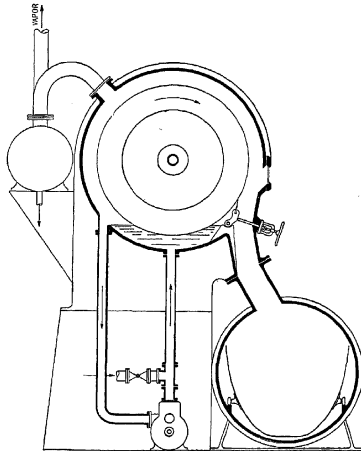



FIG. 136.—Vacuum drum drier.

to dip into a feed trough. In this case the doctor knife is on the lower part of the drum.

Vacuum Drum Driers.—For sensitive materials that cannot be heated to the boiling point at atmospheric pressure, such a drier as shown in Fig. 135 may be built so as to operate in a vacuum. An example of this drier is shown in Fig. 136. It will be noted that the roll, the method of feeding, and the doctor knife are all just as would be the case for atmospheric drum driers.

The whole apparatus is simply built inside a cast-iron casing that can be connected to a condenser and vacuum pump and operated under any desired vacuum. The removal of solids from this type of drier is the only troublesome feature. Instead of a conveyor below the doctor knife, there may simply be a chute leading to a large storage vessel. The drier operates until this vessel is full of product, when the operation is interrupted, the vacuum broken, and the discharge chamber emptied. By mak-

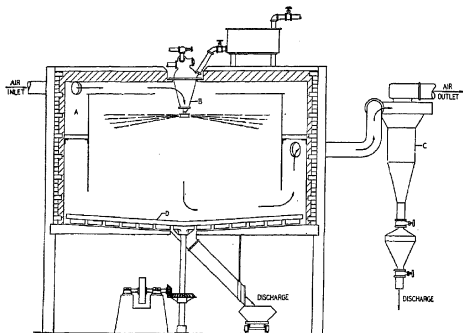



FIG. 137.—Spray drier. *A*. Spray chamber. *B*. Spray device. *C*. Separator. *D*. Product rakes.

ing the receiver large and confining this operation to cases where the volume of the product is small as compared to the volume of the feed, the length of operation can be made satisfactory. Such a drier can be operated continuously by so arranging a discharge screw conveyor that it can be driven in either direction. Two product receivers are provided, one at either end of the drier. These can be cut off from the drier without interrupting its operation, and they are filled and emptied alternately. This is the most expensive and elaborate type of drier.

Spray Driers.—If it is desired to dry a solution, it is conceivable that it might be sprayed in very fine drops into a stream of hot gas. In such a case drying would be extremely rapid and the

capacity of the apparatus should be large. Such driers have been built and have been quite successful abroad, but there are relatively few successful installations in the United States. A typical spray drier is shown in Fig. 137. The drying chamber proper is a large vertical cylinder *A* with some device *B* in the top for spraying the liquid. This may be a series of ordinary spray nozzles or it may be a flat disc, rotated at high speed, on to which the liquid is fed. The liquid is dispersed into fine drops and falls usually in parallel current, through a stream of hot gas. This may be flue gas for products that are not sensitive to heat, or it may be air if the product must be kept clean. The rake *D* draws part of the product to the discharge chute. For satisfactory operation the liquid must be so finely subdivided that much of the resultant dry material stays in suspension in the stream of gas. It passes from the main chamber to a dust separator *C* of any of the common types, where the product is separated and the stream of gas usually sent to a reheater and returned to the cycle. The principal difficulty in the design of spray driers (and this difficulty is responsible for their limited use) is so to direct the gas stream that the particles do not touch the walls of the chamber till they are completely dry. Otherwise a large part of the product adheres to the walls as a sticky, partly dried coating.

DRYING THEORY

The phenomena of drying, in common with many other processes discussed in this book, may be approached from two points of view: first, the equilibrium relationships and, second, the rate-of-reaction relationships. In all the following discussions, moisture contents will always be referred to the *dry* solid as a

Equilibrium Moisture.—In general, a given material, if brought into contact with air of definite temperature and humidity, will reach a definite moisture content that will be unchanged by further exposure to this same air. This is known as the *equilibrium moisture content* of the material under the specified conditions. If the material contains more moisture than the equilibrium value, it will dry until its moisture content reaches the equilibrium value. On the other hand, if the material is dryer than the equilibrium value and is brought into contact

with air of the stated temperature and humidity, it will absorb water until it reaches this same equilibrium point. The time required for these processes may be long or short depending upon conditions, but the final result is the same provided sufficient time is allowed for equilibrium to be reached. For air of zero humidity, the equilibrium moisture content of all materials is zero.

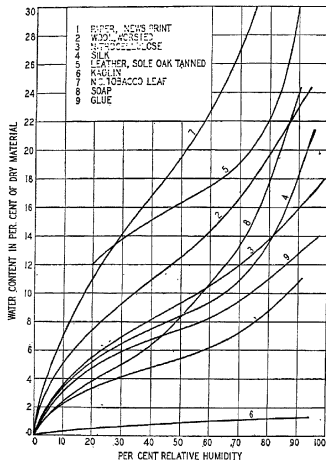



FIG. 138.—Equilibrium moisture curves. Temperature: 25°C.

For any given percentage humidity, the equilibrium moisture content varies greatly with the type of material. For example, a non-porous insoluble solid will have an equilibrium moisture content of practically zero, as far as the bulk of the solid is concerned, for any humidity and temperature. On the other hand, certain organic materials of fibrous or colloidal structure such as wood, paper, textiles, soap, and leather have equilibrium moisture contents that vary regularly and through wide ranges

as the humidity and temperature of the air with which they are in contact change. Some typical equilibrium moisture curves[1] for such substances are shown in Fig. 138. For soluble solids, the equilibrium moisture content is ordinarily negligible below a certain critical humidity. Above this critical humidity the vapor pressure of water in the air is greater than that of the saturated solution of the material in question. Beyond this critical value water will be withdrawn from the air by the soluble material and saturated solution will be formed on the surface of the material until the vapor pressure of this solution becomes equal to the partial pressure of the moisture in the air.

Bound and Unbound Water.[2]—If the equilibrium curves of Fig. 138 are continued to their intersection with the axis for 100 per cent humidity, the moisture content so defined is the least moisture that this material can contain and still exert a vapor pressure as high as that exerted by ordinary liquid water at the same temperature. If such a material contains more water than that indicated by this intersection, it can still exert only the vapor pressure of water at the given temperature. This makes possible a distinction between two types of water held in a given substance. The water up to the lowest concentration that is in equilibrium with saturated air (given by the intersection of the curves of Fig. 138 with the line for 100 per cent humidity) is called *bound water*, because it exerts a vapor pressure less than that of liquid water at the same temperature. Substances containing bound water are often called hygroscopic substances.

Bound water may exist under several conditions. Liquid water in very fine capillaries will exert an abnormally low vapor pressure because of high concave curvature of the surface; moisture in cell or fiber walls may suffer a vapor pressure lowering because of solids dissolved in it; water in natural organic structures is in physical and chemical combination, the nature and strength of which vary greatly with the nature and moisture content of the solid. *Unbound water*, on the other hand, exerts its full vapor pressure and it is largely held in the voids of the solid.

[1] "International Critical Tables," **2**, 322–325.

[2] McCready and McCabe, *Trans. Am. Inst. Chem. Eng.*, **29**, 131–160 (1933).

Some of the terms employed in the above discussion may be clarified by reference to Fig. 138. Consider, for instance, Curve 2 for worsted yarns. This intersects the curve for 100 per cent humidity at 26 per cent moisture; consequently, any sample of wool that contains less than 26 per cent moisture contains only bound moisture. Any moisture that it contains above 26 per cent is unbound moisture. A sample of wool containing, for instance, 30 per cent of water, contains 4 per cent of unbound water and 26 per cent of bound water; while a sample containing 20 per cent water contains only bound water and no unbound water. Suppose, now, that this wool is to be dried by exposure to air of 30 per cent humidity. Curve 2 shows that the lowest moisture content that can be reached under these conditions is about 9 per cent. This, then, is the equilibrium moisture content for this particular set of conditions. If a sample containing 30 per cent moisture is to be dried with air of 30 per cent humidity, it contains 21 per cent of free moisture and 9 per cent of equilibrium moisture. It may be helpful to remember that the distinction between bound and unbound water depends on the property of the material itself, while the distinction between free and equilibrium moisture depends on the particular conditions chosen for the drying operation.

In any drying process, the equilibrium moisture content represents the limit to which the material may be dried, as long as air of the corresponding temperature and humidity is used. It is the free moisture content that is amenable to removal by drying rather than the total moisture content.

Rate of Drying

The transfer of moisture from the interior of a slab into the drying medium surrounding it involves two steps: first, the transfer of the moisture as either liquid or vapor through the solid to the surface; and second, the transfer of water vapor from the surface of the slab into the main stream of the drying medium.

There are many possible cases of drying. In *adiabatic drying* the heat required for the vaporization of the water comes solely from the sensible heat of the drying medium. The following discussion will be restricted to the adiabatic drying of a solid containing a negligible amount of bound moisture.

Consider a wide sheet or slab of solid material. Assume that past this sheet there is flowing a stream of warm air of constant temperature, velocity, direction and humidity. When the above factors are constant the process is known as drying under *constant-drying conditions.*

Rate-of-drying Curve.—Before any drying problem can be attacked, the results of a drying experiment must be available for the material in question. Such results are most advantageously presented as a rate-of-drying curve,[1] the abscissa of which is the free-moisture content of the solid, and the ordinate of which is the corresponding rate of drying per unit time per unit area of

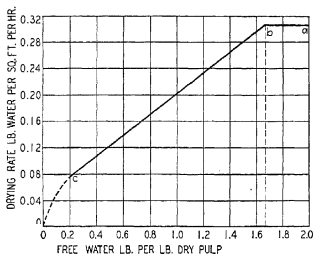



FIG. 139.—Rate-of-drying curve.

drying surface. A typical rate-of-drying curve is given in Fig. 139. The most striking feature of this curve is the fact that, when the solid has a high moisture content, the rate of drying is constant until a definite moisture content is reached, after which the rate of drying steadily decreases and approaches zero rate at zero free moisture content. This behavior in the rate-of-drying curve suggests a division of the process into two parts: first, a constant-rate period represented by the segment *ab* of Fig. 139; and second, the falling-rate period represented by the line *bc*. The abscissa of point *b*, the free moisture content at which the constant-rate period ends and the falling-rate period begins, is known as the *critical moisture content.* In general, the rate curve in the falling-rate period is straight for a consider-

[1] SHERWOOD, *Ind. Eng. Chem.*, **21**, 12, 976 (1929).

able distance beyond the critical moisture content and then curves toward the origin. The curvature may be concave upwards or concave downwards depending upon a number of conditions.

Constant-drying Period.—Experimental evidence indicates that, during the constant-rate period, liquid moisture is diffusing to the surface of the solid at a rate equal to that of evaporation from the surface. The drying process is essentially that of evaporation from a water surface. This mechanism has been discussed in detail on pages 251 to 254. The temperature of the solid during the constant-rate period tends to become the wet-bulb temperature of the drying air. If the solid, when first brought into contact with the air, is at a temperature above or below the wet-bulb temperature of the air, the first process that takes place is a cooling or heating of the slab to the wet-bulb temperature. This preliminary period is not shown in Fig. 139.

The temperature equilibrium reached during the constant-rate period may be modified by receipt of radiant heat by the solid from the warmer surroundings and by the transfer of sensible heat through the edges of the solid.

Falling-rate Period.—As the moisture content of the slab diminishes, a point is reached at which the power of the slab to deliver moisture to the surface becomes less than that of the air to evaporate such water. This causes the plane of vaporization to move from the surface of the slab into the interior of the slab.[1] The distance to which this plane of vaporization will move in a given length of time depends on a number of factors, such as the rate of liquid moisture diffusion through the slab, the rate of water-vapor diffusion through the part of the slab between the surface and the vaporization plane, and the rate of heat transfer through these same zones. Once the plane of vaporization starts moving into the interior of the solid, the rate of drying begins to diminish and the falling-rate period begins. The plane of vaporization will continue to move towards the center of the solid and the rate will continue to diminish until all of the free moisture is removed. If the solid is being dried from both faces, the plane of vaporization on the two sides will meet at the center line and the drying process will be completed.

[1] McCready and McCabe, *loc. cit.*

The mechanism of drying in the falling-rate period can be discussed in more detail with the aid of Fig. 140. In this figure, one-half of the slab is shown diagrammatically; and temperature, partial-pressure and water-concentration gradients are shown. At the instant represented by the figure, the plane of vaporization has moved into the solid a distance L. The water gradient, starting at a maximum at the center line, decreases somewhat through the undried slab, decreases very rapidly at the plane of vaporization, and is substantially zero in the dried layer (since the solid under consideration has negligible bound moisture). It is necessary that heat be supplied to the plane of vaporization.

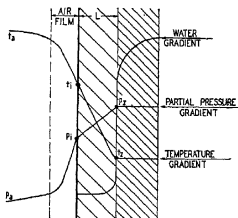



Fig. 140.—Mechanism of drying—no bound water present.

Such heat must be driven to this plane by a temperature gradient represented by $t_a—t_i—t_z$. The part of the gradient $t_a—t_i$ is the driving force for heat flow through the air film, and $t_i—t_z$ is that for heat flow through the dried layer. Also, since the water vapor formed at the plane of vaporization must diffuse from that point to the air stream, there must be a partial pressure gradient $p_z—p_i—p_a$ to act as a driving force. Here $p_z—p_i$ is the driving force for water-vapor diffusion through the dried layer, and $p_i—p_a$ is the corresponding difference for the air-film diffusional resistance.

Drying of Substances Containing Bound Water.[1]—Consider now the case of the drying of a slab of material that has a considerable bound-moisture content. The conditions in a slab of hygroscopic material during the period of falling-rate

[1] McCready and McCabe, *loc. cit.*

drying are shown in Fig. 141. The essential difference between this case and that of negligible bound-moisture content is that the zone between the surface of the solid and the vaporization plane contains a bound-water gradient. The bound water at the plane of vaporization is that corresponding to 100 per cent humidity. The bound-water concentration at the surface of the solid is that corresponding to the humidity of the air at the interface. As the distance L increases, more and more of the bound water is vaporized, so that vaporization is taking place throughout the vaporization zone. The partial-pressure gradient $p_z—p_i$ is influenced, not only by the fact that it must provide a driving force for the vapor diffusion, but also by the fact that value of p at any point in the vaporization zone will be in equilibrium with the bound moisture at that point. This same factor influences the temperature gradient $t_i—t_z$. Not all of the heat entering the solid reaches the plane of vaporization, as some of this heat is necessary to vaporize bound water in the zone L.

FIG. 141.—Mechanism of drying—bound water present.

It is apparent that, in the drying of a hygroscopic solid, when the limit of the zone of vaporization reaches the center line there is still considerable moisture that can be removed by the drying air. This moisture is the bound water left when all of the unbound water has been removed. The drying from this point on consists entirely of the vaporization of bound water and the diffusion of the water vapor so formed to the surface. Since the partial pressure for this bound water is less than that of free water at the same temperature, the driving force during this period is small, the resistance to diffusion is high (since it includes substantially half the solid slab) and the drying rate is low. This fact is shown in Fig. 139, which represents the drying of a slab of paper pulp, a material containing considerable bound moisture. After the time when the boundary of the zone of vaporization reaches the center line (low moisture content of the slab), the drying rate falls very rapidly.

Effect of Air Humidity.—If the wet-bulb temperature of the air and its velocity are fixed, but if the humidity of the air is varied, the effect is to vary p_a and to vary the gradient p_a—p_i in proportion. During the constant-rate period the rate of drying will be proportional to p_i—p_a just as in the vaporization of liquid water, and p_i will be fixed at the value corresponding to the wet-bulb temperature. Qualitatively, the effect of

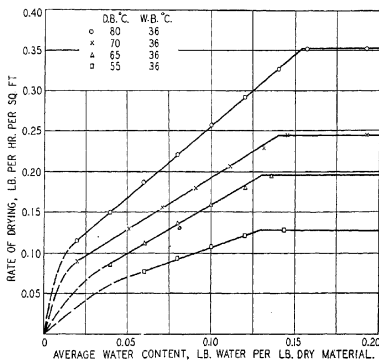



FIG. 142.—Effect of humidity on rate of drying.

variations of humidity on the falling-rate period is the same. The driving force for the drying reaction is the gradient

$$p_z—p_i—p_a,$$

and if p_a is increased, the rate of drying diminishes in proportion, and *vice versa*. This effect is shown in Fig. 142,[1] in which the wet-bulb temperature is constant, but the dry-bulb temperature (and hence the relative humidity) changes. As was predicted, during the interval of constant-rate drying, the change in the rate of drying is proportional to the change in the humidity gradient. This proportionality continues throughout the drying process.

[1] McCready and McCabe, *loc. cit.*

As the air humidity is changed, the rate curves in Fig. 142 are displaced vertically in proportion to the rate of drying through the constant-rate interval.

Effect of Air Velocity.—An increase in air velocity affects the coefficients of heat flow and diffusion of the air film in the manner discussed on page 254. This is equivalent to increasing the rate of drying during the constant-rate period in direct proportion to the increase in these film coefficients. During the

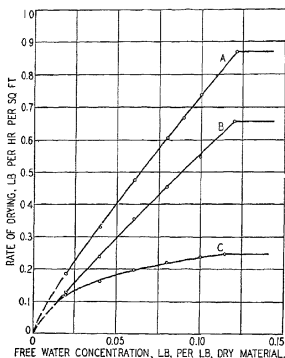



FIG. 143.—Effect of air velocity on rate-of-drying curve. Air velocities: curve *A*, 24.6 ft. per sec.; curve *B*, 13.1 ft. per sec.; curve *C*, 2.4 ft. per sec.

falling-rate period, however, the air-film resistance is only a part of the total resistance to both diffusion and flow of heat, so that as the zone of vaporization penetrates more deeply into the solid the improvement in the air-film coefficients becomes of less and less relative importance. In other words, increasing the air velocity will increase the rate of drying in proportion to the 0.8 power of the velocity during the constant-rate period, but will exercise a continually decreasing influence on the rate of drying during the falling-rate period. In the case of a hygroscopic solid, when the limit of the zone of vaporization reaches

the center line the air-film resistance is usually a negligible part of the entire resistance to heat and material flows. During this stage of the drying, air velocity has little or no effect. These effects are shown in Fig. 143, which presents rate-of-drying curves on a porous alundum slab for three different air velocities. It will be noted that the three curves touch each other at low-water concentrations at the end of the falling-rate periods, but diverge sharply from each other during the constant-rate period.

Effect of Shrinkage.—An important factor in controlling the drying rate is the shrinkage of the solid. Various materials differ considerably in this property. Rigid porous solids do not shrink appreciably during drying; but colloidal, amorphous and fibrous materials such as wood, textiles, paper pulp, soap, and clay undergo severe shrinkage as the moisture is removed from them. The general effect of such shrinkage is to increase the resistance to moisture flow in the dried layers; although such materials, even when wet, dry slowly. If such a solid is dried by hot dry air (in an attempt to increase the partial-pressure gradient $p_a—p_i$) the moisture is removed from the surface of the solid much more rapidly than the interior of the solid can deliver moisture to the surface. Either the shrinkage resulting from this drying causes warping or checking (as with lumber), or a layer of impervious material is formed (as with soap). This latter effect is usually called *case-hardening.*

For such materials it may be desirable to dry with moist air, deliberately decreasing the gradient $p_a—p_i$, and thus reducing the drying rate to the point where diffusion of water to the surface can keep up with the vaporization at the surface. This dries the material more or less uniformly throughout and minimizes the effects of local shrinkage by keeping the shrinkage uniform.

Equations for Constant-rate Period.—It has been shown that, in general, drying under constant drying conditions takes place in two phases: one where the rate of drying is constant, and a second phase where the rate of drying is variable. Consider first the phase in which rate of drying is constant. In this case the moisture on vaporizing from the surface must diffuse through the air film into the main air stream. The driving force for this reaction is, approximately, the difference between the humidity of the air at the interface and that of the bulk of the air. The humidity at the interface is the saturation humidity

of the air at the temperature of the solid. The drying rate is described by the equation

$$w = k_G' A(H_w - H_a) \tag{147}$$

where

w = the weight of moisture vaporized and picked up by the air per hour.
k_G' = the film coefficient for the diffusion process.
A = the area of the interface.
H_a = the humidity of the air.
H_w = the humidity of the air at the interface.

The factors influencing the film coefficient k_G' are, first, the diffusion constant and, second, the thickness of the air film, just as was the case in the diffusion of moisture vapor through the air film in humidification. As has been emphasized many times, the thickness of the film depends primarily on the temperature of the film, on the mass velocity of the air, and on the direction of air flow. In general, k_G' can be expressed as an empirical power function of the mass velocity as shown in the equation

$$k_G' = \alpha v^n \tag{148}$$

where

α = a constant.
v = mass velocity.
n = a constant.

Equations for the Falling-rate Period.—As shown in Fig. 139, the rate of drying during the falling-rate period is a function of the percentage of free moisture in the slab. In general, this relationship is a curve; and for the general case, especially where accurate results are desired, this curve should be determined experimentally. If this curve is at hand, the time of drying can be obtained by means of a simple integration. Assume that the equation for the rate-of-drying curve, in the falling-rate period, may be written as:

$$-\frac{dW}{A d\theta} = f(F) \tag{149}$$

where

W = the weight of free water present in the solid in pounds.
A = the drying area in square feet.
θ = the time hours.
$f(F)$ = ordinate of rate-of-drying curve where free moisture content is F pounds per pound dry solid.

Since the weight of water removed in a short interval of time is equal to the product of the weight of dry solid and the change in free moisture per unit of dry weight;

$$-dW = A[f(F)]d\theta = -DdF \qquad (150)$$

and

$$\int_{\theta=0}^{\theta=\theta_f} d\theta = \frac{D}{A}\int_{F_2}^{F_1} \frac{dF}{f(F)} = \theta_f \qquad (151)$$

where

D = the weight of dry material in the solid, pounds.
θ_f = the time of drying in the falling-rate period in hours.
F_1 = the free moisture content at the start of the falling-rate period, in pounds per pound dry solid.
F_2 = the free moisture content at the end of the falling-rate period, in pounds per pound dry solid.

If, therefore, $1/f(F)$ is plotted as ordinate against F as abscissa, the area of the integral between the limits F_2 and F_1 multiplied by the value of D/A will give the time required for the falling-rate period.

When the rate-of-drying curve during the falling-rate period is not at hand, often an approximate method can be used. In this method it is assumed that the rate-of-drying curve during the falling-rate period is a straight line connecting the origin with the critical point.

Under this assumption, Equation (150) becomes

$$-\frac{dW}{Ad\theta} = mF \qquad (152)$$

where m is a constant, and is given by Equation (153)

$$m = \frac{R_c}{F_c} \qquad (153)$$

where

R_c = the rate of drying during the constant-rate period in pounds per hour per square foot.

F_c = the free moisture content at the critical point in pounds per pound dry solid.

For this simplified case, Equation (151) becomes:

$$\theta_f = \int_{\theta=0}^{\theta=\theta_f} d\theta = \frac{DF_c}{AR_c}\int_{F_2}^{F_1}\frac{dF}{F} = \frac{DF_c}{AR_c}\ln\frac{F_1}{F_2} \qquad (154)$$

In this method it is necessary to know only the coordinates of the critical point on the rate-of-drying curve.[1]

Example 23.—A slab of paper pulp, 4 ft. by 4 ft. by ¼ in., is to be dried, under constant-drying conditions, from 66.7 per cent water to 35 per cent water (both on wet basis). It is to be dried under such conditions that the curve of Fig. 139 will represent the process. Under these conditions its equilibrium moisture is 0.5 per cent. The dry material in one slab weighs 5.0 lb. How long will the drying process take?

Solution.—The first step is to convert all moisture percentages to the dry basis.

Initial moisture $= \frac{66.7}{33.3} = 2.000$ lb. H_2O per lb. dry material

Final moisture $= \frac{35}{65} = 0.538$ lb. H_2O per lb. dry material

Equilibrium moisture $= \frac{0.5}{99.5} = 0.005$ lb. H_2O per lb. dry material

Free Moisture:

Initial $= 2.000 - 0.005 = 1.995$ lb. H_2O per lb. dry material
Critical $= 1.67$ lb. H_2O per lb. dry material
Final $= 0.538 - 0.005 = 0.533$ lb. H_2O per lb. dry material

Constant-rate Period:

Use Equation (147). Here

$A = 32$ sq. ft. (slab dried from both sides)

$k_G'(H_w - H_a) = 0.307$ lb. per sq. ft. per hr.

$w = 32 \times 0.307 = 9.82$ lb. H_2O evaporated per hr. per slab.

[1] Sherwood, in PERRY, p. 1259, gives data for the critical points of various solids.

But to dry one slab from the initial moisture content to the critical moisture content requires the evaporation of only

$$(1.995 - 1.67) \times 5.0 = 1.625 \text{ lb. per slab.}$$

Hence the drying time in the constant-rate period is $1.625/9.82 = 0.165$ hr. = 10 min.

Falling-rate period.—Use Equation (151). Here

$$A = 32.0$$
$$D = 5.00$$
$$F_1 = F_c = 1.67$$
$$F_2 = 0.533$$

The integral of $dF/f(F)$ is to be evaluated from Fig. 139 as follows:

F	$f(F)$	$1/f(F)$
1.67	0.307	3.25
1.60	0.297	3.37
1.40	0.264	3.78
1.20	0.233	4.29
1.00	0.201	4.97
0.80	0.170	5.88
0.60	0.138	7.24
0.533	0.127	7.87

When these values of $1/f(F)$ are plotted *vs.* F and the area under the curve between $F = 1.67$ and $F = 0.533$ is determined, it is found to be 5.576. Substituting in Equation (151) gives:

$$\theta_f \quad \frac{5.00}{32.0} \times 5.576 = 0.872 \text{ hr.} = 52+ \text{ min.}$$

The total drying time is therefore 52 + 10 or 62 min.

Example 24.—Suppose that, for the paper pulp slab of the previous problem, the curve of Fig. 139 were not available, but there were available the following data:

Equilibrium moisture—0.5 per cent
Critical moisture—1.67 lb. free H_2O per lb. dry pulp
Drying rate at critical point—0.307 lb. per sq. ft. per hr.

What would be the time for drying from 66.7 per cent to 35 per cent water?

Solution.—Conversion of moisture content to dry basis, calculation of free moisture, and time for drying in the constant-rate period, are all as in Example 23. For the falling-rate period use Equation (154):

$$D = 5.00$$
$$F_c = F_1 = 1.67$$
$$A = 32.0$$
$$R_c = 0.307$$
$$F_2 = 0.533$$

Substituting in Equation (154)

$$\theta_f = \frac{5.00 \times 1.67}{32.0 \times 0.307} \ln \frac{1.67}{0.533} = 0.971 \text{ hr.}$$
$$= 58.3 \text{ min.}$$

When this is compared with 52+ min. obtained by the rigorous method, the error is not serious. This method assumes that the falling-rate portion of the drying curve is a straight line between the origin and the critical point, whereas the falling-rate portion of Fig. 139 is all above such a line (*i.e.*, actual drying rates are higher than calculated). The deviation may be greater than in these two examples.

Equations for Variable Drying Conditions.—The discussion of drying rate given above has been restricted to cases wherein

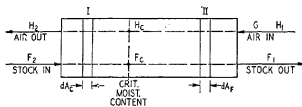



FIG. 144.—Conditions in countercurrent drier.

the process is carried out under constant-drying conditions; that is, where stock temperature, air temperature, air humidity, air velocity, and direction of air flow are unchanged. Before the integrated equations so obtained can be applied to most drying problems met in actual practice, they must be extended to cases of variable drying conditions. In actual driers, the material as it moves through the apparatus is dried by air of constantly varying temperature and humidity. Also, the stock temperature may change.

Consider the drying of a material whose initial moisture content is above the critical value. Suppose that this material is to be dried to some moisture content below the critical. Let the whole process be adiabatic; and neglect the heat necessary to change the temperature of stock. Figure 144 represents conditions in the drier diagrammatically. The stock enters at one end with a free moisture content of F_2, and as it passes through the drier

the moisture content decreases. At some point (indicated by the dotted line) the stock has reached its critical moisture content F_c. From that point until the solid issues from the other end of the drier its moisture content continues to fall and reaches F_1 at the exit. The air enters the drier at the right-hand end, with a humidity of H_1. This humidity increases to H_2 at the drier exit. When the air crosses the section where the moisture content of the solid is the critical moisture content, the air has a humidity of H_c. Any given continuous drier does not necessarily contain both zones. The material may not be dried beyond the critical moisture content, or it may enter the drier with a moisture content less than the critical. These two sections must be analyzed separately because drying obeys different laws in the two sections.

The zone to the left of the dotted line will be analyzed first. Consider a very short section (I) in this zone. The humidity of the air stream at this point is H and within this short section there is sufficient stock to offer dA_c sq. ft. of stock area. If the saturation humidity corresponding to the stock temperature is H_s, the rate of drying at this point is

$$dw = k_G'(H_s - H)dA_c \tag{155}$$

If G lb. of drying air pass through the drier per hour

$$GdH = dw \tag{156}$$

Eliminating dw from Equations (155) and (156) gives

$$G\frac{dH}{H_s - H} = k_G'dA \tag{157}$$

If H_c is the humidity of the air as it enters this zone and H_2 is the humidity of the leaving air, and if the total drying area that must be supplied in the zone is A_c:

$$\frac{G}{k_G'}\int_{H_c}^{H_2}\frac{dH}{H_s - H} = A_c = \frac{G}{k_G'}\ln\frac{H_s - H_c}{H_s - H_2} \tag{158}$$

A material balance across that part of the drier where the stock is above the critical moisture content is

$$w = G(H_2 - H_c) \tag{159}$$

Substituting for G its value from Equation (158) gives

$$w = \frac{k_G' A_c (H_2 - H_c)}{\ln \frac{H_s - H_c}{H_s - H_2}} \tag{160}$$

The quantity $(H_2 - H_c)$ may be written $(H_s - H_c) - (H_s - H_2)$, and when this is substituted in Equation (160), there results

$$w = k_G' A_c \frac{(H_s - H_c) - (H_s - H_2)}{\ln \frac{H_s - H_c}{H_s - H_2}} \tag{161}$$

which may be written

$$w = k_G' A_c (\Delta H)_m \tag{162}$$

where $(\Delta H)_m$ is the logarithmic mean humidity difference.

Consider now the section (II) in the part of the drier where the material is below its critical moisture content. Let the area of stock in this section be dA_f. If the linear relationship shown in Equation (152) between rate of drying and free moisture content can be assumed, the equations may be modified in such a way that they can be used for this section. For this purpose, Equations (152) and (153) can be combined and written as

$$dw = \frac{R_c F dA_f}{F_c} \tag{163}$$

Also, by Equation (155) R_c, the rate of drying at the critical point, can be written as

$$R_c = k_G'(H_s - H) \tag{164}$$

Eliminating R_c from Equations (163) and (164) gives

$$dw = \frac{k_G' F (H_s - H)}{F_c} dA_f = G dH \tag{165}$$

Before Equation (165) can be integrated, F must be expressed in terms of H. This can be done by writing a water balance over that part of the apparatus to the right of the area dA_f. If D is the weight of dry stock passing through the drier, in pounds per hour, a water balance states that

$$D(F - F_1) = G(H - H_1) \tag{166}$$

whence

$$F = F_1 + \frac{G}{D}(H - H_1) \tag{167}$$

Substituting the value of F from Equation (167) in Equation (165) gives

$$\frac{dH}{(H_s - H)\left[F_1 + \frac{G}{D}(H - H_1)\right]} = \frac{k_G'dA_f}{GF_c} \tag{168}$$

This equation cannot be integrated unless F_c is constant. Figure 142 shows that the critical moisture content does not vary with variations in humidity for the particular material there studied. It is not known how general this relationship is. However, if F_c be considered constant,

$$\int_{H_1}^{H_c} \frac{dH}{(H_s - H)\left[F_1 - \frac{GH_1}{D} + \frac{GH}{D}\right]} = \frac{1}{\frac{GH_s}{D} + F_1 - \frac{GH_1}{D}} \ln \frac{F_1 - \frac{GH_1}{D} + \frac{GH}{D}}{H_s - H}\Bigg|_{H_1}^{H_c} \tag{169}$$

If the right-hand side of Equation (168) be integrated and set equal to the integral of Equation (169):

$$\frac{1}{\frac{GH_s}{D} + F_1 - \frac{GH_1}{D}} \left[\ln \frac{\left(F_1 - \frac{GH_1}{D} + \frac{GH_c}{D}\right)(H_s - H_1)}{\left(F_1 - \frac{GH_1}{D} + \frac{GH_1}{D}\right)(H_s - H_c)}\right] = \frac{k_G'A_f}{GF_c} \tag{170}$$

Since Equation (167) holds for any part of the falling-rate period, it holds for the condition where $F = F_c$, and hence

$$F_c = F_1 + \frac{GH_c}{D} - \frac{GH_1}{D} \tag{171}$$

Condensing the denominator of the log term in Equation (170), substituting in the numerator of the log term from Equation (171), and rearranging, gives:

$$A_f = \frac{GF_c}{k_G'\left(\frac{GH_s}{D} + F_1 - \frac{GH_1}{D}\right)} \ln \frac{F_c(H_s - H_1)}{F_1(H_s - H_c)} \qquad (172)$$

If Equation (171) be solved for F_1, and the corresponding quantity substituted for F_1 in the first term of Equation (172), there results:

$$A_f = \frac{F_c}{k_G'\left(\frac{H_s - H_c}{D} + \frac{F_c}{G}\right)} \ln \frac{F_c(H_s - H_1)}{F_1(H_s - H_c)} \qquad (173)$$

The falling-rate section of the drier (to the right of the dotted line in Fig. 144) must contain enough stock to expose A_f sq. ft. of surface to the action of the air. The total volume of the drier is, therefore, the sum of the two volumes calculated from Equations (162) and (173).

In any case it is necessary to determine the value of the constant k_G' experimentally. It is best to determine this constant by carrying out a drying experiment under the same conditions of mass velocity, air direction, stock temperature, etc., that are to be used in the full-scale drier. The effect of reasonable variation in the mass velocity of the air can be calculated from Equation (148) if the coefficient k_G' is determined for two different values of v.

The above equations are best understood by using them in a numerical example.

Example 25.—A continuous, counterflow tunnel drier is to be designed to dry a porous cake material from 50 to 3.5 per cent moisture. The drying agent is hot air which enters at a dry-bulb temperature of 250°F. and a wet-bulb temperature of 100°F. Due to radiation, it is estimated that the stock temperature will be constant at 5°F. above the wet-bulb temperature of the air. The stock will enter the drier at 105°F. The air will leave the drier at 154°F. The equilibrium moisture content is 1.5 per cent. The input of wet material is 5,000 lb. per hr., and the surface available for drying is 0.3 sq. ft. per lb. of dry stock.

A small-scale experiment was carried out under constant drying conditions and using an air velocity and direction and stock temperature

substantially equal to those to be employed in the final drier. In this experiment, it was found that the material dried at a constant rate under constant external conditions until the moisture content was 20 per cent and that the rate then fell in accordance with Equation (152) from a moisture content of 20 to 3.5 per cent. The rate of drying during the constant-rate period was found to be 0.50 lb. per hr. per sq. ft. when dried with air of a dry bulb of 150°F. and a wet bulb of 100°F. and a stock temperature of 105°F.

Radiation from the drier can be neglected.

All analyses are on the wet basis.

Calculate

a. The weight of dry air used per hour.

b. The time the material must remain in the dryer.

Solution.—The calculation of the weight of dry air used per hour is carried out by determining, first, the weight of water picked up by one pound of dry air and, second, the total water evaporated. A simple division will give the air requirement.

The difference between the air inlet temperature and the wet-bulb temperature is too great to permit using adiabatic cooling lines for determining the humidity of the initial air. Using Equation (129) gives the initial humidity as 0.0054 (reading from the adiabatic cooling lines gives 0.0080). On adiabatic cooling to 154°F. this air will reach a humidity of 0.0276. The water picked up by 1 lb. of air is, therefore, (0.0276 − 0.0054) or 0.0222.

The entering stock contains 50 per cent of its weight as water and therefore brings to the drier 2,500 lb. of water and 2,500 lb. of dry stock per hr. The leaving material contains 3.5 per cent H_2O, or $(0.035/0.965)(2{,}500) = 91$ lb. moisture. The total moisture removed in the drier is $2{,}500 - 91 = 2{,}409$ lb. per hr.

The weight of dry air used per hour is therefore $2{,}409/0.0222 = 108{,}500$ lb.

Drying above Critical Moisture Content.—In order to calculate the time of contact of the material in the drier, it is necessary to determine the drying coefficient from the data of the small-scale experiment by the use of Equation (147). The air used had a humidity, as shown by its dry-bulb temperature of 150°F. and its wet-bulb temperature of 100°F., of 0.0306. The saturation humidity corresponding to the stock temperature of 105°F. was 0.0504. Since the drying rate was found to be 0.50 lb. per hr. per sq. ft., the drying coefficient is $k_G' = \frac{0.50}{0.0504 - 0.0306} = 25.2$ lb. per hr. per sq. ft. per unit humidity difference.

In order to determine the time of contact of the material in the large drier, it is necessary to divide the drier into two zones. The first zone will dry the material from 50 per cent moisture to the critical moisture content of 20 per cent. The second zone will dry the material from 20 to 3.5 per cent. The area that must be exposed to the air in the drier in the two zones can then be calculated by means of Equations (162) and (173). The time of contact can be calculated from the volume of stock dried per hour and the exposed area per unit volume of stock.

The moisture contained in the stock leaving the first zone is

$$(20/80)(2500) = 625 \text{ lb.}$$

The moisture removed in the first zone is then 2,500 − 625 = 1,875 lb. Since there are 108,500 lb. of dry air leaving this zone with a humidity of 0.0276, the humidity of the air leaving the first zone is calculated by the water balance

$$108{,}500(0.0276 - H_1) = 1{,}875$$

where H_1 is the humidity of the leaving air. Solution of this equation gives

$$H_1 = 0.0103$$

Since $H_s = 0.0504$ and $G = 108{,}500$, the area of contact in the first zone is given by Equation (162). In Equation (162)

$$H_s = 0.0504$$
$$H_c = 0.0103$$
$$H_2 = 0.0276$$
$$H_s - H_c = 0.0401, \qquad H_s - H_2 = 0.022$$
$$(\Delta H)_m = \frac{0.0401 - 0.0228}{\ln \dfrac{0.0401}{0.0228}} = 0.0307$$
$$W = 1{,}875$$
$$k_G' = 25.2$$
$$A_c = \frac{1{,}875}{25.2 \times 0.0307} = 2{,}430 \text{ sq. ft.}$$

Drying below Critical Moisture Content.—For the portion of the drier where the moisture content is below the critical, Equation (173) is to be used.

$$F_c = \frac{20}{80} - \frac{1.5}{98.5} \quad 0.235$$
$$F_1 = \frac{3.5}{96.5} - \frac{1.5}{98.5} \quad 0.0211$$
$$D = 2{,}500$$
$$k_G' = 25.2$$
$$H_s = 0.0504$$
$$H_c = 0.0103$$
$$H_1 = 0.0054$$
$$G = 108{,}500$$

Substituting in Equation (173) and solving gives $A_f = 1{,}294$ sq. ft.

Since each pound of dry stock offers 0.3 sq. ft. of surface, the surface offered by the hourly input of stock is (2,500)(0.3) = 750 sq. ft. The first section must contain 2,430/750 = 3.24 hr. input of stock, and the time of contact in this zone is 3.24 hr. Similarly, the time of contact in the second zone must be 1,294/750 = 1.72 hr. The total time of contact is therefore 3.24 + 1.72 = 4.96 hr.

DRIER OPERATION

Air Recirculation.—In most cases the supply of air for a drier is taken either from the room or from outdoors. In any case the temperature of such air is usually considerably below that at which the drier operates. In such cases the drier must be preceded by an air heater. If the air that has been through the drier is all discharged at its final temperature, a large amount of heat is continually used in raising air from its entrance temperature to its exit temperature, and this heat serves no useful purpose in the drier. If some of this air, which would otherwise be discharged, can be returned to the inlet of the heater without increasing the humidity of the resulting mixture to the point where drying is seriously retarded, some of the sensible heat of the exit air may thus be saved and the total heat requirements of the drier diminished. This warmer inlet mixture cools along a higher adiabatic line. If the air is discharged from the drier at a constant percentage humidity, the higher adiabatic line calls for higher absolute humidities for a given discharge temperature and therefore the same amount of moisture can be removed from the drier by discarding fewer pounds of hot air.

Humidity Control.—For sensitive products such as lumber, soap, and other colloidal or fibrous materials that exhibit skin effects, it is highly important that the humidity of the entering air be under continuous and automatic control throughout the process. This may be done by various standard devices. Automatic temperature-control valves are available in many makes, and with an accuracy of a few degrees. One such thermostatic control valve may control the steam for the heating coils, thus introducing the air at a constant dry-bulb temperature. Another such thermostatic valve, with its sensitive element kept wet by means of wicking, may be placed beside the first and set to give any desired wet-bulb temperature. This may be accomplished by having the second thermostat control the direct injection of steam into the air, or it may control the damper that returns moist air from the drier discharge to the drier inlet, or it may control the temperature of water sprayed into the air to humidify it. By any of these methods, the two control valves may be made to fix a definite wet-bulb and a definite dry-bulb temperature and thus give automatic temperature and humidity control.

It is also possible in special cases to have the setting of these valves controlled by a clock-operated cam. This cam is so designed with relation to the drying cycle that it can change the setting of the thermostat, and therefore either the humidity or the temperature of the air or both, to give different drying conditions in different parts of the drying cycle.

Movement of Air.—In Chap. III different types of fans were described. For those driers in which it is desired to circulate large volumes of air in order to give high air velocities, but in which the pressure drops are small, the propeller type of fan is the most desirable. This type, for instance, may be used on compartment driers where the passages are proportioned to give low air resistances. It is sometimes used on rotary driers, where because of the straight path and the large cross-section, the resistances are low. In other designs of driers where the cross-sections of the passages are small and the resistances to air flow are high, or where the air to be heated must be forced through banks of tempering coils with correspondingly high friction losses, fans of the steel plate or centrifugal type must be used.

Nomenclature of Chapter VIII

A = area
D = weight of dry material in the solid
F = free moisture content
G = pounds of dry air
H = humidity
$(\Delta H)_m$ = logarithmic mean humidity difference
k_G' = film coefficient for diffusion process
L = distance
m = a constant
n = a constant
p = partial pressure
R = rate of drying
t = temperature
v = mass velocity
W = weight of moisture
w = weight per hour
sub a = in bulk of air
sub c = critical moisture content; in the constant-rate period
sub f = in the falling-rate period
sub s = saturation
sub w = at the interface
sub 1 = initial conditions

sub 2 = final conditions
α = a constant
θ = time

Problems

1. The production of a certain drier is 1 ton per hr. and the percentage of moisture on the wet basis is reduced from 50 to 15 per cent. The humidity of the air passing through the drier rises from 0.0100 to 0.0200, while the air temperature falls from 155 to 100°F. The stock enters and leaves at 83°F.

Calculate the heat lost to the surroundings in B.t.u. per hour.

Latent heat of H_2O at 83°F. = 1,045 B.t.u. per lb.
Specific heat of dry air = 0.238
Specific heat of H_2O vapor = 0.48

2. Sheet material, measuring 3 ft. square and 2 in. thick, is dried from 50 to 2 per cent moisture content (wet basis) under constant-drying conditions. The dry density of the material is 30 lb. per cu. ft. and its equilibrium moisture content is negligible.

Experiment showed that the rate of drying, under the correct constant conditions, was constant at 1.0 lb. per sq. ft. per hr. between moisture contents between 50 and 25 per cent. Below 25 per cent the rate decreased.

Calculate the total time required to dry the material from 50 to 2 per cent H_2O.

3. One ton (dry weight) of a porous solid, in pieces such that its total surface available for drying is 600 sq. ft. per ton, is to be dried under constant-drying conditions from 0.10 lb. to 0.01 lb. H_2O per lb. of dry solid. The conditions are such that the drying rate is represented by the lowest curve of Fig. 143. How long will the drying process take?

4. A rotary countercurrent drier is fed with wet sand containing 50 per cent moisture and is discharging sand containing 3 per cent moisture. The entering air is at 220°F. and has an absolute humidity of 0.007 lb. per lb. dry air. The wet sand enters at 70°F. and leaves at 97°F. The air leaves at 100°F. The wet-sand input is 1,200 lb. per min. Radiation amounts to 10 B.t.u. per lb. dry air.

Calculate the pounds of dry air passing through the drier and the humidity of the air leaving the drier.

Latent heat of H_2O at 70°F. = 1,053
Specific heat of dry sand = 0.21
Specific heat of dry air = 0.238
Specific heat of H_2O vapor = 0.48

5. A continuous recirculation spray drier is concentrating 15 per cent phosphoric acid to 45 per cent. The hot discharge from the spray chamber provides the product, which is withdrawn; and the circulating acid, which is combined with incoming cold feed, is passed through a heater and is sprayed at a temperature of 220°F. into the spray chamber.

Air, dry bulb 85°F., wet bulb 70°F., is preheated and passed through the spray chamber. The air leaves at the same temperature as it enters and may be assumed to be in equilibrium with the 45 per cent acid leaving the spray chamber. The weight of the acid entering the sprays per minute is 2,500 lb. and there are 4,000 cu. ft. of air leaving the drier per minute.

Under the operating conditions given above, calculate the following quantities:

1. Exit temperature of air and acid.
2. Pounds of water evaporated per minute.
3. Pounds of dry air passing through the drier.
4. Pounds of product per minute.
5. Pounds of dilute acid added per minute.
6. Concentration of the acid fed to the sprays.
7. Humidity of leaving air.

Assume that Dühring's rule applies to the 45 per cent acid solution and that no H_3PO_4 volatilizes in the drier.

Data:

Specific heat of 45 per cent H_3PO_4 = 0.65. At 100°C. the vapor pressure of 45 per cent H_3PO_4 is 605 mm.; at 8°C. the vapor pressure is 6 mm.

CHAPTER IX

DISTILLATION

The term *distillation* is sometimes employed for those processes where a single constituent is vaporized from a solution—for example, in "distilling" water. In general, however, this term is properly applied only to those operations where vaporization of a liquid mixture yields a vapor phase containing more than one constituent, and it is desired to recover one or more of these constituents in a nearly pure state. Thus the separation of a mixture of alcohol and water into its components is distillation; whereas the separation of a brine into salt and water is evaporation, even in those cases where the salt is not desired and condensed water vapor is the only valuable product. In the recovery of glycerin from the dilute solution obtained as a by-product from soap making, the first part of the process is evaporation because in this step only water is driven off, and little or no glycerin is vaporized. This part of the process is called *glycerin evaporation* and is accomplished in ordinary evaporators. When the concentration reaches about 80 per cent glycerin, however, appreciable amounts of glycerin begin to appear in the vapor, and simple evaporation will not effect a separation. The more complicated operation employed after this point is reached is called *glycerin distillation.*

The basic requirement for a separation of components by distillation is that the composition of the vapor be different from the composition of the liquid from which it is formed. If the vapor composition is the same as the liquid composition, distillation processes will not affect a separation. Theoretically, distillation can never yield a component in absolutely pure form, although practically the product may be made of any purity that is economically warranted.

VAPOR-PRESSURE RELATIONSHIPS

The basic data of any distillation problem are the equilibria between the liquid and vapor phases of the system to be subjected

to distillation. In all that follows, two-component systems will be considered. Two-component systems may be defined as systems containing only two substances that are volatile. For such a combination there must be available what is called a boiling-point diagram.

Boiling-point Diagrams.—Figure 145 represents the boiling-point and equilibrium composition relationships, at constant pressure, of all mixtures of liquid A (boiling point t_a) and liquid B (boiling point t_b). Liquid A is the more volatile. In such a diagram temperatures are plotted as ordinates and compositions as abscissas. It will be noted that the diagram consists of two curves, the ends of which coincide. Any point (such as point y) on the upper curve has for its abscissa the composition of vapor (e) that will just begin to condense at the temperature given by its ordinate (t_1) and will give liquid of composition d. Any point (such as point x) on the lower curve has for its abscissa the composition of liquid that will just begin to boil at the temperature t_1, giving vapor of composition e. Any two points on the same horizontal line (such as x and y) represent compositions of liquid and vapor in equilibrium with each other at the temperature given by the horizontal line through them. For all points above the top line, such as point a, the mixture is entirely vapor. For all points below the bottom line, such as point b, the mixture is completely liquefied. For points between the two curves, such as point c, the system consists partly of liquid and partly of vapor.

Fig. 145.—Boiling-point diagram.

Suppose that a liquid mixture of composition d is heated slowly. It will begin to boil at t_1. The first vapor produced will have a definite composition, represented on the diagram by e. As soon as an appreciable amount of vapor has been formed, the composition of the liquid will no longer correspond to d, since the vapor is richer in the more volatile component than the liquid from which it was evolved and hence the point x tends to move toward t_b.

The boiling-point diagram must, in general, be determined experimentally. It will vary with the total pressure. Fortunately, however, many distillation processes are carried out at atmospheric pressure, and a boiling-point diagram determined at that pressure is generally satisfactory. Further, changes in pressure merely move the whole diagram up or down, but do

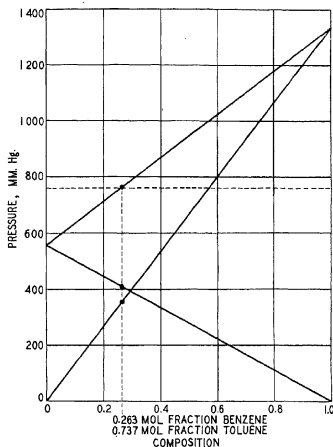



Fig. 146.—Partial-pressure curves of a system that obeys Raoult's law (benzene-toluene).

not greatly affect the relation between the vapor and liquid curves. The experimental determination of such curves is a rather difficult physicochemical procedure and will not be described here.

Raoult's Law.—For some special cases it is possible to compute the boiling-point diagram, over certain ranges, from vapor-pressure data of the pure components. Such calculations are based on Raoult's law. This law, which applies to but a few mix-

tures for all possible concentrations, states that, at any particular constant temperature, the partial pressure of one component of a mixture is equal to the mol fraction of that component multiplied by its vapor pressure in the pure state at the temperature of the liquid. According to this law the partial pressure of the component varies linearly from zero to the full vapor pressure as its mol fraction varies from zero to unity.

To illustrate Raoult's law, consider the case of benzene and toluene. At a temperature of 100°C. toluene has a vapor pressure of 557 mm. Consequently, if partial pressure is plotted against composition, the partial pressures of toluene at various compositions will fall along a straight line from 557 mm. for pure toluene to zero for pure benzene. At this same temperature benzene has a vapor pressure of 1,344 mm., and its vapor pressure will change linearly from zero for pure toluene to 1,344 mm. for pure benzene. This is shown in Fig. 146. The total pressure for any composition will be the sum of the two partial pressures at that composition, and if the partial-pressure curves are straight lines (*i.e.*, Raoult's law holds), the total pressure will be a straight line between 557 mm. for pure toluene and 1,344 mm. for pure benzene.

At the point where the total pressure is 760 mm. it is found graphically that the composition of the liquid is 0.263 mol benzene and 0.737 mol toluene. The composition of the vapor is determined from the partial pressures by Dalton's law, Equation (7). Here p_a is 351 mm. and the total pressure is 760 mm.; hence the vapor composition y (mol fraction benzene) is $^{351}/_{760}$ or 0.462. Therefore, for an ordinate of 100°C. in Fig. 145, the abscissa of the point on the liquid curve is 0.263, and on the vapor curve 0.462.

The above calculation can be carried out analytically by writing Raoult's law for component A as

$$p_a = P_a x \tag{174}$$

where p_a = the partial pressure of component A over a solution in which the mol fraction of A is x, and P_a represents the vapor pressure of A in the pure state at the temperature of the liquid. Similarly,

$$p_b = P_b(1 - x) \tag{175}$$

can be written for the second component (component B) of the mixture. Here p_b represents the partial pressure of component B over the solution, and P_b represents the vapor pressure of pure B at the temperature of the solution. If P represents the total pressure,

$$P = p_a + p_b = P_a x + P_b(1 - x) \tag{176}$$

Since y, the mol fraction of component A in the vapor, is equal to the ratio of the partial pressure of A to the total pressure,

$$y = \frac{p_a}{p_a + p_b} = \frac{P_a x}{P_a x + P_b(1 - x)} = \frac{P_a x}{P} \tag{177}$$

By choosing a series of temperatures intermediate between the boiling points of the two pure components, points can be calculated on both the vapor and liquid curves of the boiling-point diagram for any given total pressure, provided the vapor-pressure curves of the two pure components are at hand and provided it is known that the mixture obeys Raoult's law through the range of composition involved. This may be done analytically, since for any particular temperature, x can be calculated from Equation (176), and having found x, y can be evaluated from Equation (177). The solution may also be made graphically by the method of the preceding paragraph.

Example 26.—The vapor pressures of benzene and toluene are as follows:

Temperature, degrees Centigrade	C_6H_6, millimeters	C_7H_8, millimeters
80	753	290
80.6	760	
85	877	345
90	1,016	405
95	1,168	475
100	1,344	557
105	1,532	645
110	1,748	743
110.7	.....	760

Assuming that mixtures of benzene and toluene obey Raoult's law, calculate and plot the boiling-point diagram for this pair of liquids at 760 mm. total pressure.

Solution.—A series of temperatures are chosen, and the corresponding values of P_a and P_b are substituted in Equation (176). For example, at 85°:

$$P_a = 877$$
$$P_b = 345$$
$$877x + 345(1 - x) = 760$$
$$x = 0.780$$

The vapor composition is determined from Equation (177):

$$y = \frac{877x}{760}$$
$$y = 0.900$$

Similarly

Temperature, degrees Centigrade	x	y
90	0.581	0.777
95	0.411	0.632
100	0.258	0.456
105	0.130	0.263
110	0.017	0.039

These values are plotted in Fig. 147.

Raoult's law applies only to mixtures in which the components are very similar chemically and the molecules of the two substances do not interact in any way. Thus benzene and toluene obey Raoult's law closely, but mixtures of alcohol and water, acetic acid and water, or methanol and acetone do not. Most combinations used in practice deviate more or less widely from this law.

As an example of the divergence from Raoult's law shown by many systems, consider the mixtures of carbon bisulphide and acetone at 37.2°C. shown in Fig. 148. The partial pressures of each of the two components are plotted against their respective mol fractions in the liquid mixtures. By comparing this figure with Fig. 146 it is apparent that both of the components of this mixture diverge greatly from Raoult's law, represented by the straight lines in Fig. 148.

In spite of the great divergence in the middle of the diagram, the actual partial-pressure curve for either component is tangent

to the Raoult's law line for that component when nearly pure. This behavior is quite generally the case and is often summarized

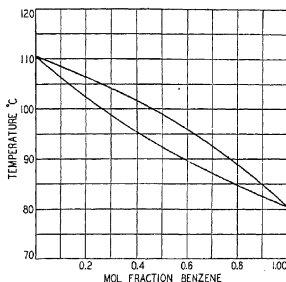



FIG. 147.—Boiling-point diagram (system benzene-toluene).

by the statement that "Raoult's law applies to the solvent." It must be emphasized, however, that the validity of Raoult's law may hold only over a very short range of compositions.

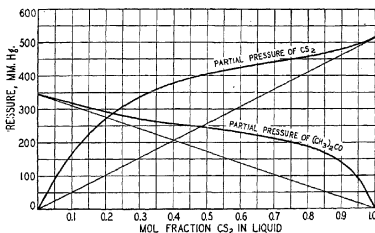



FIG. 148.—Partial-pressure curves at 37.2°C. (system carbon bisulphide-acetone).

Henry's Law.—Another law that is somewhat analogous to Raoult's law is known as Henry's law. According to this, the partial pressure of a component over a solution is proportional to its mol fraction in the liquid. This can be expressed as

$$p_a = Kx \tag{178}$$

where p_a represents the partial pressure of component A over a liquid mixture in which the mol fraction of A is x and K is the Henry's law constant. By comparing Equations (174) and (178) it will be seen that Raoult's law is essentially a special case of Henry's law wherein the constant in Equation (178) becomes the vapor pressure of the pure component.

Reference to Fig. 148 will show that the partial-pressure curve for either component is substantially straight at the end of the

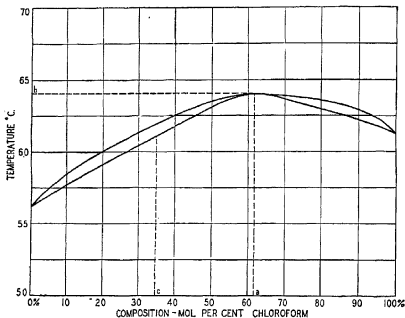



FIG. 149.—Boiling-point diagram showing maximum boiling point (system chloroform-acetone).

curve wherein the component in question is present in very small amounts. This fact is also quite generally true and is, of course, equivalent to saying that the component present in small amounts will follow Henry's law or that "Henry's law applies to the solute." In all cases it is an experimental problem to determine just how far either Henry's law or Raoult's law actually applies.

Constant Boiling Mixtures.—If the mixture follows Raoult's law it will give a boiling-point diagram of the type shown in Fig. 145. Many combinations, however, show diagrams such as Figs. 149 and 150. It will be noted from Fig. 149, which represents the boiling-point diagram for chloroform and acetone,

that the composition represented by point a has the highest boiling point of any combination of these two substances. This

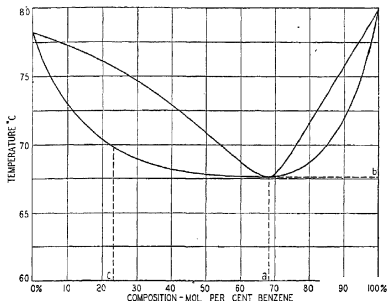



FIG. 150.—Boiling-point diagram showing minimum boiling point (system benzene-ethyl alcohol).

system exhibits a maximum boiling point, represented by point b. Figure 150 represents the system benzene-ethyl alcohol, which shows a minimum boiling temperature of b corresponding to composition a. It will be noted from both of these diagrams that the liquid and vapor curves are tangent at the maximum or minimum boiling point. This means that the composition of the vapor from minimum or maximum boiling mixture is the same as the composition of the boiling liquid. Such mixtures are therefore called *constant boiling mixtures.*

FIG. 151.—Normal equilibrium diagram.

It was stated that in order to separate two components by distillation it was necessary that the vapor be of different composition than the liquid from which it is formed. Accordingly, constant boiling mixtures cannot be separated by distillation.

Furthermore, a mixture on one side of such a constant boiling mixture, as represented by c in Figs. 149 and 150, cannot be transformed by distillation into mixtures on the other side of the constant boiling mixture. If the total pressure on the system is changed, the constant boiling mixture may shift in composition, and this principle can be utilized to obtain separations under

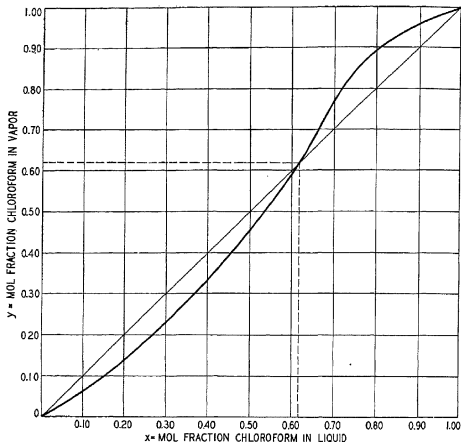



FIG. 152.—Equilibrium diagram showing maximum boiling point (system chloroform-acetone).

either pressure or vacuum that cannot be obtained at atmospheric pressure, where systems involving constant boiling mixtures are concerned. The same result may also be obtained by adding a third component.

Equilibrium Diagrams.—In the discussion of distillation problems a simplified form of Figs. 145, 149 and 150 is often used. Such a simplified diagram is called an equilibrium diagram and gives a relationship between the composition of the vapor

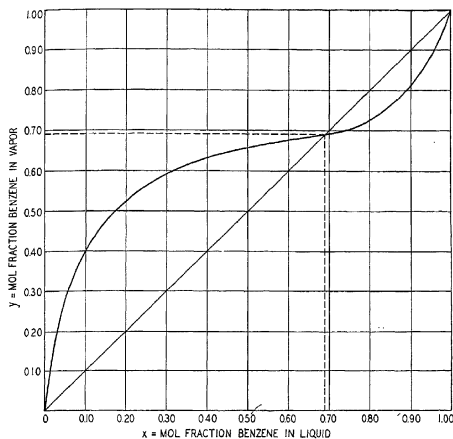



FIG. 153.—Equilibrium diagram showing minimum boiling point (system benzene-ethyl alcohol).

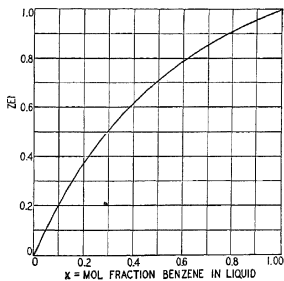



FIG. 154.—Equilibrium diagram (system benzene-toluene).

and that of the liquid in equilibrium with this vapor. Figure 151 is such a diagram for a system of the type shown in Fig. 145. Any liquid of a composition such as x_1 is in equilibrium at its boiling point with a vapor of composition y_1. Figure 152 is the equilibrium curve of the system chloroform-acetone shown in Fig. 149, and here the equilibrium curve crosses the diagonal line drawn through the origin at 45 deg. The point of intersection represents the constant boiling mixture, for here x is equal to y. Figure 153 is the equilibrium curve for the system benzene-ethyl alcohol, showing a minimum boiling point as in Fig. 150.

An equilibrium curve is easily constructed if the boiling-point diagram is at hand. It is necessary only to choose a liquid composition, proceed vertically to the first line on the boiling-point diagram, horizontally to the top line, and vertically down to the composition axis, where the value of y, corresponding to the assumed value of x, is found.

Example 27.—From the data of Example 26, plot the equilibrium diagram for the system benzene-toluene.

Solution.—The pairs of values for x and y are plotted in Fig. 154.

DISTILLATION METHODS

In practice, distillation may be carried out in either of two main methods. The first method involves the production of a vapor by boiling the liquid mixture to be separated and condensing the vapors without allowing any liquid to return to the still in contact with the vapors. The second method involves the returning of part of the condensate to the still under such conditions that this returning liquid is brought into intimate countercurrent contact with the vapors on their way to the condenser. This latter method is of such importance that it is given the special name of *rectification.*

Equilibrium Distillation.—There are two important types of distillation that do not involve rectification. The first of these is *equilibrium* or *flash distillation* and the second is *simple* or *differential distillation.* Equilibrium distillation involves vaporizing a definite fraction of a batch of liquid, keeping all of the liquid and all of the vapor in intimate contact so that at the end of the operation the vapor is in equilibrium with the liquid, withdrawing the vapor, and condensing it. The relationship

between liquid and vapor compositions at the end of the process is, therefore, that exhibited by the equilibrium diagram.

Consider a binary system whose components are A and B. A is the more volatile. Let the number of mols in the initial batch be W_0, and let the composition of the original batch, expressed as mol fraction[1] of component A, be x_0. Suppose that V mols are vaporized in an equilibrium distillation process. Then there will be left in the liquid state $W_0 - V$ mols. Let the composition of the residual liquid be x, and the composition of the vapor be y. Since there were W_0x_0 mols of more volatile component in the original liquid, and since none of this has been lost, this quantity must equal the sum of the amounts of volatile component in the final liquid and vapor:

$$W_0x_0 = Vy + (W_0 - V)x \tag{179}$$

This equation is a material balance of the more volatile component. There are two unknowns in Equation (179). These unknowns are x and y. Before the equation can be solved numerically, there must be another relationship between the two unknowns. Such a relationship is provided by the equilibrium curve, and values of x and y must be chosen so that they fit both Equation (179) and the equilibrium curve. This is most easily done by plotting Equation (179) on the equilibrium diagram, and the intersection of the two lines gives the desired solution.

Equilibrium distillation is not of great importance in the handling of two-component systems. The method is used in multicomponent systems in oil refining, however, where petroleum mixtures are heated under pressure in pipe stills, the pressure removed and the vapor flashed under approximately equilibrium conditions from the superheated liquid.

Differential Distillation.—In differential or simple distillation the vapor generated by boiling the liquid is withdrawn from contact with the liquid and condensed as fast as it is formed. Consider a batch of W_0 mols[1] of liquid. Suppose that at any given time during the distillation there are W mols of liquid left in the still. At this time let the liquid composition be x and the vapor composition y. The total amount of component A in the liquid will be xW. Suppose a very small amount of liquid dW is vapor-

[1] The following equations will hold as well if compositions, including those on the equilibrium curve, are expressed as weight fraction.

ized. During the vaporization the liquid composition will diminish from x to $x - dx$, and the weight of the liquid will diminish from W to $W - dW$. There will be left in the still $(x - dx)(W - dW)$ mols of A, while the amount ydW has been removed from the still. A material balance equation with respect to component A is, therefore,

$$xW = (x - dx)(W - dW) + ydW \tag{180}$$

Expanding Equation (180):

$$xW = xW - xdW + dxdW - Wdx + ydW \tag{181}$$

It will be noted that the e is a second-order differential in Equation (181) which may be neglected, and the equation may be written as

$$\frac{dW}{W} = \frac{dx}{y - x} \tag{182}$$

If Equation (182) is integrated between the limits of W_0, the initial weight in mols, and W_1, the final weight in mols, on the left-hand side; and the limits, x_0, the initial concentration, and x_1, the final concentration, on the right-hand side, Equation (183) results:

$$\int_{W_1}^{W_0} \frac{dW}{W} = \int_{x_1}^{x_0} \frac{dx}{y - x} = \ln \frac{W_0}{W_1} \tag{183}$$

Equation (183) is known as the *Rayleigh equation*. The function dx can be integrated graphically from the equilibrium curve, since this curve gives the relationship between x and y.

Simple or differential distillation is approached by commercial batch distillation processes where the vapor is removed as fast as it is formed without appreciable condensation. Although as a method of separation this process is not effective, many such stills are used, especially where the components to be separated have widely different boiling points and methods giving sharp separations are not necessary even if they are possible. The older design of the batch type of petroleum still, known as the "topping" still, is an example. Also, most laboratory distillations carried on without reflux columns are of this type.

Rectification.—Rectification, the second method of distillation described on page 334, has been developed in commercial practice to the point where it is extremely effective. Most of its applications were originally developed for distilling mixtures containing alcohol. In general, they are far ahead of the methods used in much laboratory work. Instead of laborious and time-consum-

ing fractional distillations involving methods closely allied to simple distillation, commercial practice has developed the use of continuous countercurrent rectification, carried out in elaborate and highly effective distillation apparatus equipped with what are known as *fractionating columns*. Essentially, a fractionating column is a device for bringing a stream of vapor and a stream of liquid into intimate countercurrent contact, while the *still* is that part of the apparatus that generates the vapor. The still must always be provided with a source of heat.

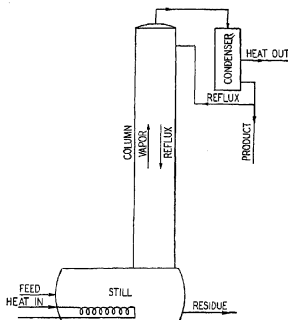



FIG. 155.—Diagram of still and fractionating column.

A distillation apparatus that includes such a column is shown very diagrammatically in Fig. 155. In this figure, heat, supplied to the still by steam coils, generates vapor, which passes up through the column. The vapor coming from the top of the column is completely condensed. Part of it is withdrawn as the product; and the rest, called the *reflux*, is put back into the top of the column to provide the downcoming stream of liquid. Inside the column, provision is made for intimate contact between the ascending vapors and the descending reflux. A simple method of providing for this contact is to fill the column with lumps of an inert material that will provide surface for the reaction between the vapor and liquid.

Bubble-cap Columns.—A more effective method of insuring contact between the vapor and reflux of a fractionating column is the use of plates and bubble caps. A sketch of a plate assembly is shown in Fig. 156. The column is divided into sections by

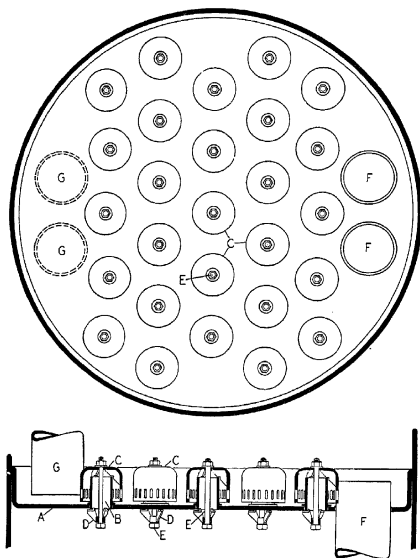



FIG. 156.—Construction of bubble-cap column. *A*. Column plate. *B*. Vapor nozzles. *C*. Bubble caps. *D*. Spider. *E*. Holding-down bolts. *F*. Downtake pipes to plate below. *G*. Downtake pipes from plate above.

means of a series of horizontal plates *A*. Each plate carries a number of short nipples *B*. Each nipple is surmounted with a bell-shaped cap *C*, that is secured by a spider *D* and bolt *E*. The edge of the cap may be serrated, or the sides may be slotted.

Vapor rises from the plate below through the nipple, is diverted downward by the cap, and bubbles out under the serrations or through the slots. A layer of liquid is maintained on the plate by means of an overflow or downpipe F, and the depth of the liquid is such that the caps are submerged. The downpipe G, from the plate above, is sealed by the liquid on the plate below, as shown, so that vapor cannot enter the downpipe.

Ordinarily the liquid is delivered at one end of a diameter by the downpipe from the plate above, flows across the plate, and is

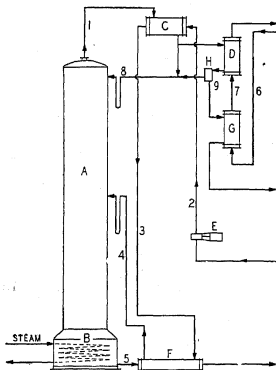



FIG. 157.—Column still and accessories.

discharged by a downpipe at the other end of the same diameter. Sometimes baffles are placed between the caps to divert the liquid and cause uniform flow across the plate. The caps are not necessarily circular; they may be rectangular with lengths several times their widths. In large columns there may be several downpipes.

A bubble-cap column for rectification, with some of its accessories, is shown in Fig. 157. The column A, contains a considerable number of such plates as shown in Fig. 156. At the bottom is a section B which is the vaporizer or boiler. This may be provided either with steam coils or with a heating element exactly like that in a vertical tube evaporator. Vapor leaves

the top of the column through pipe 1 and passes in series through condensers C and D. Feed enters from the pump E and is pumped through line 2 first through condenser C, where it is partly heated, then through line 3 to a heat interchanger F, where it is heated still further, and then through line 4 to some point part way down the column. The plate on which the feed is introduced is called the *feed plate*. The part of the column above the feed plate is called the *rectifying section*, and that below and including the feed plate is called the *stripping* or *exhausting section*. Any line, such as line 4, that enters the column is usually provided with a loop to seal it against any pressure that may exist in the column. The waste or high-boiling fraction from the still leaves the boiler B through line 5, goes through heat exchanger F and then to storage. Water for the condensers enters through line 6 and first goes through cooler G, in which it cools the overhead product of the still. It then goes through line 7 to condenser D, where it condenses the rest of the vapors, and from here leaves to the sewer.

The liquid from condenser C will be slightly leaner in the more volatile constituent than the liquid from condenser D. It may be returned directly to the still as reflux through line 8 or it may be combined with the product of condenser D. In the particular case shown, condensate from condenser D goes through line 9 to a reflux control device H, where a part is sent to be mixed with the liquid from C as reflux, and the rest of the condensate from D goes through cooler G and then to storage. The arrangement of these auxiliary condensers and the manner in which their condensates are combined to be used as reflux or removed as product, vary greatly with different installations, with the materials being separated and with the purity of product desired.

THEORY OF THE FRACTIONATING COLUMN

A number of methods have been proposed for the calculation of fractionating columns. A simple and accurate method for most practical purposes is that of McCabe and Thiele,[1] and this method will be used in the present discussion.

Definition of the Theoretical Plate.—The performance of any actual column is best understood by comparing it with an ideal

[1] *Ind. Eng. Chem.*, **17**, 605 (1925).

column. This ideal column is composed of "theoretical plates." A theoretical plate fulfills the requirement that the vapor rising from the plate and passing to the plate above is in equilibrium with the liquid leaving the plate and passing to the plate below. The degree with which an actual plate, operating on a definite mixture, will approach such an ideal plate may be determined only by experiment. This subject will be discussed later. The present discussion will deal only with perfect plates.

Mechanism of Fractionation.—Consider one plate (or section) of a column made up of perfect plates. This section involves

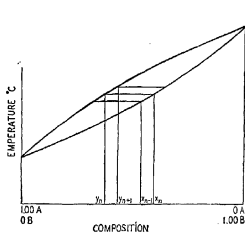



FIG. 158.—Boiling-point diagram applied to fractionating column.

four streams of material. A stream of liquid from above and a stream of vapor from below enter the section, a stream of liquid is delivered to the part of the column below, and a stream of vapor is delivered from the section to the part of the column above it. Suppose that the boiling-point diagram for the binary mixture being handled is given in Fig. 158. Component A is the more volatile, x is the composition of the liquid in mol fraction of A, and y is the composition of the vapor expressed in the same units.

The compositions of the streams are represented by x_n, x_{n-1}, y_{n+1}, and y_n in Fig. 158. The plate under consideration is receiving liquid of a composition x_{n-1} from the plate above and vapor of a composition y_{n+1} from the plate below. It delivers vapor of a composition y_n to the plate above and liquid of a composition x_n to the plate below. From the definition of the theoretical plate it follows that these last two streams are in

equilibrium, and Fig. 158 has been drawn to correspond to this assumption.

The liquid from the plate above has a composition x_{n-1} and is richer in component A than is the stream passing to the plate below. The vapor rising from the plate below has a composition y_{n+1} and is richer in component B than is the vapor rising from the plate under consideration. Since, therefore, the streams having the composition x_n and y_n are in equilibrium the streams having the compositions x_{n-1} and y_{n+1} cannot be in equilibrium. This may be seen also from Fig. 158. When these two streams are brought into contact with each other they tend to approach equilibrium. Some of component A is vaporized from the liquid, decreasing its content of component A from x_{n-1} to x_n; and some of component B is condensed from the vapor, increasing its content of component A from y_{n+1} to y_n. The heat necessary to vaporize A from the liquid is furnished by the condensation of B from the vapor. The section is acting as an interchange apparatus whereby the more volatile component is transferred from the liquid stream to the vapor stream and in this manner carried up the column, while the less volatile component is transferred from the vapor stream to the liquid stream and so carried down the column.

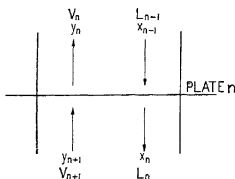



FIG. 159.—Material balance over ideal plate.

Heat and Material Balances over an Ideal Plate.—Consider a theoretical plate, as indicated in Fig. 159. This plate is the nth plate down from the top in a column consisting of ideal plates. The feed to the column is at some point below this plate. Rising to the plate is a vapor stream from the $(n + 1)$th plate below. Let the quantity of this vapor be V_{n+1} mols per hr., and let the mol fraction of component A (the more volatile) in this vapor be represented by y_{n+1}. The plate sends V_n mols of vapor per hr. to the $(n - 1)$th plate, and the composition of this vapor is y_n. A stream of liquid from the plate $(n - 1)$ descends on plate n as overflow or reflux. Let the weight of this stream expressed in mols per hr., be L_{n-1}. The mol fraction of component A in this

stream is x_{n-1}. Finally, plate n delivers, to plate $n + 1$, L_n mols per hr. of reflux of composition x_n.

The material balance equations that apply to the plate may be written down by inspection. They are

Total material:

$$V_{n+1} + L_{n-1} = V_n + L_n \tag{184}$$

Component A:

$$(V_{n+1})(y_{n+1}) + (L_{n-1})(x_{n-1}) = V_n y_n + L_n x_n \tag{185}$$

A heat balance for the plate may be written if a datum temperature is chosen. Let this be the temperature of the liquid on the plate. Since the vapor V_{n+1} is evolved from a boiling liquid leaner in component A than the liquid on the plate, the temperature of this vapor is somewhat higher than that of the datum. Also, the temperature of the liquid L_{n-1} is less than this datum temperature, since the liquid on plate $n - 1$ is richer in component A and hence possesses a lower boiling point.

A heat balance for the plate will contain several items. For convenience, each item will be designated by a letter.

Heat item	Item letter
Latent heat in vapor V_{n+1}	a
Sensible heat in vapor V_{n+1} above datum temperature	b
Sensible heat in liquid L_{n-1} below the datum temperature	c
Heat of mixing	d
Latent heat in vapor V_n	e
Radiation loss	f

The heat balance is

$$a + b - c + d = e + f \tag{186}$$

This equation may be greatly simplified by taking advantage of an approximation that is fairly close to the truth in ordinary cases. The two largest items in Equation (186) are, for most systems, the latent heat items a and e. The other four items are small; the sensible heats because the temperature change from one plate to another is small, the heat of mixing is ordinarily very small, or zero, and the radiation loss is minimized by lagging. Furthermore, two of these items (b and d) are balanced against the other two (c and f), so the net effect of neglecting all four

may be even smaller than the magnitude of any one. If the quantity $(b + d - c - f)$ be neglected in comparison with the greater quantities a and e, it follows that, approximately,

$$a = e \tag{187}$$

and latent heats alone need be considered.

Constant Molal Overflow.—The relationship shown in Equation (187) is made more useful by combining it with *Trouton's rule.* This empirical approximation states that the molal heat of vaporization divided by the absolute temperature of the boiling point is constant for large groups of chemically similar liquids. Thus, a mol of component A on plate n requires as much heat to vaporize it as a mol of component B on that plate.[1] The heat of vaporization of a mol of any mixture of A and B is therefore independent of the composition of the mixture, if the temperature changes are neglected. According to Equation (187), then,

$$V_{n+1} = V_n \tag{188}$$

since Equation (187) is obtained by multiplying each side of Equation (188) by the same quantity, namely, the molal latent heat.

Since plate n is any plate above the feed plate, this result is general (except for the feed plate and the top plate), and the mols of vapor rising to any plate are equal to the mols of vapor leaving that plate. The subscripts on the quantities V_{n+1} and V_n lose significance and may be dropped.

It is also apparent from Equations (184) and (188) that

$$L_{n-1} = L_n \tag{189}$$

Equations (188) and (189) define the terms *constant molal vaporization* and *constant molal overflow*, respectively.

Calculation of Number of Theoretical Plates in a Fractionating Column.—Consider a fractionating column that has both rectifying and stripping sections. Assume for simplicity that the column has but a single condenser in which the vapor from the

[1] If the two components of the mixture do not have the same Trouton's-rule constant (*i.e.*, if they belong to different classes, as water and acetic acid), a fictitious molecular weight may be assumed for either component, so that Trouton's rule is obeyed. In this case the boiling-point diagrams and equilibrium diagrams must be calculated on the basis of this fictitious molecular weight. PETERS, *Ind. Eng. Chem.*, **14**, 476 (1922).

top plate is completely condensed but is not cooled below its condensing temperature. Figure 160 is a diagrammatic section of the top portion of such a column, taken at any point above the feed plate. Let D be the mols of distillate drawn off as product per unit time, and let L be the mols of distillate returned to the top plate as reflux per unit time. Let x_D be the composition of the distillate from the condenser, and this will also be the composition of the product D and the reflux L. Assume that the plates of the column are numbered serially from the top plate, and that a section is taken through the column between the nth and the $(n + 1)$th plates. Since the reflux L is at its boiling point, the liquid overflow from the top plate per unit time will also equal L. Due to the fact that the molal reflux in the column is constant, L mols of liquid will also leave plate n on their way to plate $(n + 1)$. Likewise, the stream of vapor passing through the column will be constant in amount from plate to plate. Let the mols of vapor in this stream be V. Let x_n be the composition of the liquid leaving the nth plate and let y_{n+1} be the composition of the vapor arising from the $(n + 1)$th to the nth plate. In this nomenclature x is always a liquid composition and y is always a vapor composition, both as mol fractions of the more volatile component. The subscript in any case refers to the plate where the quantity involved originates.

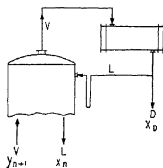



Fig. 160.—Material balance above feed plate of column.

Since all of the material entering the section in question must leave it,

$$V = L + D \tag{190}$$

Likewise, all of the more volatile constituent that enters in the vapor must leave in either the liquid reflux or the distillate and therefore

$$Vy_{n+1} = Lx_n + Dx_D \tag{191}$$

Eliminating V from Equations (190) and (191):

$$y_{n+1} = \frac{L}{L + D}x_n + \frac{D}{L + D}x_D \tag{192}$$

If the plates of the column are assumed to be perfect, the vapor rising from any plate will be in equilibrium with the liquid leaving that plate; and therefore a point having coordinates $x = x_n$, $y = y_n$ will lie on the equilibrium curve. If values of L, D and x_D are known, Equation (192) and the equilibrium curve can be used in a stepwise manner to calculate the compositions of liquid and vapor pertaining to the various plates in the column. For example, the composition of the distillate of the column of Fig. 160 is the same as that of the

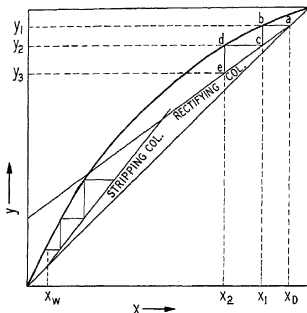



Fig. 161.—Development of McCabe-Thiele method.

vapor from the top plate, and therefore y_1 and x_D are identical. The value of x_1 is found by reading the x-coordinate of a point on the equilibrium curve, whose y-coordinate is y_1. If x_1 is substituted in Equation (192), y_2 may be calculated and the value of x_2 read from the equilibrium curve. This procedure can be continued as far as is necessary.

The above procedure is especially suited for graphical treatment. It is only necessary to plot a line representing Equation (192) where y_{n+1} and x_n are considered to be the variables. Equation (192) is the equation of a straight line, of slope $\frac{L}{L + D}$, and this line must pass through the point (x_D, y_1). Since x_D

and y_1 are equal, the point (x_D, y_1) must also lie on the diagonal of the equilibrium diagram. Substitution of x_D for x_n in Equation (192) gives the result $y_1 = x_D$, which is formal check of this fact. Figure 161 shows Equation (192) plotted on the equilibrium diagram. This line is called the *operating line*. The analytical procedure described in the last paragraph can be reproduced by starting at point a, which is the point (x_D, y_1). The horizontal segment ab gives the value of x_1. A vertical segment from b to point c on the operating line gives (as the y-coordinate of point c) y_2. A second horizontal segment cd, gives x_2, and a second vertical section de gives the value of y_3. This stepwise process is the graphical solution for determining the concentrations of liquid and vapor pertaining to any plate in the series. In any given case the steps will be continued until the feed plate is reached, where the addition of a new stream changes the flows of liquid and vapor and therefore calls for a new operating line.

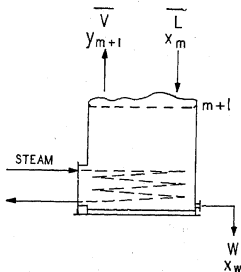



FIG. 162.—Material balance below feed plate of column.

The stripping column can be analyzed in the same way. Figure 162 represents a section of the column taken below the feed plate between any two plates such as m and $m + 1$. From the boiler, which may be considered the bottom section of the column, the residue W is removed from the system. Let the composition of the residue be represented by x_w. Let $\bar{V}$ represent the vapor leaving the $(m + 1)$th plate and let $\bar{L}$ be the liquid leaving the mth plate and descending upon the $(m + 1)$th plate. Let y_{m+1} be the composition of the vapor leaving the $(m + 1)$th plate, and let x_m be the composition the liquid leaving the mth plate. The quantities $\bar{V}$ and $\bar{L}$ will be constants throughout the stripping column, but will in general differ from L and V, the corresponding quantities in the rectifying column. Total material and more volatile component balances give the equations

$$\bar{V} = \bar{L} - W \tag{193}$$
$$\bar{V}y_{m+1} = \bar{L}x_m - Wx_w \tag{194}$$

Elimination of $\bar{V}$ from Equations (193) and (194) gives:

$$y_{m+1} = \frac{\bar{L}}{\bar{L} - W} x_m - \frac{W x_w}{\bar{L} - W} \tag{195}$$

Equation (195) is the equation of another straight line, of slope $\frac{\bar{L}}{\bar{L} - W}$, which, if $\bar{L}$ can be determined, can be plotted on the equilibrium diagram and can be used with the equilibrium curve for determining concentration changes in the stripping column in the same manner as the operating line for the rectifying column used above the feed plate. When Equation (195) is written for the last plate (by substituting x_w for x_m), the corresponding value of y is found to be x_w. Hence the line passes through the point ($y = x_w$, $x = x_w$) or a point on the diagonal of the equilibrium diagram.

Effect of Feed Condition on Operating Lines.—The relationship between L and $\bar{L}$, and V and $\bar{V}$ depends upon the condition of the feed mixture that enters the column. This feed may be cold liquid; it may be liquid at its boiling point; it may be a mixture of liquid and vapor; it may be a saturated vapor; or it may be a superheated vapor. It is desirable to develop a general rule that will take all these cases into account.

Let the quantity q be defined as the *total heat needed to convert one mol of feed into saturated vapor, divided by the molal latent heat.* If the feed is already a saturated vapor, q will be zero. If the feed is a liquid at the boiling point, q will be equal to the latent heat divided by the latent heat, or 1. If the feed is a cold liquid, q will be greater than one because heat over and above the latent heat is necessary to convert a mol of feed to the saturated vapor. If the feed is a mixture of liquid and vapor, q will be between zero and 1; and will equal the fraction of the feed that is liquid. If the feed is a superheated vapor, q will be negative.

If F represents the mols of feed entering the column per unit time, it follows from a definition of q that qF is the flow of liquid, over and above that coming from the rectifying section, that descends the stripping section, and, therefore

$$\bar{L} = L + qF \tag{196}$$

Also, by a material balance over the feed plate,

$$L + \bar{V} + F = \bar{L} + V \tag{197}$$

and, therefore,

$$\bar{V} = L + qF + V - L - F = F(q - 1) + V \tag{198}$$

If $\bar{L}$ be substituted from Equation (196) into Equation (195)

$$y_{m+1} = \frac{L + qF}{L + qF - W}x_n \qquad \frac{Wx_w}{L + qF - W} \tag{199}$$

If a value of L has been chosen, and if the amount of feed and waste is known, the operating line for the stripping column can be plotted by the aid of Equation (199) because the line passes through the point (x_w, x_w) and has a slope of $\frac{L + qF}{L + qF - W}$, all the quantities in which are now known.

Intersection of the Operating Lines.—There is a more convenient method of plotting the operating line for the stripping section than the direct use of Equation (199). The method is found by eliminating the reflux L from Equations (192) and (199). This gives the equation of a line on which the operating lines must intersect, whatever the value of L may be. For this purpose, the subscripts can be dropped from the x and y values of Equations (192) and (199) and each equation solved for the quantity $L(y - x)$. Thus, from Equation (192)

$$L(y - x) = D(x_D - y) \tag{200}$$

and from Equation (199)

$$L(y - x) = qF(x - y) + W(y - x_w) \tag{201}$$

Therefore,

$$D(x_D - y) = qF(x - y) + W(y - x_w) \tag{202}$$

But, by overall total material and volatile component balances:

$$W = F - D \tag{203}$$

$$Wx_w = Fx_f - Dx_D \tag{204}$$

If W and Wx_w are eliminated by means of Equations (203) and (204) from Equation (202) the result is:

$$D(x_D - y) = qF(x - y) - Fx_f + Dx_D + Fy - Dy$$

or, simplifying:

$$y = \frac{q}{q-1}x - \frac{x_f}{q-1} \tag{205}$$

Equation (205) represents a straight line called the q-line, on which the operating lines intersect. The position of the line depends only on q and x_f. The slope of the line is $q/(q-1)$, and, as may be seen by substituting x_f for x in Equation (205),

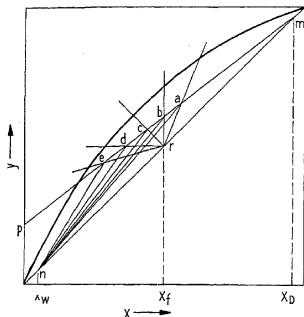



Fig. 163.—Effect of condition of feed on q-line.
ra = q-line when feed is cold liquid
rb = q-line when feed is liquid at boiling point
rc = q-line when feed is mixture of liquid and vapor
rd = q-line when feed is saturated vapor
re = q-line when feed is superheated vapor

the line intersects the 45-degree diagonal ($x = y$) at the point $x = x_f$, $y = x_f$.

If the feed is a cold liquid, the q-line has a slope upwards and to the right; if the feed is a liquid at the saturation temperature, the line is vertical; if the feed is a mixture of liquid and vapor, the line slopes upward and to the left and the slope is equal to the ratio of the liquid to the vapor; if the feed is a saturated vapor, the line is horizontal; and, finally, if the feed is a superheated vapor, the slope is downward and to the left. All of these cases are shown in Fig. 163 where operating lines

are shown for various thermal conditions of the feed. The value of L is the same for all of the cases shown in Fig. 163.

The line mp is the operating line for the top part of the column and is determined by Equation (192). The operating line for the bottom part of the column must pass through the point (x_w, x_w), and the two operating lines must intersect on the q-line. Hence, when the q-line is ra, the lower operating line is na, and so for all the other positions of the q-line.

Reflux Ratio.—Although the equations already presented are sufficient to carry out the graphic analysis of fractionating column design and performance, the procedure is simplified by the introduction of a new quantity, the *reflux ratio*. This is the ratio between the amount of liquid passing down through the rectifying column and the amount drawn off as product. If the reflux ratio be represented by R it can be defined as

$$R = \frac{L}{D} = \frac{V - D}{D} \tag{206}$$

If both the numerator and denominator of both terms of the right-hand side of Equation (192) be divided by D, the resulting equation is

$$y_{n+1} = \frac{R}{R+1}x_n + \frac{x_D}{R+1} \tag{207}$$

This also is the equation of the operating line for the upper part of the column. The value of x_D is fixed by the conditions of the problem; and R, the reflux ratio, may be set at will during the operation of the still by merely adjusting the reflux control device H of Fig. 157.

It follows from Equation (207) that the y-intercept of the operating line for the rectifying section is $\frac{x_D}{R+1}$. This operating line is plotted most easily by passing it through this intercept and the point on the $x = y$ diagonal where $x = x_D$.

Calculation of Number of Theoretical Plates in a Fractionating Column.—The discussion above may be summarized by the following problem: Consider a continuous column fed with liquid of composition x_f and of known molal heat content, making a product of composition x_D, discharging a waste of composition x_w, and operating with a reflux ratio of R mols

per mol of product. How many perfect plates are needed? On what plate should the feed be introduced?

1. The equilibrium curve and the $x = y$ diagonal are plotted (Fig. 164).

2. The value of q is calculated from the molal heat content of the feed and the q-line plotted by passing a straight line through

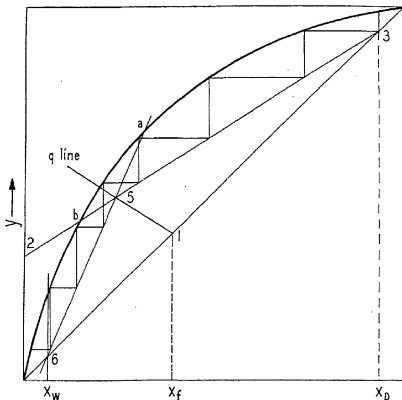



FIG. 164.—Typical solution for fractionating column by McCabe and Thiele's method.

the intersection of the vertical line $x = x_f$ with the diagonal (point 1) and having a slope of $\frac{\ }{q - 1}$

3. The intercept $\frac{x_D}{R + 1}$ is calculated and plotted on the y-axis as point 2 in Fig. 164. A straight line is drawn between point 2 and point 3, which latter is the point on the $x = y$ line corresponding to x_D. This establishes the point 5 on the q-line.

4. Connect point 5 with point 6, which is the point on the diagonal corresponding to the value of x_w.

5. Starting at point 3 draw a series of rectangular steps between the equilibrium curve and the operating line for the rectifying column. The steps are continued until the point 5 is passed. When point 5 is passed the verticals are dropped to the operating line for the stripping section, and this line is utilized until the vertical line of a step coincides with x_w[1] or a lesser value.

The total number of steps is the total number of perfect plates in the column, and the step straddling the q-line is the plate on which the feed should be introduced. Consequently, in the case shown in Fig. 164 there will be seven plates and the feed will be on the fourth plate from the top. The transfer from one operating line to the other may be made at any point between a and b, but any other method than the one described will result in smaller steps (*i.e.*, more plates in the column).

Example 28.—A continuous fractionating column is to be designed to separate 30,000 lb. per hr. of a mixture of 60 per cent toluene and 40 per cent benzene into an overhead product containing 97 per cent benzene and a waste containing 98 per cent toluene. (All the above are percentages by weight.) A reflux ratio of 3.5 mols reflux per mol of product is to be used. The molal latent heat of vaporization of benzene and toluene is 7,240 cal. per g.-mol.

a. Calculate the weight of product and waste per hour.

b. Determine the number of perfect plates and the position of the feed plate

(1) if the feed is liquid and at its boiling point

(2) if the feed is liquid and at 20°C. (Sp. ht. = 0.44)

(3) if the feed is a mixture of ⅔ mol of vapor and ⅓ mol of liquid per mol of feed.

c. If steam at 20 lb. gage is to be used for heating, how much steam is required per hour for each of the above three cases (assuming that radiation losses are negligible)?

Solution.—*Part a*—Let

D = pounds product per hour.
W = pounds waste per hour.

[1] To fulfill the conditions of the problem literally, the last vertical must coincide with x_w. This is equivalent to saying that the four quantities x_f, x_D, x_w, and R may all be fixed arbitrarily; but in general, such a choice will not result in an integral number of plates. Thus, in the case of Fig. 164, an integral number of plates can be obtained only by a slight change in one of the above quantities. As actually drawn in Fig. 164, the composition of the waste would be leaner than x_w. Practically, however, it is not necessary to make this adjustment because the plates in an actual column are seldom equivalent to theoretical plates.

$$30{,}000 = D + W$$

Benzene balance:

$$(30{,}000)(0.40) = 0.97D + 0.02W$$

Solving these two equations simultaneously,

$$D = 12{,}000$$
$$W = 18{,}000$$

Part b(1).—It will be necessary to convert the composition of feed, overhead, and waste from weight per cent to mol fraction:

Feed

$$\begin{aligned} 40/78 &= 0.513 \text{ mol benzene} \\ 60/92 &= \underline{0.652} \text{ mol toluene} \\ & 1.165 \text{ total mols} \\ x_f &= \frac{0.513}{1.165} = 0.440 \end{aligned}$$

Product

$$\begin{aligned} 97/78 &= 1.2450 \text{ mols benzene} \\ 3/92 &= \underline{0.0326} \text{ mol toluene} \\ & 1.2776 \text{ total mols} \\ x_D &= \frac{1.2450}{1.2776} = 0.974 \end{aligned}$$

Waste

$$\begin{aligned} 2/78 &= 0.0256 \text{ mol benzene} \\ 98/92 &= \underline{1.066} \text{ mols toluene} \\ & 1.092 \text{ total mols} \\ x_w &= \frac{0.0256}{1.092} = 0.0234 \end{aligned}$$

The first step (page 352) is to plot the equilibrium diagram and on it erect verticals at x_D, x_f, and x_w. These should extend to the diagonal of the diagram. (See Fig. 165.)

The second step is to plot the q-line (Equation 205). Here, q is 1, and the q-line is vertical; that is, a continuation of the line $y = x_f$.

Step 3 is to plot the operating line for the upper or rectifying part of the column. This is done by calculating its intercept on the y-axis from Equation (207). Here

$$x_D = 0.974$$
$$R = 3.5$$

and the intercept is 0.217. From the intersection of this operating line with the q-line, the operating line for the lower or stripping part of the column is drawn (step 4).

Step 5 is to draw the rectangular steps between the two operating lines and the equilibrium diagram. By counting the rectangular steps it is found that twelve perfect plates are required, and the feed is on the seventh plate from the top of the column.

Part b(2).—This solution will be the same except for the q-line. The specific heat given is calories per gram, and this must be converted to

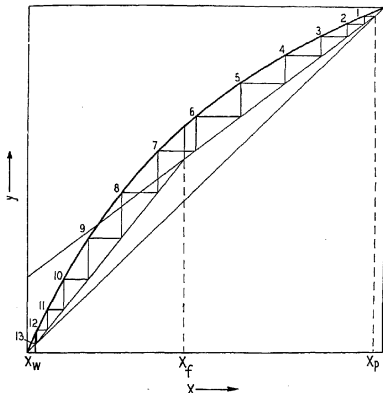



Fig. 165.—Solution of Example 28, Part *b*(1).

calories per gram-mol. From the solution of Part *b*(1) it is seen that 100 lb. of feed contain 1.165 mols, hence the average molecular weight of the feed is 85.8. The specific heat in calories per gram-mol is therefore $0.44 \times 85.8 = 37.7$.

It is next necessary to know the boiling point of the feed. From Fig. 147 it is seen that a mixture containing 0.44 mol-per cent benzene boils at 93°C. To convert 1 gram-mol of feed to saturated vapor will take

Sensible heat . . $37.7 \times (93 - 20)$ =	2,752 calories
Latent heat	7,240
Total	9,992

The value of q is therefore $9{,}992/7{,}240 = 1.38$ and the slope of the q-line is $\dfrac{1.38}{1.38 - 1} = 3.63$.

If the steps outlined above be carried out for this case, it is found that the total is now eleven perfect plates with the feed on the fifth from the top (Fig. 166).

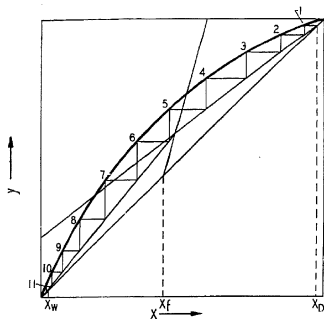



FIG. 166.—Solution of Example 28 Part b(2).

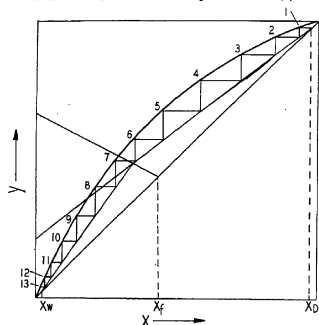



FIG. 167.—Solution of Example 28, Part b(3).

Part b(3).—From the definition of q on page 348 it follows that for this case $q = 0.333$ and the slope of the q-line is -0.5. The solution is shown

in Fig. 167, and calls for 13 perfect plates with feed on the seventh from the top.

Part c.—The total feed is 30,000 lb. per hr., and 100 lb. contains 1.165 mols. Therefore the total feed is

$$30{,}000/100 \times 1.165 = 349.5 \text{ mols}$$

The total product is 12,000 lb. and 100 lb. of product contain 1.2776 mols. The total vapor going up the rectifying part of the column is 4.5 mols per mol of product, or

$$V = \left(\frac{12{,}000}{100}\right) \times 1.2776 \times 4.5 = 690.2$$

From Equation (198)

$$\bar{V} = 349.5(q - 1) + 690.2$$

The latent heat of vaporization is 7,240 cal. per g.-mol, or

$$7{,}240 \times 1.8 = 13{,}032 \text{ B.t.u. per lb.-mol.}$$

The heat given up by 1 lb. steam at 20 lb. gage in condensing is 939 B.t.u. per lb. The steam needed is, therefore,

$$\left(\frac{13{,}032}{939}\right)\bar{V} = \left(\frac{13{,}032}{939}\right)[349.5(q - 1) + 690.2] \text{ lb. per hr.}$$

Case	q	Pounds of steam per hour
$c(1)$	1.00	9,576
$c(2)$	1.38	11,420
$c(3)$	0.33	6,331

Minimum Reflux.—Since the slope of the operating line is $\frac{R}{R + 1}$, the slope increases as R increases, until when R is infinite the slope is 1; in other words, the operating line coincides with the diagonal. This gives the minimum number of plates, but zero capacity for any finite still. On the other hand, there is a value for R below which the still cannot theoretically operate. This is called the *minimum reflux*. Any actual still must use a reflux ratio greater than the minimum but less than infinity.

In Fig. 168, the lines *ac*, *ad*, *ae*, and *af* show the effect of increasing the reflux ratio. In the case of small reflux ratios such as

represented by the operating line *ac*, the rectangular steps between this line and the equilibrium curve cannot pass the point *g*, and an infinite number is required to reach *g*. This shows that operation under the conditions assumed is theoretically impossible, for the steps never reach the *q*-line. If the line *ad* be considered, it is obvious that the point *d* could be

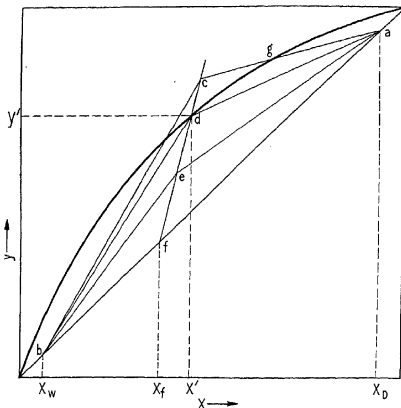



Fig. 168.—Minimum reflux ratio.

reached, but it would require an infinite number of plates both below and above the feed. This is the least reflux that would permit of operation under any conditions theoretically possible. The slope of operating line *ad* is such that it passes through the points (x', y') and (x_D, x_D). Let this minimum reflux ratio be R'. Then,

$$\frac{R'}{R'+1} = \frac{x_D - y'}{x_D - x'} \tag{208}$$

$$R' = \frac{x_D - y'}{y' - x'} \tag{209}$$

The line *ae* represents operation with a reflux ratio greater than the minimum. This corresponds to operating conditions not only theoretically but also actually possible, since it calls for a finite number of plates. The line *af* represents the other limiting condition (infinite reflux ratio), but this is impossible practically as discussed above.

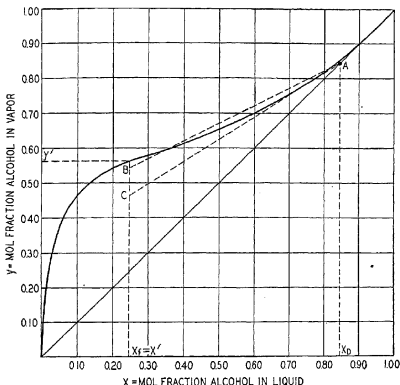



Fig. 169.—Equilibrium diagram (system ethyl alcohol-water).

Equation (209) cannot be applied in all cases for calculating the minimum reflux. Thus, if the equilibrium curve shows a concavity upward (as, for example, that for ethyl alcohol-water mixtures as shown in Fig. 169), it will be seen that the operating line must not cut across the equilibrium curve between the abscissas x_f and x_D, or the column will not operate, even if the operating line, such as AB, intersects the vertical q-line $x = x_f$ below the point (x', y'). In such a case, the minimum reflux must be determined from the slope of the operating line AC that is tangent to the equilibrium curve.

Example 29.—What is (*a*) the minimum reflux ratio and (*b*) minimum number of plates for cases *b*(1), *b*(2), and *b*(3) of Example 28?

Solution.—(*a*) For minimum reflux ratio, use Equation (209). Here

$$x_D = 0.974.$$

Case	x'	y'	R'
b(1)	0.440	0.658	1.45
b(2)	0.521	0.730	1.17
b(3)	0.300	0.513	2.16

(*b*) For minimum number of plates, the reflux would be infinite, the operating lines will coincide with the diagonal, and there is no difference

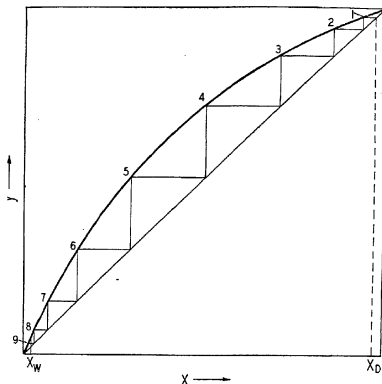



Fig. 170.—Solution of Example 29, Part (*b*).

between the three cases. The plot is given in Fig. 170 and calls for nine perfect plates.

Plate Efficiency.—Thus far the discussion has dealt only with theoretical plates. An actual column is constructed, however, of plates that usually do not act perfectly. The vapor leaving an actual plate is usually weaker in volatile constituents than vapor in equilibrium with the liquid leaving the plate. To apply the McCabe and Thiele method to an actual

case, it is necessary to convert the number of theoretical plates to actual plates. The factor for this purpose is known as the *plate efficiency*. The bottom plate (or the still pot if a separate still is used) delivers vapor directly from the boiling liquid and is therefore considered a perfect plate.

Two kinds of plate efficiency are recognized. In the first kind, an average efficiency for the entire column is obtained by dividing the number of theoretical plates by the number of

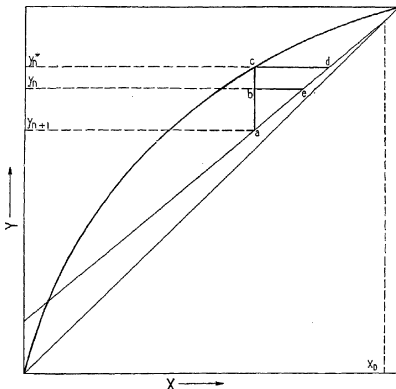



FIG. 171.—Definition of individual plate efficiency.

actual plates found necessary to do the same work in practice. The second type of plate efficiency considers the individual plates, and is derived as follows:

The diagram of an actual plate as compared with that of a theoretical plate is shown in Fig. 171. The triangle *acd* represents the theoretical plate and the triangle *abe* represents the actual plate. The theoretical plate enriches a mol of vapor passing through it by the amount $y_n^* - y_{n+1}$, represented by the distance *ac*. The actual plate, however, due to the finite time of contact of liquid and vapor on the plate, enriches the vapor only by the

amount $y_n - y_{n+1}$, represented by the distance *ab*. The efficiency of the plate is, then, represented by the ratio of the line *ab* to the line *ac*.

This definition of plate efficiency is due to Murphree[1] and is represented by the equation

$$E = \frac{y_n - y_{n+1}}{y_n{}^* - y_{n+1}} 100 \tag{210}$$

where

y_{n+1} = composition of the vapor rising to the plate.
y_n = composition of the vapor leaving the plate.
$y_n{}^*$ = composition of vapor in equilibrium with the liquid leaving the plate.

In general, the first definition (or average plate efficiency) is somewhat more convenient to use and in most cases the two methods do not give greatly different results. Plate efficiencies by either definition must be determined experimentally.

Example 30.—If the average plate efficiency for the still of Example 28 is 60 per cent, how many actual plates will be used in cases *b*(1), *b*(2), and *b*(3); and on what actual plate will the feed be introduced?

Solution.—In case *b*(1) the number of theoretical plates may be approximated as 12.3. The actual bottom plate is a perfect plate, hence there will be, above it, 11.3 theoretical plates or 11.3/0.60 = 18.8 or 19 actual plates. Hence the column will be 20 plates high. There are to be 6 theoretical plates, or 10 actual plates above the feed plate. Hence the feed will be on the 11th plate from the top. In the same way, for case *b*(2) there will be a total of 18 actual plates with feed on the 8th from the top. For case *b*(3) there will be 21 actual plates and 10 plates above the feed plate.

Factors Influencing Plate Efficiency.—Plate efficiencies are best studied by focusing attention on the mechanism of the interaction of the vapor and liquid on an actual plate. The bubble caps are submerged by liquid. The depth of submergence can be controlled by the height of the downpipes above the plate. The vapor passing through the slots forms a multitude of small bubbles which rise through the liquid. Ordinarily the caps are spaced quite closely together and the bubbles issuing from one cap impinge on those from adjacent caps,

[1] *Ind. Eng. Chem.*, **17**, 747–750 (1925).

increasing the violence of contact. Above the level of liquid there is a layer of froth from which the vapor disengages. Above the froth layer there will be small particles of liquid floating in the vapor space, some of which may be carried to the plate above. The liquid flow takes place across the plate and to produce this there must be a small hydraulic gradient from downpipe to downpipe.

The factors that influence the effectiveness of contact (and hence the plate efficiency) can be classified in three groups; first, those that have to do with the design and construction of the plate and the flow of the liquid across it; second, those that control the inter-diffusion of components between liquid and vapor as the bubbles pass through the liquid layer on the plate; third, those that control the entrainment of fine particles from one plate to that above it.

Considering the first group of factors, plate efficiencies will be improved if the liquid is not allowed to short-circuit across the plate; if each bubble cap is adequately supplied with liquor; if the hydraulic gradient across the plate is low; and if the stream of liquid passes across the plate with little mixing in the direction of flow. In the second group of factors, plate efficiencies are improved by small slots, which give fine bubbles; by increased submergence, which gives longer time of contact; and by the colliding of bubbles against each other, which tends to develop new surfaces of contact. Entrainment, which tends to decrease plate efficiency by undoing the processes occurring on the plate, is high for close plate spacing or high vapor velocities, and is somewhat dependent on surface tension.

In addition to the above design factors, plate efficiencies will depend upon the materials being distilled, although the exact relationships involved are still unknown.

It is not practical to attempt to obtain maximum plate efficiency. Such a result can be obtained only by wide plate spacing, low velocities, very small slots, large submergence and elaborate baffles and entrainment separators. This would result in very tall columns, large pressure drops for the passage of vapor, and expensive construction. In practice, the emphasis is on simplicity of construction and low pressure drop, in spite of the resulting lowering of plate efficiency. Efficiencies of 50 to 70 per cent on binary mixtures in ordinary sized columns

are the rule, although efficiencies as high as 90 per cent in small columns have been reported, and with poor design efficiencies may drop well below 50 per cent. In very large columns operating on light petroleum fractions, efficiencies of 100 per cent and over have been reported. This is due to the fact that in such columns, the liquid flows across the plate without longitudinal mixing. As a result, liquid coming on the plate at one side is richer in the more volatile constituents than that leaving the plate at the other side (assuming that inlet and outlet downpipes are on opposite sides of the plate). Vapor coming in contact with the first-mentioned liquid can be enriched considerably above the equilibrium determined by the composition of the leaving liquid; but it is on the basis of this latter equilibrium that plate performance is calculated.

Operating Vapor Velocity.—If the vapor velocity through a distilling column is made too high, the vapor will tend to carry liquid with it from plate to plate. This may, if the vapor velocity is moderate in magnitude, result only in a low plate efficiency due to the entrainment of fine drops of liquid. In extreme cases, however, the vapor may carry large volumes of liquid and cause so much priming that the column cannot operate. Ordinarily, the vapor velocity, based on the cross-section of the empty column, and measured in feet per second, should equal numerically the distance between plates in feet.[1] Velocities of about 10 ft. per sec. through the slots of the bubble caps are good practice.

Calculation Methods for Packed Columns.—On page 337 it was stated that a column filled with inert solid shapes, that offer a large amount of surface per unit tower volume, can be used instead of bubble-cap plates. A column of this type is known as a *packed column* and is sometimes used either because of its cheapness or because it can be easily constructed of acid-resistant materials. The height of a packed column can be calculated by using a factor known as the *height equivalent to one theoretical plate* or the H. E. T. P.[2] This method divides the packed column into units, each unit of such a height that the vapor and liquid leaving it are of equilibrium compositions. Each unit acts as a theoretical plate. If the H. E. T. P. is known for the mixture and

[1] Perry, p. 1196.

[2] Peters, *Ind. Eng. Chem.*, **14**, 476 (1922).

column construction under consideration, the total height of the column is calculated by multiplying the number of theoretical plates required for the separation in question by the H. E. T. P.

Optimum Reflux.—From Fig. 168 it will be seen that as the reflux ratio is increased, the distance between the operating lines and the equilibrium curve is increased, and hence the number of perfect plates necessary for the desired separation is decreased. Since the vapor ascending the column is $(R + 1)$ mols per mol of product, increasing the reflux ratio increases the diameter of the column for a given output. With a reflux just greater than the minimum, the diameter of the column is small but the number of plates is very large. As the reflux ratio is increased slowly, the diameter of the column increases slowly but the number of plates decreases rapidly, thereby decreasing the cost of the column. As the reflux ratio continues to increase, the diameter of the column increases proportionally, but the number of plates is not greatly decreased, and hence the cost of the column increases again. One factor in the cost of operation of the column is fixed charges, which are a definite per cent of the first cost. These are illustrated as curve 1 in Fig. 172.

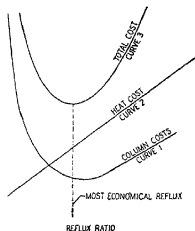



FIG. 172.—Determination of optimum reflux.

The reflux is obtained by condensing vapors that have left the still, and the heat so transferred to the cooling water is ordinarily lost. This heat must be supplied by steam coils in the bottom of the still. The cost of heat, therefore, increases practically in proportion to the reflux ratio plus one, as shown in curve 2. The total cost of operation is the sum of the fixed charges (curve 1) and the steam cost (curve 2). This total cost (curve 3) will show a decided minimum, and this minimum determines the most economical reflux ratio. In practice, column stills operate with reflux ratios varying from 1.5 to 4 or 5 times the minimum with a fair average at about twice the minimum.

SYSTEMS OF MORE THAN TWO COMPONENTS

It is beyond the scope of this book to discuss quantitatively the distillation of mixtures containing more than two components. In general, the method of handling such mixtures is to send the mixture to one column where one of the components is separated in a practically pure state. The rest of the material is then sent to another column, another component removed, and the residual mixture sent to a third column if necessary. It might be thought that one column could carry out the separation, removing one component from the top, another from the bottom, and a third from the middle. If the theory of rectification of a two-component mixture is understood, however, it will be seen that a mixture withdrawn from the middle of a column will contain large amounts of all three components. In general, for an N-component mixture, $N - 1$ columns will be required for the separation.

Systems of Incomplete Mutual Solubility.—An exception to this statement is represented by systems such as alcohol, fusel oil, and water. Alcohol and water are miscible in all proportions, and alcohol and fusel oil are miscible in all proportions. Fusel oil, however, is very slightly miscible with water. The alcohol has the greatest volatility, the water next, and the fusel oil is the least volatile (if the boiling points are taken as the criteria of volatility). When such a mixture is fed to a distilling column, the alcohol concentrates in the top of the column in the usual way, while the fusel oil and water, being less volatile than alcohol, remain in the bottom of the column. Since fusel oil is immiscible with water, however, a process of steam distillation (see page 367) takes place, which tends to drive the fusel oil up the column. This means that the fusel oil is trapped in the center of the column, and in such columns the fusel oil-water mixture containing alcohol must be withdrawn from this point. This product will separate into two immiscible layers—a water-alcohol layer and an alcohol-fusel oil layer. The first may be returned to the column, but to recover the alcohol from the second layer, it must be sent to a separate column.

Petroleum Distillation.—Crude petroleum contains a large number of substances, all mutually soluble, with boiling points extending over a wide range but with exceedingly small steps between the boiling points of individual components. From such a mixture it is difficult to separate, even approximately,

individual constituents. The various fractions recognized in the oil industry (such as naphtha, gasoline, kerosene, gas oil, wax distillate, etc.) are not pure substances but mixtures of a large number of substances. There are present, in each, small amounts of very low-boiling constituents and small amounts of very high-boiling constituents, with the majority of the fraction boiling in a certain temperature range. If a mixture such as crude oil be distilled through a fractionating column, the low-boiling constituents will be largely in the top of the column and the high-boiling ones at the bottom. Hence such fractions as are desired may be taken off at intermediate points in the column. In such a case there is no real separation into individual constituents, and hence this very complex multicomponent system may be separated into such fractions as are desired commercially by the use of a single column. Even in this case, however, the sharpness of the separations is improved by sending each *side stream* to a small individual column and giving it a further rectification.

Heat Recovery.—The use of heat exchangers is common practice in distilling systems. For example, the feed to the apparatus is often preheated by hot waste from the bottom of the column, or by using cold feed as the cooling medium in the condensers. Various arrangements of condensers are often used; for example, the vapor from the top of the column may be partially condensed in one condenser, thus providing the reflux, and the remainder condensed in a second condenser, which provides the product. Smaller heat exchangers are often used after the condensers to cool the product well below its boiling point. Many combinations of heat exchangers, condensers, and columns are possible; and many control devices may be used, such as steam-pressure regulators, regulating valves for the reflux, thermometers, safety pressure vents, and testers for sampling the materials leaving the apparatus.

Columns are usually lagged to prevent heat loss, and such lagging also improves the separation obtained for a given vaporization, since it is advantageous to remove in the condenser all of the heat that is to be removed from the column.

STEAM DISTILLATION

A simple type of process, known as *steam distillation*, is often used to distill a material that has a relatively high boiling point under atmospheric pressure or that is liable to decompose at this

temperature. This method is often used to separate high-boiling materials from non-volatile impurities. It can be applied only when the product is almost or completely immiscible with water.

Theory of Steam Distillation.—Assume that the material to be distilled is placed in a batch still and that steam is blown directly into the material. This process can be done in either of two ways. In the first method, the only heat input is that due to the latent heat of the steam, and a certain proportion of the steam must condense to provide heat of vaporization for the product that is taken overhead. As a result, there is formed in the still a layer of water. In the second case, either the steam is highly superheated or another external source of heat is supplied, so that the steam does not condense but passes out with the product. In either case, when the sum of the partial pressures of the steam and of the material distilled reaches the total pressure in the apparatus, both materials will pass over in the molecular ratio of their partial pressures. If these mixed vapors are condensed, two layers will be formed in the condensate receiver, the two materials can be separated by gravity and any non-volatile impurity will be left behind in the still. In either case the ratio by weight of component A to component B in the vapor is

$$\frac{W_a}{W_b} = \frac{p_a M_a}{p_b M_b} \tag{211}$$

where

W_a = weight of component A in the vapor.
W_b = weight of component B in the vapor.
p_a = partial pressure of A.
p_b = partial pressure of B.
M_a and M_b = molecular weights of the materials.

If two liquid layers collect in the still, it is possible to fix either the temperature or the total pressure, but not both. For example, if the temperature is fixed, each liquid will exert a partial pressure equal to its vapor pressure at the given temperature. The sum of these partial pressures will necessarily equal the total pressure, which is thus fixed.

Fig. 173.—Vapor-pressure curves for steam-distillation calculations.

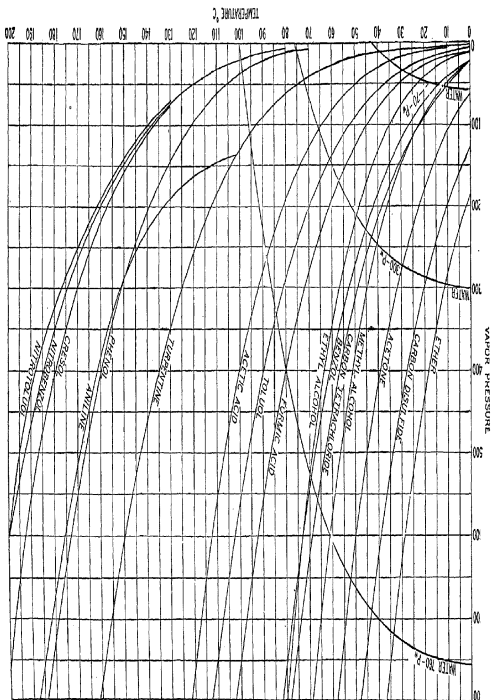



Figure 173[1] is useful in the calculation of steam distillations by the first method. The water curve is plotted as 760 minus the vapor pressure of the water, *vs.* temperature, and hence cuts across the vapor-pressure curves of the materials that are to be distilled. The other curves are ordinary vapor-pressure curves. The intersection of the water curve with the curve of the material in question represents the temperature at which the steam distillation can take place. At this temperature the vapor pressure of the material is y and the vapor pressure of water is $(760 - y)$. The molal ratio of water to product in the material passing over is obviously $\frac{760 - y}{y}$. This diagram, therefore, not only makes possible a quick determination of the temperature at which steam distillation takes place, but it also presents graphically the richness of the vapor. It must be remembered, however, that the proportions represented by this chart are by mol fractions, or volume fraction in the vapor phase. Since most of the substances on the chart have a molecular weight considerably greater than that of water, the composition of the distillate by weight is much richer than would appear from the diagram.

There are also shown in Fig. 173 the curves for water vapor for distillation at total pressures of 300 and 70 mm. By comparing the intersection of the 760-mm. curve for water with, say, benzol, and the 300-mm. curve for water with the same substance, it appears that lowering the total pressure from 760 to 300 mm. has lowered the temperature of distillation from 69 to 46°C. At the same time it has changed the ratio of water to benzol by volume from 230:530, or 1:2.3, to 75:225, or 1:3.3. Decreasing the total pressure at which steam distillation is carried out does not necessarily improve the yield per pound of steam, because the yield depends on the relative slope of the vapor-pressure curves for water and the substance desired, in the range that is being considered, provided a layer of liquid water is present.

In the case where but one liquid layer is present, both temperature and pressure can be independently fixed just so long as the total pressure is less than the sum of the vapor pressures of the two materials at the temperature of the still. In this case the partial pressure of the steam will be less than the vapor

[1] Hausbrand, "Verdampfen, Kondensieren, und Kühlen," 6th ed., p. 44, 1918.

pressure of water at the temperature of the distillation and therefore the steam will be superheated and its pressure and temperature can be varied independently.

Equation (211) becomes, in this case

$$\frac{W_a}{W_b} = \frac{(P - p_b)M_a}{p_b M_b} \tag{212}$$

where P is the total pressure, and p_b is the vapor pressure of the material at the temperature chosen.

Inspection of Equation (212) shows that if the total pressure is progressively decreased and the temperature kept constant, the steam consumption becomes less and less, and vanishes when the total pressure equals the vapor pressure of material B. In this limiting case the distillation becomes a simple vacuum distillation. The tendency in large installations, such as those for distilling lubricating oils, is toward vacuum distillation rather than its equivalent, steam distillation with superheated steam.

Steam-distillation Practice.—Steam distillation is used on such substances as nitrobenzene and anilin that boil at temperatures so high that it would require high-pressure steam to vaporize them. It is used on substances like fatty acids, that would decompose at the boiling point; although in this case the boiling points are so high that it is necessary to carry out steam distillation at the highest possible vacuum. It is also used for substances like turpentine, where the only other means of distillation available—namely, heating by direct fire—would be dangerous or would impair the quality of the product.

In operation by the first method (where the steam is the sole source of heat), if the batch is started cold, the steam must condense, giving up its latent heat until the batch has been heated to the distillation temperature. After distillation begins, a very small amount of heat is obtained by the cooling of the steam from the pressure at which it is supplied to the pressure at which distillation takes place, but this is often negligible. If there is any radiation loss from the apparatus, not only must this heat be made up by the condensation of further amounts of steam, but the substance being distilled will also condense and be returned to the still, thus greatly decreasing the yield per pound of steam. It is, therefore, necessary that such apparatus be thoroughly insulated and that its capacity be large

in comparison to the initial batch. It is not impossible to have the amount of steam condensed in the still several times as great as the volume of material distilled from it.

Example 31.—One hundred gallons per hour of turpentine are to be steam-distilled at atmospheric pressure. The latent heat of vaporization of turpentine is 74 cal. per g., its molecular weight is 136, and its specific gravity is 0.865. No external heat is supplied. How much steam at 5 lb. gage will be used per hour? What would be the effect of operating at 300 mm. absolute pressure instead of atmospheric? Neglect heat required to heat up the charge and heat lost by radiation.

Solution.—From Fig. 173 it is seen that the curves for turpentine and for water at atmospheric pressure cross at 95.5°C. At this temperature the vapor pressure of turpentine is 114 mm., and that of water is 646 mm. From Equation (211) the ratio by weight of turpentine to water in the product is

$$\frac{W_a}{W_b} = \frac{114 \times 136}{646 \times 18} = 1.333$$

One hundred gallons of turpentine is 100 × 8.33 × 0.865 or 720.5 lb. Since the ratio of turpentine to water is 1.333 by weight, the steam passing over into the product will be 720.5/1.333 or 541 lb. The total heat needed is

$$720.5 \times 74 \times 1.8 = 96{,}000 \text{ B.t.u. per hr.}$$

Steam at 5 lb. gage has a temperature of 108.3°C. The steam that goes over with the product cools from 108.3 to 95.5°C. as superheated steam (specific heat = 0.48) and therefore furnishes

$$541 \times 0.48 \times (108.3 - 95.5) \times 1.8 = 6{,}000 \text{ B.t.u. per hr.}$$

The rest of the heat needed, or 90,000 B.t.u., must be supplied by steam that condenses and then cools to 95.5°C. This steam furnishes, per pound,

As latent heat	960.4 B.t.u.
As sensible heat (108.3 − 95.5) × 1.8	23.0
Total	983.4

The weight of steam condensed is

$$\frac{90{,}000}{983.4} = 91.5 \text{ lb.}$$

The total steam used per hour is 541 + 91 = 632 lb.

At 300 mm. absolute pressure the results are

Temperature of distillation	72°C.
Vapor pressure of water	255 mm.

Vapor pressure of turpentine...........	45 mm.
Ratio of turpentine to water by weight.	1.334
Weight of steam in product...........	541 lb.
Total heat needed[1]...................	96,000 B.t.u. per hr.
Heat from steam in product...........	17,070 B.t.u. per hr.
Weight of steam condensed............	77 lb.
Total steam used......................	618 lb.

Nomenclature of Chapter IX

A, B = components of mixture to be separated by distillation
a = latent heat in vapor V_{n+1}
b = sensible heat in vapor V_{n+1} above datum temperature
c = sensible heat in liquid L_{n-1} below datum temperature
D = product
d = heat of mixing
E = plate efficiency
e = latent heat in vapor V_n
F = mols of feed per unit time
f = radiation loss
K = Henry's law constant
L = weight of reflux, in mols per hour
M = molecular weight
m or n = theoretical plate
P = total pressure
p = partial pressure
q = total heat needed to convert 1 mol of feed into saturated vapor, divided by the molal latent heat
R = reflux ratio
V = number of mols vaporized per hour
W = number of mols, or residue, or weight
x = mol fraction of A in solution, or composition
y = mol fraction of A in vapor, or composition
sub a = of component A
sub b = of component B
sub D = distillate from condenser
sub n = on plate n
sub o = in the original batch
sub w = residue

A bar over a symbol signifies its value in the part of the column below the feed plate.

Prime refers to minimum reflux.

[1] No data are available for the change in latent heat of turpentine with temperature. The order of magnitude of this change, however, is such that it must be very small as compared with the degree of accuracy needed.

Problems

1. A mixture containing 70 mol-per cent benzene and 30-mol per cent toluene is vaporized at 760 mm. total pressure until one-third of the mols in the original mixture are in the vapor phase. The vapor is kept in intimate contact with the liquid until the process is complete, and then the two phases are separated and the vapor condensed.

a. What is the composition of the condensate?
b. Of the residue?

2. A mixture containing 70 mol-per cent benzene and 30 mol-per cent toluene is distilled under differential conditions and 760 mm. total pressure. One-third of the mols in the charge are vaporized. Calculate the composition of distillate and residue.

3. A continuous still, consisting of a still body surmounted with a bubble-cap column, is to be fed with a mixture containing 50 per cent by weight of benzene and 50 per cent by weight of toluene. It is desired to obtain an overhead product containing 95 per cent by weight of benzene and a residue from the still containing 95 per cent of toluene by weight. The plate efficiency is 65 per cent. The feed is all liquid, and heated to the boiling point before entering the column.

a. What is the minimum reflux for this case?
b. What is the minimum number of actual plates that can be used?
c. If the reflux ratio used is 50 per cent greater than the minimum, how many actual plates should be used?
d. If the feed rate is 20,000 lb. per hr., what is the hourly production of product and residue?
e. If the latent heat of benzene is 92.9 cal. per g., and if the still is heated with steam at 25 lb. per sq. in. gage, how much steam is required per hour? Neglect radiation.

4. The same still as in Problem 3 is to be fed with the same mixture, but in the form of saturated vapor. Product and waste are to be as in Problem **3.** Twice the theoretical minimum reflux is to be used.

If steam costs $0.30 per 1,000 lb., what difference in operating cost is caused by the change?

5. At 240°C. the vapor pressure of stearic acid is 19 mm. If superheated steam is blown into a molten mass of stearic acid at this temperature and atmospheric pressure, and if the still and vapor main are so well lagged that no condensation takes place until the vapors reach the condenser, what is the minimum steam consumption per pound of stearic acid distilled?

Barometer = 760 mm.
Molecular weight stearic acid = 284.4

6. The equilibrium boiling-point data for acetic acid and water are as follows:

Temperature, degrees Centigrade	Per cent by weight of H_2O in liquid	Per cent by weight of H_2O in vapor
100.00	100	100
......	95	96.4
100.25	90	92.6
100.45	80	85.2
100.75	70	77.8
101.00	60	70.5
101.25	55	66.7
101.50	50	63.0
101.85	45	58.5
102.25	40	53.8
102.75	35	48.5
103.40	30	42.9
104.00	25	36.9
105.00	20	30.4
106.25	15	23.6
108.50	10	16.3
112.00	5	8.45
116.50	1	2.17
118.00	0	0

The average latent heat of water in this range of temperature is 530 cal. per g.; that of acetic acid is 76 cal. per g.

A continuous still is to be used to remove water from an 80 per cent acetic acid solution. The apparatus consists of a still surmounted by a column containing bubble-cap plates. Mixing in the still is complete enough so that the vapor rising to the column is in equilibrium with the waste.

The product drawn from the still is to contain at least 90 per cent acid, while the condensate is to contain not more than 4 per cent acid. The plate efficiency is 65 per cent.

a. What is the minimum reflux for this case?

b. What is the minimum number of theoretical plates required?

c. If a reflux ratio of 1.3 times the minimum is used, what will be the actual number of plates required?

CHAPTER X

GAS ABSORPTION

This chapter will discuss those processes in which one constituent is removed from a gas by treatment with a liquid. Similar operations are sometimes performed by treating the gas with a finely divided solid, but the theory of such cases is rather obscure and consequently is not discussed in this book. It may happen in practice that more than one constituent of the gas will be removed by the liquid that is used, but the discussions which follow will assume that but one substance is dissolved.

In any case in which a gas is to be treated with a liquid, it is found that the rate at which the soluble constituent passes into solution is ordinarily rather slow. Consequently the largest possible surface must be provided for the interaction between the two phases. This might conceivably be done by causing the gas to rise through a layer of liquid in the form of very fine bubbles. In practice, however, it is almost always found more convenient to subdivide the liquid or to give it an extended surface and to pass the gas over this surface. In some cases where the rate of absorption is high, it is not necessary to adopt any special means for bringing the two phases into extensive contact, but this is relatively uncommon.

GAS-ABSORPTION EQUIPMENT

The apparatus in commercial use for gas absorption has never become standardized, so that there are no specific types, or characteristic designs of individual makers, to be described. Gas absorption is almost always carried out either in spray chambers or, more commonly, in towers packed with some solid material over which the liquid is distributed and up through which the gas passes.

Properties of Tower Packing.—The properties that a satisfactory tower packing should have are as follows:

1. *Low Weight per Unit Volume.*—This affects not only the total weight to be carried by the tower but also the design of the

tower shell itself. A packing that is dumped into the tower at random may exert a side thrust against the walls, and if the packing has a high unit weight this may affect the cost of tower construction.

2. *Large Surface per Unit Volume.*—This is so obvious as to need no comment.

3. *Large Free Cross-section.*—This is of importance because it affects the frictional drop through the tower and therefore the power that is required to circulate the gas. Also, a small free cross-section means a high velocity for a given through-put of gas, and above certain limiting velocities there is a tendency to blow the liquid out of the tower.

4. *Large Free Volume.*—In some cases, such as the absorption of oxides of nitrogen where time must be allowed for reactions in the gas phase, this factor may be of importance. In other cases it is of no special significance.

5. *Small Weight of Liquid Retained.*—This is generally an advantage, since it decreases the load on the tower and removes the liquid from the tower as rapidly as possible. In some cases it may be a disadvantage, especially where the reaction between gas and liquid is slow, or where the solubility of the gas in the liquid is not great. In acid towers it is usually desirable to have a small amount of liquid retained to lower the hazard when the tower is being emptied.

In addition to the above are the obvious requirements that the packing must be cheap, must have a reasonable mechanical strength, and must be chemically inert toward the materials to be handled.

Types of Tower Packing.—A wide variety of types of packing have been suggested at one time or another. These may be classified as follows:

1. *Broken Rock.*—This at once suggests itself, since such material is always at hand. It is not always easy to find material that will be inert. This packing has various disadvantages, chief of which are its great weight, its relatively small surface per unit of area, and its small free cross-section. It is now employed in only two important cases: first, the use of crushed quartz for the packing of Glover towers in sulphuric acid manufacture; and, second, in one of the systems for making the liquor for use in sulfite pulp manufacture where broken limestone is used. In

this latter case it is desired to have the solution produced in the tower react with limestone, and the two operations are thus combined in one.

2. *Coke.*—Coke has the advantage of being light in weight and having a large surface per unit weight. Its disadvantages are a small free cross-section and a tendency for some slightly soluble constituents of the coke to pass into solution. It is also rather friable. The surface is not so large as might be expected, since many of the pores are so small that they are completely filled or filmed over with liquid and therefore are not effective in furnishing surface at which reaction with the gas phase could take place. Coke is usually cheap and generally available, and in many small and simple operations its use is justified.

3. *Stoneware Shapes.*—So many of the operations of gas absorption are carried out with acid liquids as the solvent, that chemical stoneware is a common material. This has been employed in the most diverse and elaborate forms.

Towers may be packed with ordinary rectangular brick set on edge, but this packing has a large weight and small surface per unit volume, though it may be arranged to give large free volume and large free cross-section. Every conceivable kind of specially shaped brick has been suggested and made at one time or another, but it is not necessary to discuss these forms here. A new type of tower packing that has become quite popular is the *Berl saddle* which is a saddle-shaped porcelain unit that is piled at random. The advantage of this type of packing is the comparatively low frictional resistance that it offers to the flow of the gas while maintaining adequate gas-liquid surface.

4. *Raschig Rings.*—This is the most widely used form of tower packing. It consists of cylindrical rings, of the same length as the diameter of the cylinder, and with the walls as thin as the material will permit. Stoneware Raschig rings will vary from 2 to 6 in. in diameter and will have a wall from ⅜ to ⅝ in. thick. Where the rings can be made of metal they are correspondingly lighter and give a larger free cross-section and a larger free volume. Raschig rings are always dumped into the tower at random and not stacked regularly. They offer the best combination of low weight per unit volume, free volume, free cross-section, and total surface of any type of packing. Stoneware Raschig rings are sometimes made with one or two

interior webs which increase the surface without greatly decreasing free cross-section.

5. *Spiral Rings.*—Machines have been devised for making a stoneware packing having the general dimensions of a Raschig ring, but with an internal helix which may partly or completely fill the cross-section of the cylinder. Such rings are always stacked and never dumped at random. It is claimed that the helix gives more thorough contact between gas and liquid. It increases the surface without greatly decreasing either free cross-section or free volume. It greatly increases the cost of the packing and especially the labor for installing.

Friction Drop through Tower Packing.—The frictional resistance of a column of packing is of practical importance, because if the frictional resistance is high, considerable power will be consumed in forcing the gas through the system.

The calculation of friction through beds of solids and columns of tower packing may conceivably be based on a modification of Equation (26). Such attempts have been made,[1] but they require a considerable modification of the equation and involve the use of factors which are, in general, not known. The only reasonable approach to this problem at present is a purely empirical one.

The best work at present is that of Mach.[2] He gives the following equations for the calculation of the frictional resistance of columns of *dry* packing:
For Raschig rings:

$$\Delta P = 0.0846\frac{hu^{1.85}\rho^{0.83}}{D^{1.27}} \tag{213}$$

and for Berl saddles:

$$\Delta P = 0.0492\frac{hu^{1.82}\rho^{0.83}}{D^{1.40}} \tag{214}$$

where

ΔP = pressure drop in pounds per square foot.
h = height of packing in feet.

[1] CHILTON and COLBURN, *Ind. Eng. Chem.*, **23**, 913–919 (1931); *Trans Am. Inst. Chem. Eng.*, **26**, 178–195 (1931); A. M. WHITE, *Trans. Am. Inst. Chem. Eng.*, **31**, 390–408 (1935).

[2] *Forschung*, **6**, Forschungsheft 375 (1935).

u = gas velocity, feet per second, calculated on empty tower.
ρ = gas density in pounds per cubic foot.
D = nominal diameter of one packing unit in feet.

For calculating the friction drop when the packing is wet with water, the following equations hold:
For Raschig rings:

$$\Delta P = 0.0000894\frac{\rho^{0.83}hu^{1.8}}{D^{1.27}}\left[1{,}405 + \frac{\rho^{0.42}B}{D^{0.64}}\right] \qquad (215)$$

For Berl saddles:

$$\Delta P = 0.0000422\frac{\rho^{0.83}hu^{1.8}}{D^{1.40}}\left[1{,}779 + \frac{\rho^{0.42}B}{D^{0.70}}\right] \qquad (216)$$

where B is rate of water flow over the packing in gallons per hour per square foot of tower cross-section.

The above equations are valid only over a particular range of pressure drops. For any particular packing and rate of water flow, if the pressure drop is plotted *vs.* the gas velocity in logarithmic coordinates, there are two plainly marked breaks in the curve. The lower break has been called the *loading point* and represents that velocity at which drops of liquid are carried up out of the tower by the gas. The second break has been called the *flooding point* and represents the gas velocity at which the liquid cannot penetrate the tower but is held as a layer on top of the packing. In practice, absorption towers should never, under any circumstances, be run above the flooding point; and preferably, never above the loading point. It has been found from these experiments that these two points are independent of shape and size of packing and rate of water flow. The loading point comes where the pressure drop is from 10 to 15 lb. per sq. ft. per ft. of packing, and the flooding point comes when the pressure drop is from 40 to 50 lb. per sq. ft. per ft. of packing. Equations (215) and (216) are good only below the loading point. If a calculation using Equations (215) and (216) gives a pressure drop greater than these limits, the tower is probably wrongly designed and should not be operated at such high velocities. It has not yet been possible to develop equations for the friction drop in towers operating under such conditions that the pressure drop is greater than that at the loading point.

If the rate of water flow over the packing is as low as 25 gal. per sq. ft. per hr., the packing will probably be incompletely wetted and the pressure drops will be much lower than the calculated friction.

Example 32.—What pressure will be necessary to force 250 cu. ft. air per min. at 60°F. and 30 in. Hg. through an absorption system consisting of five towers in series, each 2 ft. in diameter and packed to a depth of 10 ft. with 2-in. Raschig rings dumped at random? Water is circulating over the packing at the rate of 5 lb. per sq. ft. tower cross-section per min. Neglect friction in connecting piping.

Solution.—The data for substituting in Equation (215) are

$$h = 5 \times 10 = 50 \text{ ft.}$$

$$u = \frac{250}{(\pi \times 60)} = 1.326 \text{ ft. per sec.}$$

$$B = 5 \times \frac{60}{8.33} = 36 \text{ gal. per sq. ft. per hr.}$$

$$\rho = {}^{29}\!/\!_{378} = 0.0767 \text{ lb. per cu. ft.}$$

$$D = {}^{2}\!/\!_{12} = 0.1667 \text{ ft.}$$

Substituting in Equation (215) gives

$$\Delta P = 12.4 \text{ lb. per sq. ft.}$$

Tower Construction.—Many gas-absorption processes either employ or produce an acid solution and therefore the columns for such purposes must be made of acid-resistant material. In the past acid-proof stoneware has usually been used. This is supplied in the form of rings, in diameters up to 48 in., and with bell-and-spigot joints. Special top and bottom sections are provided with inlet and outlet connections for both gas and liquid. In recent years Duriron has been available for the same purpose, and very recently it has become possible to construct such apparatus entirely of stainless steel. Steel or lead towers may be lined with acid-proof brick set in a cement of either powdered silica in water glass, or of litharge and glycerin. Many cases permit the use of ordinary sheet-steel construction, which calls for no special comments. In some cases wood towers are used, and these are merely long narrow tanks made up with tongued-and-grooved staves held together with tie-rods. The introduction of stainless steel promises to simplify tower construction to a remarkable extent.

In a tower of more than a few inches in diameter some device is necessary to distribute the liquid so that it shall completely wet the packing. This may take the form of perforated plates,

spray nozzles, or a variety of other constructions limited only by the ingenuity of the designer. A large number of small streams of liquid is in all cases preferable to a few large streams. In the case of very tall towers with heavy packing, it may be necessary to introduce horizontal perforated partitions at intervals to transfer the weight of the packing to the walls of the tower. If the total height of tower needed to complete the reaction is too great to be convenient in a single tower, several short towers may be used in series. Gas and liquid should flow through these towers in countercurrent. It also happens sometimes that the amount of liquid necessary to dissolve the soluble gas is too small to wet the total surface of packing necessary. In this case a

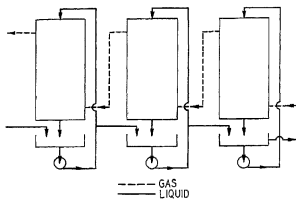



FIG. 174.—Absorption towers in series with liquor recirculation.

large amount of liquid may be circulated over each tower, and only a fraction advanced to the next tower in the series. Figure 174 shows such an arrangement.

Tourills.—In the absorption of hydrochloric acid the reaction is so rapid, and so much heat is evolved, that it becomes necessary to decrease the rate of reaction by decreasing the area of contact, to prevent an undue rise in temperature. This is done by passing the gas in a small stream over practically stationary water surfaces. The vessel in which this is usually carried out is called a tourill. Two forms are shown in Fig. 175. The form in Fig. 175*a* is sometimes called a *Woulffe bottle.* The absorbing liquid flows from vessel to vessel by pipes *A*, connected by rubber sleeves *B*. The gas flows in the opposite direction by pipes *C*. The form in Fig. 175*b* was designed to give a larger surface for radiating heat and is known as the *Cellarius tourill.* Liquid enters at *A* at the front of the tourill, and the web *B* causes it to flow

to the back, then forward, to leave by pipe *C*. The level of the liquid is such that web *B* projects above the surface. Gas enters at *D* and leaves at *E*. Water connections between tourills are made by glass return bends and rubber sleeves, and gas connections are standard stoneware pipe.

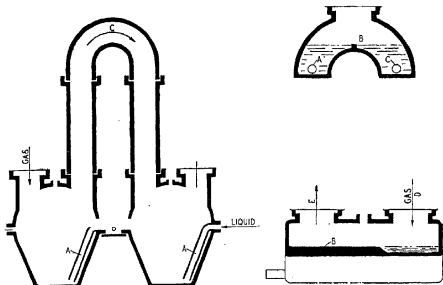



FIG. 175*a*.—Simple tourills or *Woulffe* bottles. *A*. Liquid inlet. *B*. Liquid connections. *C*. Gas connections.

FIG. 175*b*.—*Cellarius* tourill. *A*. Liquid inlet. *B*. Web. *C*. Liquid outlet. *D*. Gas inlet. *E*. Gas outlet.

THEORY OF GAS ABSORPTION

The general mechanism of the absorption of a constituent from a gas phase by a liquid must be analyzed from the twofold point of view of equilibrium and reaction rate.

Equilibrium (Solubility).—A solution of a gas in a liquid exerts, at a definite temperature and concentration, a definite partial pressure of the dissolved gas. The solvent liquid also exerts its own partial pressure, but this has no bearing on the mechanism of gas absorption.

Different gases will exhibit very great differences in the partial pressures that their solutions exert at equilibrium. For example, if the gas forms a stable chemical compound with the absorbing liquid, its partial pressure will be zero over wide ranges of concentration, provided only that some free absorbing constituent remains. On the other hand, when (for instance)

oxygen dissolves in water, large partial pressures of oxygen are exerted by very dilute solutions. This is also expressed by saying that oxygen is slightly soluble in water. Other cases are intermediate, and, in general, the lower the partial pressure of the dissolved gas from a solution of given concentration the more soluble it is said to be. Figures 176, 177, and 178 show the vapor pressures exerted at different concentrations by a very soluble gas (ammonia),[1] a moderately soluble gas (sulfur dioxide),[1] and a slightly soluble gas (oxygen).

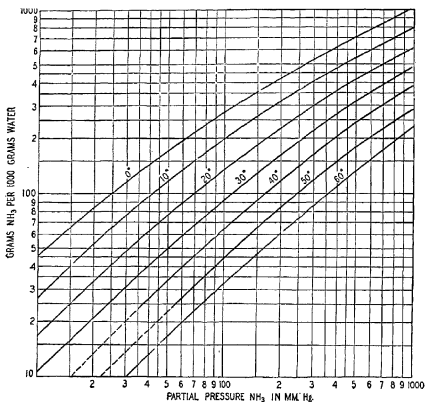



FIG. 176.—Solubility of ammonia in water.

It should be noted that a given quantity of liquid will dissolve any amount of any gas, whether highly soluble or slightly soluble, according to the partial pressure of the dissolved gas in the phase in contact with the liquid. The term "slightly soluble" merely means that it requires a higher partial pressure of the gas in contact with a liquid to yield a solution of given

[1] SHERWOOD, *Ind. Eng. Chem.*, **17**, 745–747 (1925).

concentration; while a "very soluble" gas will give the same concentration at a lower partial pressure. For example, a 32 per cent solution of hydrochloric acid at 30°C. will exert a partial pressure of hydrogen chloride of 44.5 mm. Hg. If there is a gas in contact with this solution that contains enough hydrogen chloride so that its partial pressure is more than 44.5 mm., hydrogen chloride will dissolve in the liquid until the partial pressure of hydrogen chloride in the gas becomes equal to 44.5 mm.

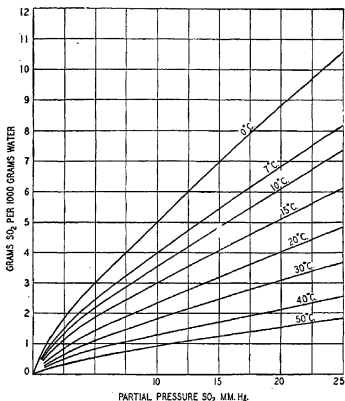



FIG. 177*a*.—Solubility of sulfur dioxide in water.

Hg (provided the amount of gas is small and the amount of the solution large). In general, if a gas-absorption process is allowed to reach equilibrium, this will be at the point where the partial pressure of the solute in the gas phase is equal to its partial pressure exerted by the liquid phase. The strongest solution that can possibly be produced in any gas-absorption process, even if the process attains complete equilibrium, is that in which the partial pressure of the solute is equal to the initial partial pressure of the solute in the gas phase.

The solubility of a gas is changed by the total pressure if the latter is large. This effect is small at pressures below about 5 atmospheres and is neglected in all but high pressure work.

An important special case of gas solubility is where the solubility curve is a straight line. In such a case the gas follows

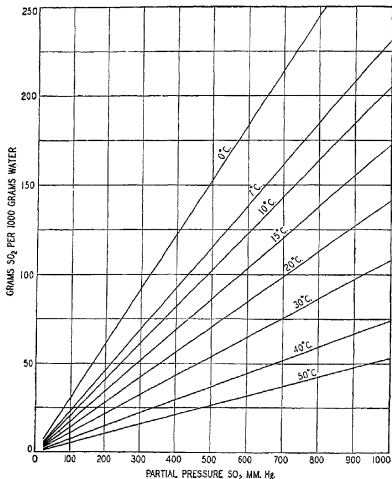



Fig. 177*b*.—Extension of curves of Fig. 177*a*.

Henry's law (see page 329) and the solubility curve is represented by the equation

$$p = Hc \tag{217}$$

where

p = the partial pressure of the dissolved gas.
c = the concentration of the dissolved gas.
H = a constant.

The quantity H is the Henry's law constant.

RATE OF ABSORPTION

From the discussion of diffusional processes given in Chap. VI it will be apparent that, in general, a material that is being dissolved from an inert gas by a liquid must diffuse in series through two films, first a gas film and second a liquid film. In Chap. VI it was shown that the rate of diffusion of component a

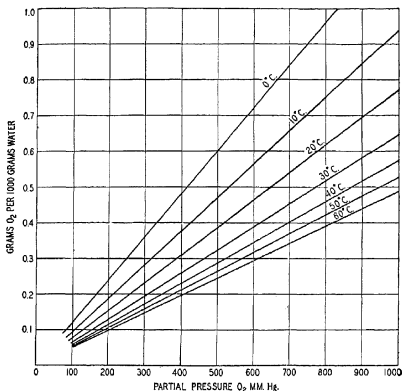



FIG. 178.—Solubility of oxygen in water.

through a stationary gas film made up mainly of an inert component b is given by the equation

$$\frac{w}{A} = \frac{b_w(p_{ag} - p_{ai})}{(p_b)_m B_g} \tag{112}$$

where

w = pounds of component a per hr.
A = area measured perpendicular to direction of diffusion of component a in square feet.

B_g = thickness of film in feet.
$(p_b)_m$ = average partial pressure of component b in the film in atmospheres.
p_{ag} = partial pressure of component a at one boundary of the film in atmospheres.
p_{ai} = partial pressure of component a at the other film boundary.
b_w = coefficient of diffusion.

The corresponding equation for the diffusion of a single component through a liquid film is

$$\frac{w}{A} = \frac{b_{Lw}(c_{ai} - c_{aL})}{B_L} \tag{117}$$

where

c_{ai} = concentration of diffusing component at one boundary of the film in pounds per cubic foot.
c_{aL} = concentration of diffusing component at other boundary of the film in pounds per cubic foot.
b_{Lw} = diffusion coefficient.
B_L = thickness of the film in feet.

The Two-film Theory of Gas Absorption.[1]—Equations (112) and (117) can be used in the case of gas absorption by considering that they apply to a gas and a liquid film in contact. Then p_{ag} is the partial pressure of the diffusing component in the main gas stream, p_{ai} is the partial pressure of a at the interface, c_{ai} is the concentration of a in the liquid at the interface, and c_{aL} is the concentration of a in the main liquid stream. In accordance with the assumption of interfacial equilibrium, p_{ai} and c_{ai} are coordinates on the solubility curve of component a in the liquid at the temperature in question. Furthermore, since the amount of a flowing through the gas film is equal to that flowing through the liquid film, the quantity w/A is the same in both cases, and

$$\frac{w}{A} = \frac{b_w(p_{ag} - p_{ai})}{(p_b)_m B_g} = \frac{b_{Lw}(c_{ai} - c_{aL})}{B_L} \tag{218}$$

Since it is impractical to measure the film thicknesses B_g and B_L it is convenient to define the two film coefficients k_L and k_g by the equations

[1] WHITMAN, *Chem. Met. Eng.*, **29**, 147 (1923).

$$k_g = \frac{b_w}{(p_b)_m B_g} \tag{219}$$

$$k_L = \frac{b_{Lw}}{B_L} \tag{220}$$

Equation (218) then becomes

$$\frac{w}{A} = k_g(p_{ag} - p_{ai}) = k_L(c_{ai} - c_{aL}) \tag{221}$$

It is apparent from Equation (221) that the rate of gas absorption for any given values of p_{ag} and c_{aL} depends on the solubility relationship of the gas and liquid under consideration and the magnitudes of the two coefficients k_g and k_L. If the solubility data are at hand and if k_g and k_L are known, the only unknowns are p_{ai} and c_{ai}. There are two relationships available for solving for these unknowns: Equation (221) and the solubility curve.

Example 33.—At 20°C. the solubility of ammonia in water can be represented by the equation

$$p = 0.0135c \tag{222}$$

where p is in atmospheres and c is in pounds per cubic foot. (This equation is followed over the range $p = 0$ to $p = 0.03$.)

A mixture of 10 per cent ammonia and 90 per cent air by volume at a total pressure of 760 mm. is in contact with an aqueous ammonia solution containing 2.5 g. NH_3 per l. The air velocity is such that

$$\frac{k_L}{k_L} = 1.0$$

where

$$k_g = \frac{\text{lb.}}{\text{(hr.)(sq. ft.)(atm. partial-pressure difference)}}$$

$$k_L = \frac{\text{lb.}}{\text{(hr.)(sq. ft.)(unit concentration difference, lb. per cu. ft.)}}$$

a. Calculate the concentration of ammonia, and the partial pressure of ammonia, at the interface.

b. Repeat, if the concentration of ammonia in the liquid is zero.

Solution.—*Part a.* From Equation (221) it follows that

$$\frac{k_g}{k_L} = \frac{c_{ai} - c_{aL}}{p_{ag} - p_{ai}} \tag{223}$$

Since k_g is based on a partial-pressure difference in atmospheres, p_{ag} and p_{ai} must be expressed in atmospheres. Also, since k_L is based on a concen-

tration difference measured in pounds per cubic foot, c_{ai} and c_{aL} must be measured in this same unit. Therefore

$$c_{aL} = \frac{2.5}{453.6} \times 28.32 = 0.156 \text{ lb. per cu. ft.}$$

$$p_{ag} = 0.100 \text{ atm.}$$

$$\frac{c_{ai} - 0.156}{0.100 - p_{ai}} = 1.00 \qquad (224)$$

For equilibrium conditions at the interface:

$$p_{ai} = 0.0135c_{ai} \qquad (222)$$

Solving Equations (222) and (224) simultaneously gives:

$$c_{ai} = 0.253 \text{ lb. per cu. ft.}$$
$$p_{ai} = 0.00341 \text{ atm.}$$

The partial-pressure difference over the gas-film is

$$0.100 - 0.00341 = 0.0966 \text{ atm. or } 73.4 \text{ mm.}$$

The concentration difference over the liquid film is 0.253 − 0.156 = 0.097 lb. per cu. ft.

b. If the concentration of ammonia in the liquid is zero, Equation (224) becomes

$$\frac{c_{ai}}{0.100 - p_{ai}} = 1.00 \qquad (224a)$$

Solution of Equations (224*a*) and (222) gives

$$c_{ai} = 0.0986 \text{ lb. per cu. ft.}$$

$$p_{ai} = 0.00133 \text{ atm.}$$

In this case the partial-pressure difference over the gas film is

$$0.100 - 0.00133 = 0.0987 \text{ atm. or } 74.0 \text{ mm.}$$

Effect of Gas Solubility on Driving Forces in Gas Absorption.—As compared with heat transfer, the driving forces in gas absorption are complex, in that one driving force (partial-pressure difference) is active over the gas-film resistance, while another driving force (concentration difference) is active over the liquid film resistance. The relationship between these two driving forces depends upon the solubility of the gas and upon the relative magnitude of the two film coefficients k_g and k_L. The action of these factors is best illustrated by the use of a Δc-Δp triangle.

In Fig. 179 the solubility curve of the gas is plotted in the usual way, with partial pressures as ordinates and concentrations as abscissas. This is the curve relating p_{ai} to c_{ai}. Plot on this same diagram point *a* which has coordinates c_{aL} and p_{ag}. If Equation

(223) is interpreted geometrically on Fig. 179, it is seen that the distance *ad* is p_{ag}-p_{ai}, and the distance *bd* is c_{ai}-c_{aL}. Further, Equation (223) states that a right triangle with sides equal to (p_{ag}-p_{ai}) and (c_{ai}-c_{aL}) has a hypotenuse of slope $-k_L/k_g$. Therefore, if a line be drawn through point *a* with a slope of

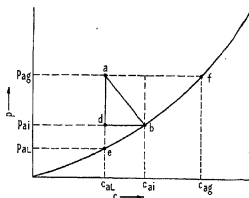



FIG. 179.—The Δc-Δp triangle.

$-k_L/k_g$, it will intersect the solubility curve at a point *b* whose coordinates are p_{ai} and c_{ai}. Such a diagram is a graphical

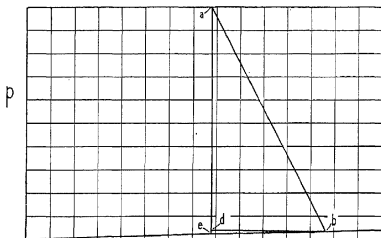



FIG. 180.—Δc-Δp triangle. Gas-film controlling.

solution of the problem of determining c_{ai} and p_{ai} from known values of p_{ag}, c_{aL}, and the solubility curve.

Soluble Gases—Gas-film Controlling.—In Fig. 180 is plotted the driving force triangle for Example 33. It will be seen from this figure that the high solubility of ammonia gives a solubility

curve that tends to hug the c-axis. The driving force Δp, which is represented by the line ad, is nearly equal to the distance ae which is the full vertical distance from point a to the solubility curve. It is therefore apparent that the slope of the line ab, which is equal to k_L/k_g, can be considerably changed without making an appreciable difference in the driving force Δp.[1] For all cases wherein a very soluble gas is absorbed, the Δp-Δc triangle will have this same characteristic. In such cases it is not necessary to construct the hypotenuse ab at all, and the driving force Δp can be taken as the vertical distance ae. In such a case the gas film is said to control; and the Δc driving force has little or no influence on the magnitude of the Δp driving force. Physically, this means that the solute gas, when it once gets through the gas film, is sucked in so avidly by the liquid that the liquid-film resistance is of negligible importance.

FIG. 181.—Δc-Δp triangle. Liquid-film controlling.

Gases of Low Solubility—Liquid-film Controlling.—Although the absorption of gases of low solubility is, by its very nature, of limited industrial importance, nevertheless treatment of this case yields a diagram of the type shown in Fig. 181. Here the slope of the line ab is such that the true Δc driving force measured by the distance bd is very nearly equal to the horizontal distance af from point a to the solubility curve. In this case the distance af can be used for Δc, and again it is unnecessary to construct the hypotenuse ab. In this case the liquid film controls the rate of absorption, and the driving force Δp is of little importance. Physically this case is the result of the fact that the tendency for the solute gas to move into the liquid is so low, because of its low solubility, that it is this process that is of controlling effect.

[1] In making and interpreting such diagrams as Figs. 179 and 180, it is obvious that the slope of the line ab may be measured directly from the drawing *only* when the units on the p- and c-axes are the same. In the case of Fig. 180, the difference in the scales on the p- and the c-axes must be taken into account.

Gases of Moderate Solubility—General Case.—In the general case of intermediate solubility, both driving forces are of comparable importance and the general diagram, such as given in Fig. 179, should be used.

Overall Coefficients.—Just as it is convenient in heat-transfer problems to use overall coefficients that are built up of film coefficients, so in gas absorption, overall coefficients can be used that are functions of the individual film coefficients. There is a complication, however, that is not present in the use of overall heat-transfer coefficients. In heat flow, the driving force is difference in temperature, regardless of whether the heat is flowing through solid, liquid, or gas. In diffusion flow, however, the driving force through the liquid film is a concentration difference, and that through the gas film is a partial-pressure difference.

Due to the fact that these two driving forces are used, two overall coefficients can be calculated. Define these coefficients by the equations

$$K_g = \frac{w}{A(p_{ag} - p_{aL})} \tag{225}$$

$$K_L = \frac{w}{A(c_{ag} - c_{aL})} \tag{226}$$

where

p_{aL} = the equilibrium partial pressure corresponding to the concentration c_{aL}.

c_{ag} = the equilibrium concentration corresponding to the partial pressure p_{ag}.

The other terms in Equations (225) and (226) are as previously defined.

Reference to Fig. 179 will show that the overall coefficient K_g is based on the driving force represented by the distance *ae*, where the coordinates of point *e* are c_{aL} and p_{aL}. It is also apparent that the overall coefficient K_L is based on the driving force represented by the distance *af* where the coordinates of point *f* are c_{ag} and p_{ag}.

The overall coefficients can be expressed in terms of the individual film coefficients k_g and k_L. For this purpose it is necessary to express c_{ai} in terms of p_{ai}. The simplest case where

this is possible is when Henry's law represented by Equation (217) applies. For this purpose Equation (217) may be written

$$p_{ai} = Hc_{ai} \tag{227}$$

where H is the Henry's-law constant and depends on the temperature. It has been shown in Chap. IX that Henry's law is not of general application and must be experimentally checked for each system before it can be used. Nevertheless, it applies to many cases well enough to be of considerable importance.

Since c_{ag} is the concentration corresponding to the partial pressure p_{ag}

$$p_{ag} = Hc_{ag} \tag{228}$$

also, since p_{aL} is the partial pressure corresponding to the concentration c_{aL},

$$p_{aL} = Hc_{aL} \tag{229}$$

Eliminating p_{ai}, c_{ai}, and c_{aL} from Equations (221), (227), and (228), it follows that

$$\frac{w}{A} = \frac{1}{\frac{H}{k_L} + \frac{1}{k_g}}(p_{ag} - p_{aL}) \tag{230}$$

while if p_{ai}, c_{ai}, and p_{ag} are eliminated from Equations (221), (227), and (229),

$$\frac{w}{A} = \frac{1}{\frac{1}{k_L} + \frac{1}{Hk_g}}(c_{ag} - c_{aL}) \tag{231}$$

If Equations (230) and (231) are compared with Equations (225) and (226), it will be seen that

$$K_g = \frac{1}{\frac{H}{k_L} + \frac{1}{k_g}} \tag{232}$$

$$K_L = \frac{1}{\frac{1}{k_L} + \frac{1}{Hk}} \tag{233}$$

also,

$$K_L = HK_g \tag{234}$$

In those cases where the solubility of the gas is great and the gas film controls, K_g is practically equal to k_g; and similarly, for cases of low solubility where the liquid film controls, K_L is nearly equal to k_L.

In the case of packed towers (as in the case of spray chambers) it is impractical to estimate the actual area of contact and the coefficients can be calculated in terms of the active volume of the absorption apparatus (page 270). Thus the coefficient K_ga is the gas-film overall coefficient per unit volume of tower, while K_La is the corresponding overall liquid-film coefficient per unit volume. In using these coefficients, instead of the term A in Equation (221) there is used V, where V is the active absorption volume.

Absorption Coefficients

It is to be expected that the magnitude of the gas-film coefficient will be influenced by almost the same variables that affect the rate of heat flow through a film resistance. This has been shown to be true.

For wetted-wall towers; that is, where the absorbing liquid flows as a continuous film down the inside wall of a tube and the gas is in turbulent flow, the following equation has been shown to hold:[1]

$$\frac{D}{B_g} = 0.023\left(\frac{Du\rho}{\mu}\right)^{0.83}\left(\frac{\mu}{\rho b_{gv}}\right)^{0.44} \tag{235}$$

where

D = inside diameter of the tower.
u = linear velocity of the gas calculated on the empty tower.
ρ = density of the gas.
μ = viscosity of the gas.
b_{gv} = diffusion coefficient in volumetric units (see page 244).

Equation (235) is in consistent units.

Although Equation (235) holds only for wetted-wall towers, which are of little practical importance, the behavior of the

[1] Sherwood, *Ind. Eng. Chem.*, **26**, 516–523 (1934).

effective film thickness with the variables as shown in Equation (235) can be used to estimate the effect of changes in operating variables in a packed tower.

Numerical Values of Gas-absorption Coefficients.—A quantity of data, several empirical equations and a few correlations of experimental data of various types have been made for absorption coefficients.[1] As examples of such equations the following may be mentioned.

Absorption of ammonia in quartz packed towers.

$$K_ga = 45{,}600[1 - 0.79(10^{-0.02L'})]\left[\frac{G'}{31G' + 294} + 0.00033\right] \quad (236)$$

where

K_ga = overall gas-film absorption coefficient in pounds per hour per cubic foot per atmosphere driving force.

L' = rate of flow of liquid down the tower in pounds of water per square foot of tower cross-section per minute.

G' = Pounds of inert gas per square foot of tower cross-section per minute.

This equation is for solid quartz packing that passes through a $1\frac{3}{4}$-in. screen but is retained on a $1\frac{1}{4}$-in. screen.

Absorption coefficients for sulphur dioxide and water.—For a tower packed with 1-in. coke

$$\frac{1}{K_ga} = \frac{0.0022}{u^{0.8}} + 0.0047 \quad (237)$$

while for a tower packed with 3-in. coke

$$\frac{1}{K_ga} = \frac{0.00114}{u^{0.8}} + 0.00875 \quad (238)$$

where

u = gas velocity in feet per second calculated on the cross-section of the empty tower.

ABSORPTION-COLUMN CALCULATIONS

If the coefficients k_g and k_L are known, Equation (221) gives a basis for the numerical calculation of absorption apparatus.

[1] PERRY, pp. 1024–1036.

Since the composition of gas and liquid varies from point to point in the apparatus, however, the rate equation must be written in the differential form and integrated.

Thus, consider a countercurrent absorption tower of the packed type, shown diagrammatically in Fig. 182. Let the symbols be defined as follows:

G = pounds of dry inert gas passing through the column in unit time.
L = pounds of absorbing liquid passing through the column in unit time.
y_1 = pounds of absorbable material in entering gas per pound of inert gas.
y_2 = pounds of absorbable material in leaving gas per pound of inert gas.
x_1 = pounds of absorbable material in leaving liquid per pound pure absorbing liquid.
x_2 = pounds of absorbable material in entering liquid per pound pure absorbing liquid.
V_1 = tower volume in cubic feet.
a = effective contact surface in square feet per cubic foot of tower volume.

FIG. 182.—Diagram of absorption column.

Consider the small volume dV of the column of Fig. 182. The interfacial area in this section is adV sq. ft. The material transferred from gas to liquid is, by a material balance,

$$Gdy = Ldx = dw \tag{239}$$

The rate equation (Equation (221)) becomes

$$dw = k_g(p_{ag} - p_{ai})adV = k_L(c_{ai} - c_{aL})adV \tag{240}$$

While, if Equations (239) and (240) are combined,

$$Gdy = k_g a(p_{ag} - p_{ai})dV = Ldx = k_L a(c_{ai} - c_{aL})dV \tag{241}$$

Graphical Integration of Rate Equations

The use of Equation (241) for actual calculation is much simplified by using a graphical method. In this method, instead of using a plot of the equilibrium curve in terms of partial pressure, p, and concentration, c, the curve is plotted as y *vs.* x, where

x = pounds of absorbable material in the liquid per pound of pure absorbing liquid.
y = pounds of absorbable material in the gas per pound of inert gas.

For these conversions, the following equations are used:

$$x = \frac{c}{\rho - c} \tag{242}$$

$$= \frac{M_a/M_b p}{P - p} = N\frac{p}{P - p} \tag{243}$$

where

ρ = the density of the solution in pounds per cubic foot.
M_a = the molecular weight of the absorbable gas.
M_b = the molecular weight of the inert gas.
$N = M_a/M_b$.

The equilibrium curve in $x - y$ units is a curve of x_i *vs.* y_i, where the subscript i refers to the interface composition of liquid and gas phases.

Determination of the Operating Line.[1]—The equation of the operating line is obtained by a solute balance over that part of the column up to the section dV. Thus, referring to Fig. 182, the equation stating that the solute lost by the gas stream in the lower part of the column equals that gained by the liquid in the same part of the column is

$$G(y_1 - y) = L(x_1 - x) \tag{244}$$

This is the equation of a straight line, of slope L/G, and passing through the points (x_1, y_1) and (x, y). If it is written for the

[1] "Principles of Chemical Engineering," 2d. ed., pp. 675–698 (1927).

terminal conditions (*i.e.*, a solute balance over the entire column), it becomes

$$G(y_1 - y_2) = L(x_1 - x_2) \tag{245}$$

From this it is apparent that the same straight line also passes through the point (x_2, y_2). Thus, if the four terminal compositions x_1, x_2, y_1, and y_2 are known, the operating line is easily plotted. The slope of the line is the ratio $L:G$.

Figure 183 shows the equilibrium distribution curve for water vapor between the phases caustic soda solution and air at 20°C.

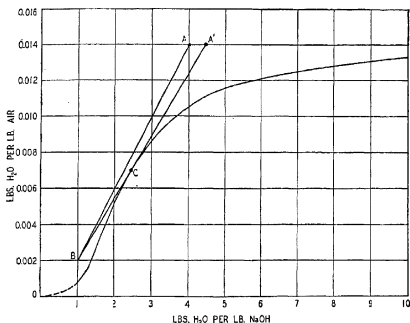



FIG. 183.—Equilibrium between solutions of caustic soda and moist air.

In this plot the abscissas are pounds of water per pound of caustic soda, and the ordinates are pounds of water per pound of dry air. If a mixture of air and water vapor of a particular composition, say 0.010 lb. water per lb. dry air, is exposed to a caustic soda solution at 20°C., water will be removed by any solution that contains less than 3.72 lb. H_2O per lb. NaOH. Water vapor will be given up from the solution to this air-water mixture for any concentration of more than 3.72 lb. water per lb. caustic; and this air-water mixture will be in equilibrium with a solution containing 3.72 lb. water per lb. caustic.

Minimum Liquid-gas Ratio.—If the operating line be plotted on Fig. 183, a line such as AB results. This is for the particular case where $x_1 = 4.0$, $y_1 = 0.014$, $x_2 = 1.0$, $y_2 = 0.002$. The coordinates of any point on the line AB represent liquid- and gas-phase compositions at a particular point in the apparatus. Thus air containing 0.010 lb. water per lb. dry air is in contact with a solution containing 3 lb. water per lb. caustic. This is a stronger solution than the one in equilibrium with this air, and hence absorption can take place. In the same way, any point on the line AB is in contact with stronger solutions than the equilibrium concentrations, as long as the line AB is entirely to the left of, and above, the equilibrium curve. Wherever the line AB becomes tangent to the equilibrium curve, as the line $A'B$ does at C, the liquid and gas phases are in equilibrium and the process stops at the point of tangency. In fact, it would require an infinitely tall tower even to reach such a point as C, since as the line $A'B$ approaches the equilibrium curve, the driving force is approaching zero, and hence the tower volume approaches infinity. Hence no actual tower could ever reach a point of tangency. Since conditions represented by even a tangent point cannot exist, it is apparent that the situation represented by an operating line that crosses the equilibrium curve is also impossible. The slope of the line AB is L/G, that is, the ratio of absorbing liquid to gas. This indicates that, for any given values of three of the four terminal concentrations (for example x_2, y_1, and y_2), there is a minimum ratio of absorbing liquid to dry gas that can be employed, and a maximum value of x_1 that can be reached. Thus, if x_2, y_1, and y_2 are fixed at 1.0, 0.014, and 0.002, respectively, the smallest ratio of liquid to gas that can be employed is that given by the slope of the operating line $A'B$. The value of x_1 corresponding to the minimum liquid flow is 4.40 lb. water per lb. caustic soda. The maximum ratio of dry air to NaOH is $\frac{4.40 - 1.00}{0.014 - 0.002} = 283.3$.

Integration of Rate Equation—Gas-film Controlling.—If p_{ag} and p_{ai} are eliminated from Equation (241) by the use of Equation (243) it follows that:

$$\frac{(y + N)(y_i + N)dy}{y - y_i} = \frac{PNk_g adV}{G} \tag{246}$$

It has been shown that for soluble gases the gas film controls, and the driving force at any point in the tower can be found by measuring the vertical distance between the point (c_{aL}, p_{ag}) and the equilibrium curve. On the $x - y$ diagram, the corresponding construction for locating the value of y_i for any given value of y is to drop a perpendicular from point (x, y) to the equilibrium curve. Thus, in Fig. 184, if the gas film controls, the ordinate of point b is the value of y_i corresponding to the value of y represented by the ordinate of point a.

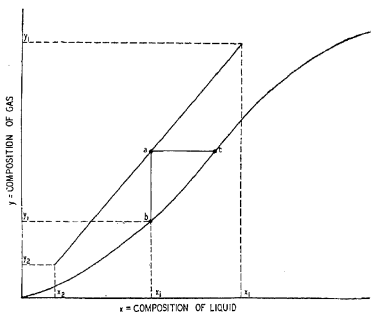



FIG. 184.—Absorption-column diagram.

Equation (246) can be written in the form

$$\int_{y_2}^{y_1} \frac{(y + N)(y_i + N)}{y - y_i} dy = \frac{PNk_ga}{G} \int_0^{V_1} dV = \frac{PNk_gaV_1}{G} \quad (247)$$

and the left-hand side can be integrated graphically by plotting values of

$$(y + N)(y_i + N)/(y - y_i)$$

as ordinates against the corresponding values of y as abscissas and determining the area under the curve between the ordinates $y = y_2$, $y = y_1$. If k_ga can be determined either on the basis of the performance of equipment of the same type as that con-

sidered or on the basis of data obtained from a small-scale experiment, V_1 can be calculated.

If N is large compared to y and y_i, Equation (247) reduces to

$$\int_{y_2}^{y_1} \frac{dy}{y - y_i} = \frac{Pk_g a V_1}{NG} \tag{248}$$

Example 34.—One hundred thousand cubic feet per hour of a gas containing 23.2 per cent NH_3 by volume (remainder air) are to be scrubbed with water in a tower 6 ft. in diameter packed with 1.5 in. quartz. 95 per

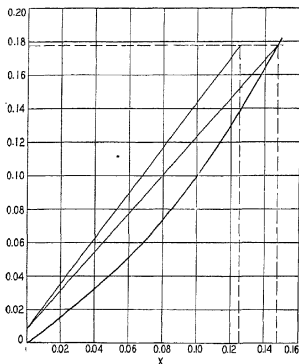



Fig. 185.—Solution of Example 34.

cent of the NH_3 is to be recovered. The process will be isothermal at 30°C. and the calculation is to be made for a barometer reading of 760 mm. What is the minimum amount of water that can be used? If sufficient water is used to give an exit solution containing 12.5 lb. NH_3 per 100 lb. water, how deep should the bed of packing be?

Solution.—The solubility curve for 30° in Fig. 176 must be converted to x and y coordinates by the use of Equation (243). The y-axis of Fig. 176 is already in the units desired for x, so Equation (242) is not needed. The results of this conversion are plotted as the equilibrium curve in Fig. 185.

The composition of the initial gas is given by

$$y_1 = \frac{23.2}{76.8} \times \frac{17}{29} = 0.1775$$

The final gas will contain 5 per cent as much ammonia per pound of air, or $0.0089 = y_2$. The concentration of ammonia in the entering liquid (x_1) is zero. Therefore the lower end of the operating line must pass through the point ($y_2 = 0.0089$, $x_2 = 0$).

The minimum water consumption will be for the case where the line through this point intersects the equilibrium curve at $y_1 = 0.1775$. Such a line is drawn on Fig. 185.

The density of air at 30°C. is 0.0727 lb. per cu. ft. The total quantity of gas handled contains 76,800 cu. ft. of air per hr., hence

$$G = 76{,}800 \times 0.0727 = 5{,}585 \text{ lb. per hr.}$$

From Equation (245)

$$5{,}585(0.1775 - 0.0089) = L(0.1475 - 0)$$

from which

$$L = 6{,}380 \text{ lb. water per hour (minimum).}$$

If the exit solution is to contain 12.5 lb. NH_3 per 100 lb. H_2O, this corresponds to $x_2 = 0.125$, and the operating line for the practical case passes through the points ($y_2 = 0.0089$, $x_2 = 0$) and ($y_1 = 0.1775$, $x_1 = 0.125$). This line is also drawn on Fig. 185. From Equation (245), $L = 7{,}530$ lb. water per hr.

The coefficient is to be calculated from Equation (236). Here L' and G' are pounds per minute per square foot tower cross-section. A 6-ft. circle has an area of 28.27 sq. ft., hence

$$L' = \frac{7{,}530}{(60 \times 28.27)} = 4.44$$

$$G' = \frac{5{,}585}{(60 \times 28.27)} = 3.29$$

Substituting in Equation (236) gives $k_g a = 140.3$.

Equation (247) may now be integrated

$$N = \frac{M_a}{M_b} = \frac{17}{29} = 0.586$$

$$P = 1 \text{ atm.}$$

To evaluate the integral, for various values of y the corresponding values of y_i are read from the curve, the function under the integral sign is evaluated for each value of y and plotted *vs.* y as in Fig. 186. The area under the curve between $y = 0.1775$ and $y = 0.0089$ is found to be 2.42 units.

Substituting in Equation (247)

$$2.42 = \frac{1 \times 0.586 \times 140.3 \times V_1}{5{,}585}$$

$$V_1 = 164.4 \text{ cu. ft.}$$

$$\frac{164.4}{28.27} = 5.8 \text{ ft. height of packing needed.}$$

Example 35.—A packed absorbing tower is absorbing moisture from air by means of a caustic soda solution. The partial pressure of water vapor in the entering air is 14.4 mm., and this is lowered to 4.9 mm. in the exit air. The solution entering the column is 50 per cent NaOH. The solution leaving the column is 25 per cent NaOH.

It is desired to increase the height of the column enough to reduce the partial pressure of water vapor in the exit air to 2.4 mm., without changing the amount or compositions of entering air and entering solution. The temperature is 20°C.

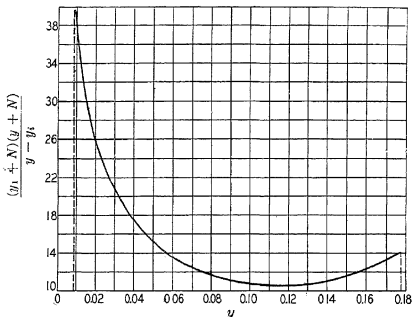



FIG. 186.—Integration for Example 34.

a. What is the composition of the solution leaving the column under the new conditions?

b. What per cent increase in column height will be necessary?

Solution.—*a.* The partial pressures must be changed to values of y, but y is humidity as defined in Chap. VII and may be calculated by Equation (118). Then

$x_1 = {}^{75}/_{25} = 3$ lb. H_2O per lb. NaOH.
$y_1 = 0.012$ lb. H_2O per lb. air.
$x_2 = {}^{50}/_{50} = 1$ lb. H_2O per lb. NaOH.
$y_2 = 0.004$ lb. H_2O per lb. air (present conditions).
$y_2 = 0.002$ lb. H_2O per lb. air (proposed conditions).

The diagram for this problem is given in Fig. 187. The line AB is the operating line for present conditions. Under the proposed conditions, the operating line passes through the point $C(x_1 = 1, y_1 = 0.002)$ and has the same slope as the present operating line, since the ratio of NaOH to air has

not been changed. These two facts determine the new operating line *CD*, and the abscissa of the intersection of this line with the line $y = 0.012$ determines the new concentration of the leaving liquor. This intersection is point *D* of Fig. 187, the abscissa of the point is 3.5, and the new concentration of the leaving solution is $(1.0/4.5)(100) = 22.2$ per cent NaOH.

b. It will be seen that N ($^{18}\!/_{29} = 0.589$) is large compared with the values of y (0.012 to 0.002). Hence Equation (248) will be sufficiently accurate. This problem can then be solved by writing Equation (248) twice, once for the present conditions and once for the new.

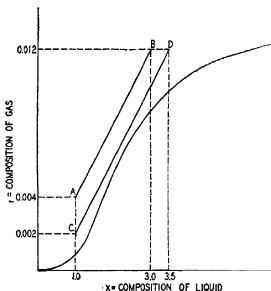



FIG. 187.—Conditions of Example 35.

For the present conditions, Equation (227) becomes

$$\int_{0.004}^{0.012} \frac{dy}{y - y_i} = \frac{Pk_g a V_1}{NG} \text{ (old)} \tag{249}$$

and for the new conditions

$$\int_{0.002}^{0.012} \frac{dy}{y - y_i} = \frac{Pk_g a V_1}{NG} \text{ (new)} \tag{250}$$

Dividing Equation (250) by (249)

$$\frac{(V_1) \text{ new}}{(V_1) \text{ old}} = \frac{\int_{0.002}^{0.012} \frac{dy}{y - y_i}}{\int_{0.004}^{0.012} \frac{dy}{y - y_i}}$$

and the solution reduces to the ratio of two integrals that can be graphically integrated by plotting the appropriate values of $\frac{1}{y - y_i}$ *vs.* y and integrat-

ing in the usual manner. The process is indicated in the following table, and Fig. 188 shows the plot for the integrations:

Present conditions			New conditions		
y	$y - y_i$	$\frac{1}{y - y_i}$	y	$y - y_i$	$\frac{1}{y - y_i}$
0.012	0.0034	292	0.012	0.0023	435
0.010	0.0028	357	0.010	0.0014	715
0.008	0.0028	357	0.008	0.00085	1,175
0.006	0.0034	294	0.006	0.00085	1,175
0.004	0.0032	312	0.004	0.0014	714
			0.002	0.0012	834

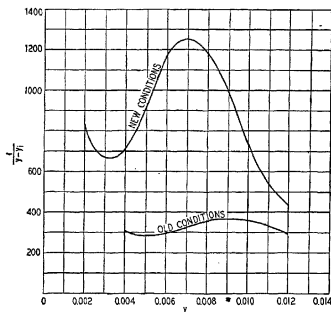



FIG. 188.—Integration of Example 35.

The areas under the upper and under the lower curves are 8.76 and 2.60, respectively, and therefore:

$$\int_{0.002}^{0.012} \frac{dy}{y - y_i} = 8.76 \text{ (new)}$$

$$\int_{0.004}^{0.012} \frac{dy}{y - y_i} \quad 2.60 \text{ (old)}$$

and

$$\frac{(V_1) \text{ new}}{(V_1) \text{ old}} = \frac{8.76}{2.60} \quad 3.37$$

The new volume or height is therefore, a 237 per cent increase over the old. This large increase in height is a result of moving the operating line so close to the equilibrium curve that the average driving force in the center of the column is very small. This shows as a peak in the graphical integration curve for the new conditions in Fig. 188. This problem also illustrates how much information may sometimes be obtained without knowing the numerical values of the coefficients.

Log Mean Partial-pressure Difference.—Under certain conditions it is unnecessary to integrate graphically Equation (241). If the solute gas obeys Henry's law (Equation (217)); if the partial pressure of the inert gas $(P - p_{ag})$ does not change greatly in the apparatus; and if the quantity $\rho - c$ of Equation (242) can be considered nearly constant, it can be shown[1] that Equation (241) integrates to:

$$w = K_g a V_1 (\Delta p)_m \qquad (251)$$

where

$$(\Delta p)_m = \frac{(p_{ag1} - p_{ai1}) - (p_{ag2} - p_{ai2})}{2.303 \log \frac{p_{ag1} - p_{ai1}}{p_{ag2} - p_{ai2}}} = \frac{\Delta p_1 - \Delta p_2}{2.303 \log \frac{\Delta p_1}{\Delta p_2}} \qquad (252)$$

where

p_{ag1} = the partial pressure of the solute gas in the entering gas.
p_{ai1} = the equilibrium partial pressure of the solute gas over the leaving liquid.
p_{ag2} = the partial pressure of the solute gas in the leaving gas.
p_{ai2} = the equilibrium partial pressure of the solute gas over the entering liquid.

and

$$\Delta p_1 = p_{ag1} - p_{ai1} \qquad \Delta p_2 = p_{ag2} - p_{ai2} \qquad (253)$$

Liquid-film Controlling.—An analogous derivation for the case where the liquid-film controls can be made by eliminating

[1] KOWALKE, HOUGEN, and WATSON, "Transfer Coefficients of Ammonia in Absorption Towers", *Bull. Univ. Wisconsin Expt. Station*, No. **68**, p. 20 (1925).

c_{ai} and c_{aL} from Equation (241) by means of Equation (242), and using an average value of the liquid density (ρ_{av}). The result is:

$$\int_{x_2}^{x_1} \frac{(1 + x_i)(1 + x)dx}{x_i - x} = \frac{k_L a \rho_{av}}{L} V_1 \tag{254}$$

The integral of the left-hand side of Equation (254) is found by plotting $(1 + x_i)(1 + x)/(x_i - x)$ vs. x and obtaining the area below the curve and between ordinates $x = x_1$, $x = x_2$. The numerical value of x_i for any value of x is found graphically from the $x - y$ plot by measuring the horizontal segment between

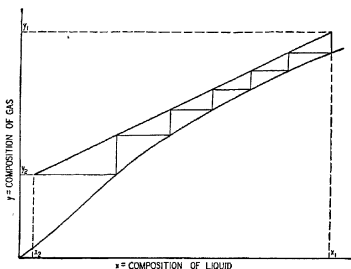



FIG. 189.—Bubble-cap absorption-column diagram.

the operating and the equilibrium lines. Such a construction is shown by the line ac of Fig. 184.

In cases where x_i and x are small in comparison with 1 (and this will almost certainly be the case in gases of low solubility where the liquid film controls) Equation (254) simplifies to

$$\int_{x_2}^{x} \frac{dx}{x_i - x} \quad k_L a \rho_a \cdot V_1 \tag{255}$$

Calculations for Bubble-cap Absorbing Columns.—If a bubble-cap tower is used as an absorption apparatus, the theoretical plate concept can be applied,[1] and the number of theoretical plates calculated by the graphical step-wise method. Figure 189

[1] BAKER, *Trans. Am. Inst. Chem. Eng.*, **15**(I), 195–216 (1923).

shows an example of such a calculation. The interpretation of this diagram will be apparent from the discussion of rectifying column calculations given in Chap. IX.

Stripping.—A process that is the reverse of absorption is that of stripping. In this case a liquid containing a substance in solution is stripped of this material by bringing the liquid in contact with a gas phase that is low in the material transferred, so that the latter passes from liquid to gas. The calculations and diagrams for such a case differ from those of gas absorption only in that the operating line lies below the equilibrium curve rather than above it.

Nomenclature of Chapter X

A = area measured perpendicular to direction of diffusion
a = component of mixture to be absorbed; active surface per unit volume
B = rate of water flow over packing in gallons per square foot; thickness of film in feet
b = coefficient of diffusion; inert component
c = concentration
D = nominal diameter of one packing unit in feet; inside diameter of tower
G = weight of inert gas per unit time
G' = pounds of inert gas per square foot of tower cross-section
H = Henry's law constant
h = height of packing, feet
K = overall coefficient
k = film coefficient
L = pounds of absorbing liquid passing through the column in unit time
L' = rate of flow of liquid down the tower in pounds of water per square foot of tower cross-section per minute
M = molecular weight
N = M_a/M_b
p = partial pressure
ΔP = pressure drop in pounds per square foot
u = gas velocity in feet per second, calculated on empty tower
V_1 = tower volume in cubic feet
w = weight per unit time
x = pounds of absorbable material per pound pure absorbing liquid
y = pounds of absorbable material per pound of inert gas
sub a = component a (component to be absorbed)
sub b = component b (inert component)
sub g = gas phase
sub i = interface

sub L = liquid phase
sub m = mean
sub v = volumetric units
sub w = at interface
sub 1 = end of tower where gas enters and liquid leaves
sub 2 = end of tower where gas leaves and liquid enters
μ = viscosity of the gas
ρ = density in pounds per cubic foot

Problems

1. One hundred cubic feet of a mixture of 20 per cent sulfur dioxide and 80 per cent air is brought into contact with 5 gal. of water at 20°C. and 760 mm. pressure. If equilibrium is attained at this temperature and pressure, calculate

a. The concentration of sulfur dioxide in the resulting solution.
b. The composition of the gas phase at equilibrium.
c. The volume of the gas phase.

2. Repeat parts *a* and *b* of Problem 1 on the assumption that equilibrium is reached without changing the volume of the gas phase. What will be the total pressure at equilibrium in this case?

3. A tower, 4 ft. in diameter, is packed with 2-in. Raschig rings. The packed section is 20 ft. high. Water flows down over the packing at the rate of 8 g.p.m. There is blown in, at the bottom, 1,500 c.f.m. of a gas containing 2 per cent HCl and 98 per cent air. The process is isothermal at 60°F. and the calculations are to be based on a barometer of 760 mm. If the blower efficiency is 35 per cent, what is the power input to the blower?

4. The liquid-film resistance for absorption of NH_3 in water may be taken as negligible when the solution produced is dilute. If the gas-film coefficient, k_g, for the absorption of NH_3 in water is 0.071 mol NH_3 per hr. per sq. ft. per atmosphere Δp under the following conditions:

NH_3 entering = 40 per cent by volume
NH_3 leaving = 2 per cent by volume
Pressure in tower = 3 atmospheres
Gas velocity = 1.4 ft. per second

what value of k_g should be used under the following conditions?

NH_3 entering = 4 per cent by volume
NH_3 leaving = 0.05 per cent by volume
Pressure in tower = 1 atmosphere
Gas velocity = 1.4 ft. per second

5. A packed column is to remove water vapor from air by means of strong sulfuric acid. Ten thousand cubic feet of wet air are to be handled per hour. The wet air is at a temperature of 90°F. and has a humidity of 0.015 lb. H_2O per lb. dry air. The dried air is to have a humidity of 0.003 lb. H_2O per lb. dry air. The entering acid is 66.7 per cent H_2SO_4 by weight. One-inch

Raschig rings are to be used for packing, a for these rings $= 58$, $P = 14.7$ lb. per sq. in., and $k_g = 0.15$ lb. mol per sq. ft. per hr. per atmosphere partial pressure difference.

The following equilibrium data are available:

Pounds H_2O per pound dry air	Pounds H_2O per pound H_2SO_4
0	0
0.0001	0.4
0.0028	0.6
0.005	0.8
0.007	1.0
0.009	1.2
0.0108	1.4
0.0123	1.6
0.0138	1.8
0.0147	2.0
0.0154	2.2
0.0158	2.4
0.0162	2.6

Assume $u = 1.0$ ft. per sec.

Determine:

a. The minimum concentration of H_2SO_4 in the discharged acid.

b. The quantity of H_2SO_4 when the exit concentration is 40 per cent H_2SO_4.

c. Calculate the diameter of the tower and the height of the packed section when the tower is operated under the conditions in *b*.

6. Air containing 12.5 per cent HCl by volume is to be prepared by blowing fresh air up through a packed tower fed with muriatic acid containing 37 per cent HCl by weight.

The $x - y$ diagram for solutions of HCl in water is as follows:

x	y
0.35	0.003
0.40	0.012
0.45	0.027
0.50	0.065
0.525	0.105
0.55	0.165
0.575	0.243
0.60	0.375

What would be the minimum exit concentration of acid in per cent HCl by weight?

The tower is to be of stoneware, 2 ft. inside diameter. 10,000 cu. ft. per hour of fresh air at 1 atm. and at 20°C. will be blown in. The exit acid is to be 32 per cent HCl by weight. If $k_g a$ for this case is 200, what should be the height of the packed section?

CHAPTER XI

EXTRACTION

This chapter will discuss those cases where a soluble constituent is removed from a solid or a liquid by the use of a solvent. This involves operations that are in wide use not only in chemical engineering but also in other arts. Water will usually be assumed as the solvent, but it is obvious that any other solvent may be used if necessary. The theory is quite inadequate, and few important quantitative studies have ever been made. Consequently, the apparatus has developed along lines dictated by convenience and experience, rather than by a theoretical analysis of the problem. Although a wide variety of devices has been used, they may be separated roughly into three groups. The first involves those devices that are used on relatively coarse solids, the second, those devices used where the solid is sufficiently finely divided to be held in suspension in the solution, and the third, those devices used for extracting liquids with liquids. The limiting case of extraction is the dissolving of a completely soluble solid.

APPARATUS

The devices used for leaching coarse solids fall into three main classes: namely, open tanks, diffusion batteries, and drag classifiers. The apparatus used for the extraction of finely divided solids usually consists of agitators and thickeners. Apparatus for extracting liquids with liquids is not yet standardized.

Apparatus for Leaching Coarse Solids

Open Tanks.—The earliest and simplest form of extraction apparatus, and one that is still used to a considerable extent, is an open tank containing a false bottom or filter of some sort. Into this tank material is charged; the solvent is applied at the top, is allowed to percolate down through the charge, and is

drawn off below the false bottom. In such tanks the solvent is not simply distributed over the solid, but the entire tank is flooded with solvent.

Filter bottoms are constructed in an endless variety of forms. Figure 190 shows a few constructions. In *a*, perforated boards are laid on notched bearer strips on the bottom of the tank. In *b*, triangular pieces are laid on top of notched bearers, and the space between them filled with gravel to act as a filter medium. In *c*, simple bearer strips are used, spaced closely enough to retain

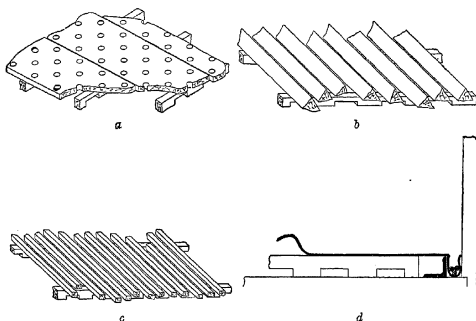



FIG. 190.—Tank filter-bottom constructions. *a*. Perforated boards on bearers. *b*. Support for gravel filter. *c*. Support for coarse material or filter cloth. *d*. Method of securing filter cloth.

the average lump of solid. In *d*, there is shown a method of constructing a cloth filter bottom that is often convenient. Notched bearer strips are laid on the bottom of the tank at intervals of a few inches. A band or hoop is fastened to the bottom of the tank from half an inch to an inch inside the wall. The filter cloth is cut large so that it lies over the space between this hoop and the side of the tank and is then made tight by caulking a hemp rope into the groove.

In some cases the rate of solution is sufficiently great so that one passage of the water down through the material gives a satisfactory extraction. After a wash with fresh water to remove

adhering solution, the solid can be discarded. Such tanks have often been built for manual discharge, but it is more satisfactory to provide either side or bottom doors through which the solid can be flushed with a hose.

In the one-tank method of extraction, the most concentrated solution that could conceivably be made would be the soluble contents of one tankful of solid dissolved in one tankful of water. If it is necessary to prepare a stronger solution than this, countercurrent operation is used. In this case there is a series of tanks, each such as the one described above, containing solid in various stages of extraction. This arrangement is called an *extraction battery.* Fresh water is introduced into the tank containing the solid that is most nearly extracted, flows through the several tanks in series, and is finally withdrawn from the tank that has been freshly charged. The material in any one tank is stationary until it is completely extracted. By means of suitable piping connections, so arranged that fresh water can be introduced to any tank and strong solution drawn off from any tank, it is possible to charge and discharge one tank at a time. The remainder of the battery is kept in strict countercurrent by advancing the inlet and draw-off tanks one at a time as the material is charged and removed. Such a process is sometimes called the *Shanks process* and was first applied to the leaching of black ash in the LeBlanc soda process. Its most important representative at present is the extraction process used in the Chilean nitrate fields, although it is often used in many other cases, especially in the production of tan-bark extracts.

Diffusion Battery.—The diffusion battery is nothing more than a Shanks countercurrent extraction system applied to materials through which water will not percolate by gravity head, but through which it must be pumped by pressure. Consequently it differs from the Shanks process in employing closed vessels instead of open tanks. Due to the use of the diffusion battery in the beet-sugar industry, the specific arrangement of valves and piping has become standardized. Although the beet-sugar industry is the most important field in which the diffusion battery is used, it is also used for the manufacture of tanning and dye-wood extracts, for the leaching of black ash in the soda and sulphate process for paper pulp, for steeping corn preparatory to the manufacture of starch, and similar purposes.

Figure 191 is a diagrammatic illustration of the principle of a diffusion battery. It will be discussed on the basis of the extraction of beet chips, but the operation is the same on any substance. For every vessel or *cell* there is a heater, because the diffusion process takes place more rapidly at higher temperatures. In some cases the heater may be dispensed with, and a simple pipe takes its place. Two main headers are necessary. One handles water and the other handles solution; and for every cell there

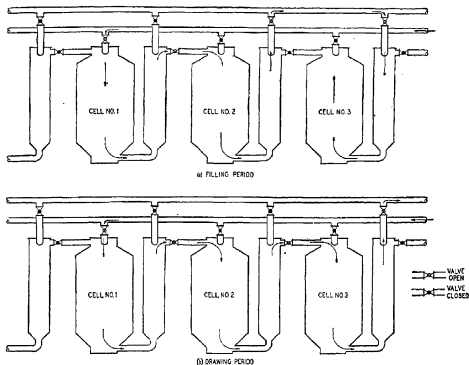



FIG. 191.—Diagram of diffusion battery.

must be three valves. In Fig. 191 the valves that are open are shown as circles and the valves that are closed are shown in solid black.

Consider Fig. 191*a*. Cell 1 is nearly exhausted and cell 3 has just been charged. The space between the chips in cell 3 is therefore filled with air. Water is introduced into cell 1 and flows down through the cell, up through the heater, down through cell 2, and up through its heater. It would not be convenient to pass the solution down through cell 3 because of the air which would be entrapped; and supposedly the charge is cold, therefore additional heating is desirable. Consequently, the liquid flows

from the heater of cell 2 through the solution line, down through the heater of cell 3, and up through cell 3. A vent at the top of this cell discharges air. When liquid appears at this vent, the valves are quickly changed to the position shown in Fig. 191*b*. Liquid now flows down through cell 3, up through its heater, and out to the process. The operation shown in Fig. 191*b* is continued until cell 1 is completely extracted. By this time another cell to the right of those shown has been filled, cell 1 is dumped, water is introduced to cell 2, and the process continued.

The actual arrangement of a diffusion battery is shown in Fig. 192 in which the valves and pipe lines may be identified by reference to Fig. 191. Sometimes the cells are arranged in a circle. In case the battery is built in a straight line to save floor space, there must be a third pipe line, called the *return line*, to carry solution from one end of the battery to the other when the first and last cells are in various intermediate positions.

The *Dorr classifier* (see page 580) may be used for leaching granular materials that are not fine enough to remain in suspension in the solvent and which contain the solute in such a form that it may be extracted by surface washing. When used for this purpose, several decks in series are usually employed and the solid and liquid flow in countercurrent.

Extraction of Fine Material

When the solid to be extracted may be ground so fine that it can be kept in suspension in the solvent by reasonable agitation, extraction apparatus becomes quite simple. In practice, this usually requires that the solid be ground to approximately 200 mesh. This can be accomplished only on relatively hard materials, and therefore such methods as are to be described may be employed for the leaching of ores or precipitates. Typical processes are the extraction of gold by cyanide solutions, the preparation of alum solutions from bauxite by leaching with sulphuric acid, the leaching of caustic soda from calcium carbonate precipitate, and similar operations. All the methods to be described under this heading originated in the cyanide process for the extraction of gold and were long used in that process before they were applied to other chemical operations.

These extraction processes require only an apparatus in which the finely divided solid may be kept suspended in the liquid.

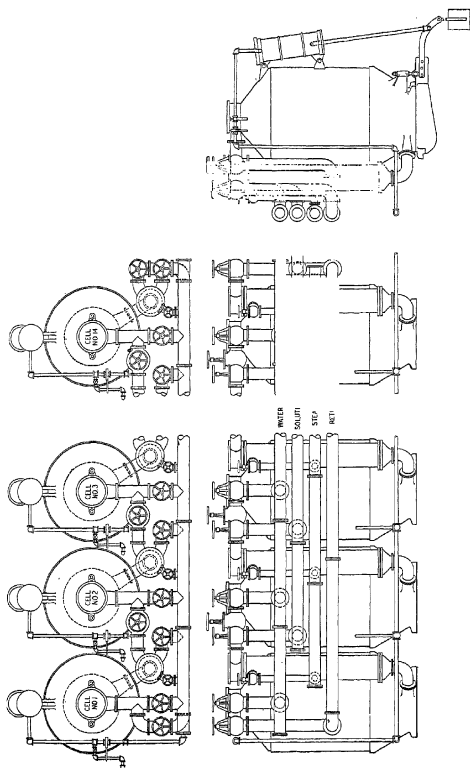



Fig. 192.—Diffusion battery.

They are usually followed by special devices for settling the solid and recovering the clear solvent, but these are separation devices and, properly speaking, do not form part of the extraction apparatus. Any type of tank or other container provided with agitators in the form of paddles, propellers, or air jets may be employed for the extraction process proper. Only two special constructions have found sufficiently wide use to warrant specific descriptions.

FIG. 193.—Pachuca tank. *A*. Lift tube. *B*. Air-jet nozzle. *C*. Air pipes for use after shut-down.

The Pachuca Tank.—This device (Fig. 193) is a tall cylindrical tank with a conical bottom. The straight part of the tank is from 3 to 5 diameters high. Through the center of the tank extends a vertical tube *A*, open top and bottom, and the nozzle of an air-jet lift *B* is located in the bottom of this tube. Liquid and solid together are lifted through the central tube by the air-jet lift, and sufficient circulation is thus produced to keep the solid in suspension. If there should be a temporary interruption in the air supply, sufficient solids may settle in the point of the cone to clog the air-jet nozzle; and therefore a number of supplementary air pipes *C* are introduced at different points in the cone to loosen such settled material.

The Dorr Agitator.—This agitator (Fig. 194) is the most widely used of the standard types. It also depends on a central tube *A* with an air-jet lift in the bottom, but it employs a flat-bottomed tank. The central tube serves also as a shaft and

carries a set of arms both at the top *B* and at the bottom *C*. The bottom arms are provided with scraper blades set at an angle so that they carry any settled material toward the central air-jet lift. The upper set of arms are in the form of launders and receive the discharge from the air-jet lift. They have a number of perforations so that they distribute the suspension over the surface of the tank as they rotate. The lower set of arms are

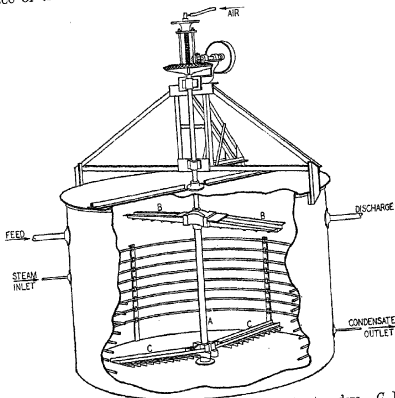



FIG. 194.—Dorr agitator. *A*. Lift tube. *B*. Distributing launders. *C*. Rakes.

usually hinged and provided with means for lifting them from the bottom of the tank when the power is interrupted, so that they may not be blocked with settled material that would prevent their starting again. The agitator shown in Fig. 194 is provided with a steam coil.

Any type of agitator used for extraction may operate either continuously or discontinuously. The simplest method is to charge a batch of solid and solvent into the agitator, agitate this batch until solution is complete, and then remove the entire batch. In large-scale operations it is more desirable to operate

continuously. In this case the agitator is provided with inlet and outlet pipes at opposite sides as indicated in Fig. 194.

Continuous Leaching of Fine Solids.—The previous discussion has been based on the assumption that a solid containing a soluble constituent was to be treated for the extraction of that constituent. When this has been done, the solid is suspended in the solution so produced, and if the solution is merely drained off, it is usually desirable to introduce a second step for recovering the solution that adheres to the solid. This may be considered an extraction process but is more often thought of as a washing process. In many cases a chemical reaction produces a precipitate that must be washed free from solution. In this case there has been no soluble constituent of a solid to be removed but it is merely a case of removing the adhering film of solution from the solid particles and is therefore not strictly an extraction process.

Such operations may be carried out by stirring up the precipitate with water, allowing it to settle, draining off the supernatant solution, and repeating the process as many times as may be desirable. This may be done with a fresh batch of solvent each time or it may be done in countercurrent—that is, fresh water may be used only for the last wash on the precipitate that is about to be discarded. The resultant solution is saved and used for the next to the last wash on the next batch, and so on. On a small scale this operation may be carried out intermittently in the same agitator tanks that were used for the main extraction. In large-scale operations where it is desirable to operate continuously, a modification of the apparatus is necessary and the resulting system is known as *countercurrent decantation.*

The Dorr Thickener.—The most widely used settling apparatus is the Dorr thickener (Fig. 195). This consists of a flat-bottomed tank of large diameter compared to its depth, and having a central shaft on which are a set of slowly revolving arms. A mixture of liquid and finely divided solid is fed into a central well with as little disturbance of the liquid in the tank as possible. The diameter of the tank is made such that the time any particular quantity of liquid is in the tank is sufficient for the solids to settle from it. Clear liquid overflows into a launder around the circumference of the tank, and the slowly moving arms at the bottom gradually scrape the settled solids to the center where

they are withdrawn. The apparatus shown in Fig. 195 is a two-deck thickener, *i.e.*, two separate and independent thickeners placed in one tank to save floor space. The Dorr thickener is merely a device for separating solids from liquids, but it is extremely useful in the washing processes that are under discussion.

Countercurrent Decantation Systems.—The flow sheet of a typical Dorr countercurrent decantation system is shown in

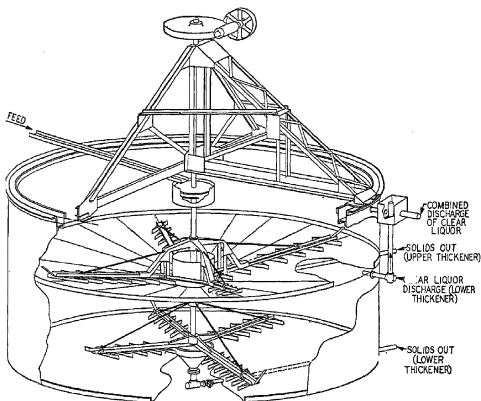



Fig. 195.—Dorr thickener.

Fig. 196, as applied to the causticizing of soda ash. One or more agitators are arranged in series, and the capacity of these agitators is so related to the quantity of material handled that any one particle of solid remains in the agitators long enough to be completely extracted or to allow the reaction that may be taking place to come to completion. The mixture of solid and liquid from the last agitator goes to the first thickener. The overflow from this thickener is the product of the operation. The underflow from this thickener is sent to a second thickener

where it meets the overflow from the third thickener. These two mix in the feed well of the second thickener, the overflow from the second thickener goes to the agitation tanks and serves as solvent in them, while the underflow from the second thickener goes to the third thickener where it is met by a stream of fresh water. More or less than three thickeners may be used in series, according to the difficulty with which the film of solution is removed from the surface of the precipitate and the completeness of recovery desired. In operating such a system, it is usually desirable to control rather accurately the rate at which solids are removed from the bottom of the thickeners, and this is usually done by diaphragm pumps (see page 95) driven by adjustable eccentrics.

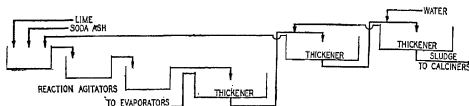



FIG. 196.—Flow sheet of countercurrent decantation system.

APPARATUS FOR EXTRACTING LIQUIDS WITH LIQUIDS

One class of equipment used for extracting liquids with liquids consists of relatively tall vertical towers. These towers contain a large number of horizontal baffles, either of the disc-and-ring type, or of staggered plates. The heavier liquid is fed in at the top and cascades from plate to plate. The lighter liquid is fed at the bottom and flows in countercurrent to the heavier liquid. Unless the difference in density between the two liquids is very great, velocities must be kept low to prevent carrying the heavier liquid over with the lighter one.

Another type of extractor is in the form of a horizontal cylinder. Vertical baffle plates at right angles to the axis of the cylinder divide it into a number of sections. The lighter liquid is fed in at one end, passes through the extractor from compartment to compartment, and leaves at the other end. The heavier liquid is pumped through a spray nozzle into the last compartment, is sprayed into the lighter liquid, and falls to the bottom of that compartment. From the bottom of this compartment it

is drawn by another pump and sprayed into the next compartment, and so on in countercurrent to the light liquid.

The field of extraction of liquids by liquids is a relatively new one, and it is not possible to mention many typical uses. Two processes in which such equipment is already in operation are the solvent refining of lubricating oils, and the extraction of acetic acid from wood distillation liquors.

THEORY OF EXTRACTION

The theory of extraction will be discussed under two heads: first, under the assumption that time enough has been provided for equilibrium conditions to be reached in the process; and, second, the rate of extraction before equilibrium is reached.

Calculations Based on Attainment of Equilibrium.—When certain further assumptions are made regarding equilibrium in the

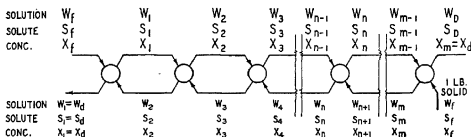



FIG. 197.—Diagram of countercurrent extraction system.

extraction and washing apparatus, the calculations reduce to straight material balances. These assumptions are: (1) the wash liquid is completely and uniformly mixed with the solution adhering to the surface of the solid; and (2) the solid exerts no selective adsorbing action on the solute but is completely inert.

Consider the flow sheet of a continuous extraction battery as shown in Fig. 197. The units are numbered serially, beginning at the unit where the fresh solvent enters. Unit n is the nth unit from the solvent end. Let the number of units in the battery be m. The underflows consist of the inert solids passing through the system plus the solution retained by the solids, while the overflows consist of clear solutions. The basis will be one pound of inert solid passing through the system, and the following notation for the solutions entering, flowing through, and leaving the system, will be used:

Stream	Weight of stream, pounds solution per pound inert solid	Weight of solute in stream, pounds per pound inert solid	Concentration of solute in stream, pounds solute per pound solution
Solution on fresh solids fed to system.........	w_f	s_f	x_f
Solution fed to system as fresh wash............	W_f	S_f	X_f
Solution adhering to solids discharged from system...............	w_D	s_D	x_D
Solution discharged from system as strong liquid.	W_D	S_D	X_D
Solution in underflow from *n*th unit.........	w_n	s_n	x_n
Solution overflowing from *n*th unit..............	W_n	S_n	X_n

In this notation the weight of a stream is represented by W or w, the solute in the stream by S or s, and the composition of the stream by X or x. Capital letters pertain to overflow, or clear liquid streams; while lower-case letters refer to underflow, or streams of solution that accompany the inert solids. Subscripts show the point of origin of the stream.

In all cases,

$$S = WX$$
$$s = wx \tag{256}$$

Also, if equilibrium is reached in all units,

$$x_n = X_n \tag{257}$$

Two methods of utilizing the material balances available for the system of Fig. 197 will be discussed. The choice of method to be used in a given case depends upon the constancy of the underflow from unit to unit. If factors such as the viscosity and density of the solutions change considerably with their compositions, the solids from the higher numbered units may retain more liquid than do those from the lowered numbered units, and the values of w will change from one unit to the next.

If, on the other hand, the weight of solution retained by one pound of inert solid is independent of the concentration of the solution, it can be expected that the solids discharged from all units of the series will retain the same weight of solution, so that

$$w_n = w_{n-1} = w_{n+1} = \text{etc.} \tag{258}$$

In this case, where w is a constant, an analytical method of calculation is the quickest. In the first case, where w_n depends on x_n, a graphical method is more satisfactory.

Extraction Equations for Case of Constant Underflow.—For unit n

$$W_{n-1} + w_{n+1} = W_n + w_n \tag{259}$$

If

$$w_{n+1} = w_n,$$

then

$$W_{n-1} = W_n \tag{260}$$

and the overflow to unit n equals the overflow from unit n. If a_n be defined as

$$a_n = \frac{W_n}{w_n} \tag{261}$$

then a_n is a constant as long as Equation (258) is true.

For unit m, however, Equation (258) is not usually true, because the solids to this unit originate outside of the extraction system and will not, in general, carry w_n lb. of solution. In fact, a very common case is where the solids fed to the system are dry, so that s_f and w_f are equal and x_f is unity.

For the case of constant retention of liquid by the solids *within the system*, a constant value of a can be used except for the last unit, and for this let a_m be used.

Since X_n and x_n are equal,

$$a = \frac{W_n}{w_n} = \frac{S_n}{s_n} \tag{262}$$

For the first unit,

$$S_1 = as_D \tag{263}$$

A solute balance over this unit is:

$$s_2 = S_1 + s_D - S_f = s_D(a + 1) - S_f \qquad (264)$$

For the second unit:

$$S_2 = as_2 = s_Da(a + 1) - aS_f \qquad (265)$$

and from a solute balance over units 1 and 2

$$\begin{aligned} s_3 &= s_D(a + a^2) - S_fa + s_D - S_f \\ &= s_D(1 + a + a^2) - S_f(1 + a) \end{aligned} \qquad (266)$$

Inspection of Equation (266) shows that

$$\begin{aligned} s_m = s_D(1 + a + a^2 + a^3 + \cdots a^{m-1}) \\ - S_f(1 + a + a^2 + \cdots a^{m-2}) \end{aligned} \qquad (267)$$

For unit m

$$\begin{aligned} S_m &= S_D = a_m s_m \\ S_D &= s_D a_m(1 + a + a^2 + a^3 + \cdots a^{m-1}) \\ &\quad - S_f a_m(1 + a + a^2 + \cdots a^{m-2}) \end{aligned} \qquad (268)$$

and

$$\begin{aligned} s_f &= S_D + s_D - S_f \\ &= s_D[1 + a_m(1 + a + a^2 + \cdots a^{m-1})] \\ &\quad - S_f[1 + a_m(1 + a + a^2 + \cdots a^{m-2})] \end{aligned} \qquad (269)$$

Let F be the fraction of the solute entering on the fresh solids that is unextracted. Then[1]

$$F = \frac{s_D}{s_f} = \frac{1}{1+a_m(1+a+a^2+\cdots a^{m-1})-\frac{S_f}{s_D}[1+a_m(1+a+a^2+\cdots a^{m-2})]} \qquad (270)$$

Very often, the fresh solvent is solute-free, and S_f is zero. Equation (270) becomes:

$$F = \frac{1}{1 + a_m(1 + a + a^2 + \cdots a^{m-1})} \qquad (271)$$

[1] BAKER, *Chem. Met. Eng.*, **42**, 669–671 (1935).

If, in addition, the solution entering with the fresh solids equals w_n, then a_m becomes equal to a and[1]

$$F = \frac{1}{1 + a + a^2 + a^3 + \cdots a^m} \tag{272}$$

Example 36.[2]—By extraction with kerosene 2 tons of waxed paper is to be dewaxed per day in a continuous countercurrent extraction apparatus, consisting of a number of cells. It may be assumed that equilibrium is attained in each cell before the underflow and overflow are taken off. The waxed paper contains, by weight, 25 per cent paraffin wax and 75 per cent paper pulp. The extracted paper pulp is put through a drier to evaporate the kerosene. The pulp, which retains the unextracted wax after evaporation, must not contain over 0.2 lb. wax per 100 lb. wax-free pulp. The kerosene used for the extraction contains 0.05 lb. wax per 100 lb. wax-free kerosene. Experiments show that the pulp will retain 2 lb. of kerosene per lb. of pulp as it is transferred from cell to cell. The strong solution from the battery is to contain 5 lb. wax per 100 lb. wax-free kerosene. Calculate the number of cells required.

Solution.—In the discussion on page 424 the x's were defined as pounds per pound of solution. The statement of this problem gives concentrations as pounds per pound of solvent. If the x's are to be defined in terms of the *solvent*, it is only necessary to define the w's as pounds solvent per pound solid (instead of pounds solution per pound solid) in order to have the final equations identical with Equations (269) to (272). The solution of this example will be worked in these units. The basis of the solution will be 100 lb. of wax-free pulp. A material balance is as follows:—

Wax in—on pulp	$100 \times {}^{25}\!/_{75}$	=	33.3 lb.
with solvent			$0.0005W_f$
	Total		$33.3 + 0.0005W_f$ lb.
Wax out—on pulp	100×0.002	=	0.20 lb.
in solution	$(W_f - 200)(0.05)$	=	$0.05W_f - 10$
	Total		$0.05W_f - 9.8$

Therefore

$$33.3 + 0.0005W_f = 0.05W_f - 9.8$$
$$W_f = 871 \text{ lb. solvent fed to system per 100 lb. wax-free pulp.}$$

By the conditions of the problem,

$$F = \frac{100 \times 0.002}{33.3} = \frac{1}{166.7}$$

Equation (270) will be used.

[1] HAWLEY, *J. Ind. Eng. Chem.*, **9**, 866–871 (1917); **12**, 492–496 (1920).
[2] BAKER, *loc. cit.*

$$a = 871/200 = 4.355$$

$$a_m = \frac{871 - 200}{200} = 3.355$$

$$\frac{S_f}{s_D} = \frac{871 \times 0.0005}{100 \times 0.002} = 2.178$$

These values will be substituted in Equation (270) and the value of the denominator of the right-hand side will be calculated for various numbers of cells, until a number of cells is found that gives a value for the denominator greater than 166.7. The results are

No. of cells	Value of denominator
1	4.35
2	9.48
3	41.32
4	176.5

Hence 3 cells will not fulfill the conditions of the problem, and 4 cells are a little better than required.

Extraction Diagram for Case of Varying Underflow.—When the weight of solution retained by the solid varies with the concentration of the solution, the graphical method of Ravenscroft[1] can be used. It is necessary to determine, experimentally, the relationship between w and x. The results are plotted with values of w as ordinates and corresponding values of x as abscissas, as shown by curve ab of Fig. 198. The graphical construction can be divided into two parts. The first part shows the quantities involved in the overall balances, and pertaining to the streams entering and leaving the system. The second part shows the concentrations inside the system, and is used to determine the number of units needed. For clarity of presentation, the two constructions are shown in separate figures, but in practice, a single diagram is used for both.

Figure 198 shows the graphic relations among the various terminal quantities. After the curve of x *vs.* w has been plotted, a curve of x *vs.* wx is drawn. This is curve oc. Since $s = wx$, curve oc gives the relationship of s *vs.* x. Locate point d and horizontal line de, by measuring a distance equal to s_f along the left-hand end of the chart. Locate point f and horizontal ling fg by measuring back from point d, toward the origin, a distance equal to S_D. Locate point h, and horizontal line hj by measuring up from point f a distance equal to S_f. Then the distance ho is equal to s_D, since the equation

[1] RAVENSCROFT, *Ind. Eng. Chem.*, **28**, 851–855 (1936).

$$s_f - S_D + S_f = s_D. \tag{273}$$

is a solute balance over the entire system.

The x-coordinate of point j, which is on the s *vs.* x curve, must be the value of x corresponding to $s = s_D$, or x_D, which is the concentration of the solute in the solution discarded with the exhausted solids. Furthermore, if a vertical line, jk, is drawn to curve ab, and the horizontal line km established, the

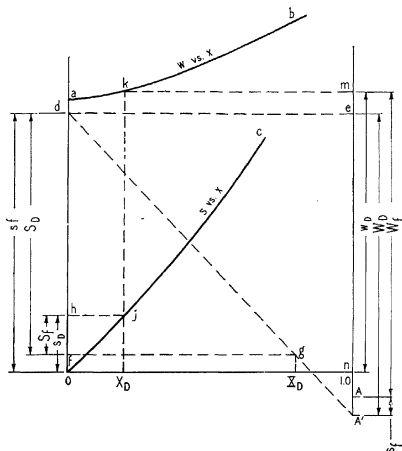



FIG. 198.—Ravenscroft method—overall quantities.

distance mn is equal to w_D, which is the weight of solution discarded with the exhausted solids. Locate point A' by measuring down from point e along the $x = 1.0$ line a distance equal to W_D, the weight of concentrated overflow leaving the system. For future use in determining the number of units required, establish point A by measuring down from point m a distance equal to W_f, the weight of fresh wash to the system.

The distance $\overline{AA'}$ is, from the line segments read from the drawing, given by the equation:

$$w_D - W_f - \overline{AA'} + W_D - s_f = 0 \tag{274}$$
$$\overline{AA'} = w_D - W_f + W_D - s_f \tag{275}$$

By an overall solution balance,

$$w_D + W_D - W_f = w_f \tag{276}$$

and

$$\overline{AA'} = w_f - s_f \tag{277}$$

The distance $\overline{AA'}$ represents, therefore, the weight of solvent in the unextracted solids entering the system. If the entering solids contain no solvent, points A and A' coincide.

As a final construction, connect points d and A' by a straight line. Let this line intersect line fg at point g. Then the abscissa of point g is X_D, the composition of the concentrated overflow leaving the unit. This fact is proved as follows:

The equation of line $A'd$ is, from inspection,

$$y = s_f - xW_D \tag{278}$$

The equation of line fg is

$$y = s_D - S_f \tag{279}$$

Eliminating y from Equations (278) and (279) gives

$$xW_D = s_f + S_f - s_D \tag{280}$$

or, by Equation (273)

$$xW_D = S_D \tag{281}$$

and therefore,

$$X = X_D \tag{282}$$

The second part of the construction, which gives the number of units required, is shown in Fig. 199. The s *vs.* x curve, and the line fg $(y = s_D - S_f)$ are shown. Draw, also, a straight line through points o and A. Assume, now, that the composition x_n of the liquid leaving in the underflow of the nth unit is known, and that it is desired to determine X_{n-1}, the composition of the liquid overflow from the next weaker unit. This quantity is determined as follows: Draw a vertical line through point x_n, intersecting line oc at p and line oA at q. Draw a horizontal

line through point p, intersecting the y-axis at r. Draw a straight line through points q and r, intersecting the line fg at point s. Then the abscissa of point s is X_{n-1}. The proof of this construction is:

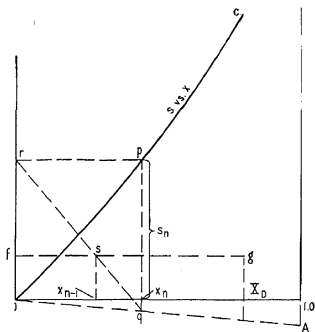



FIG. 199.—Ravenscroft method—determination of number of units.

The equation of line $\overline{oA}$ is (Fig. 198)

$$y = -(W_f - w_D)x \tag{283}$$

so that of line rsq is

$$y = s_n - \frac{[s_n + (W_f - w_D)x_n]x}{x_n} \tag{284}$$

The abscissa of point s is found by eliminating y from Equations (279) and (284)

$$x = \frac{s_n - s_D + S_f}{\frac{s_n}{x_n} + W_f - w_D} \tag{285}$$

But

$$\frac{s_n}{x_n} = w_n \tag{286}$$

and solute and total solution balances around the weak end of the battery, including units 1 to $n - 1$ inclusive, are:

$$S_{n-1} = S_f + s_n - s_D \tag{287}$$
$$W_{n-1} = W_f + w_n - w_D \tag{288}$$

From Equations (285) to (288), inclusive

$$x = \frac{S_{n-1}}{W_{n-1}} = X_{n-1} = x_{n-1} \tag{289}$$

When x_{n-1} is known, x_{n-2} is found by repeating the construction starting with a vertical through x_{n-1} and continuing as long as necessary.

To determine the number of units required, the first vertical is erected through point X_D, and triangles are constructed until a hypotenuse passes through, or to the left of, point x_D. The number of triangles so found plus one equals the number of units that must be used.

Example 37.—Oil is to be extracted from meal by means of benzene, using a continuous countercurrent extractor. The unit is to treat 2,000 lb. of meal (based on completely exhausted solid) per hr. The untreated meal contains 800 lb. of oil and 50 lb. of benzene. The fresh wash solution consists of 20 lb. of oil dissolved in 1,310 lb. of benzene. The exhausted solids are to contain 120 lb. of unextracted oil. Experiments carried out under conditions identical with those of the projected battery show that the solution retained by the solid depends on the concentration of the solution, as shown in the table:

Concentration, Pounds Oil per Pound Solution	Solution Retained, Pounds per Pound Solid
0.0	0.500
0.1	0.505
0.2	0.515
0.3	0.530
0.4	0.550
0.5	0.571
0.6	0.595
0.7	0.620

Find: *a.* The composition of the strong solution.

b. The composition of the solution adhering to the extracted solids.

c. The weight of solution leaving with the extracted meal.

d. The weight of the strong solution.

e. The number of units required.

Solution.—The first step is to plot the data given in the table, with compositions as abscissas and solution retained as ordinates. This gives

curve ab in Fig. 200, and is a plot of w *vs.* x. Next, multiply the corresponding values of the two columns of the table, and plot the product against the concentration. This gives curve oc of Fig. 200 and is a plot of s *vs.* x.

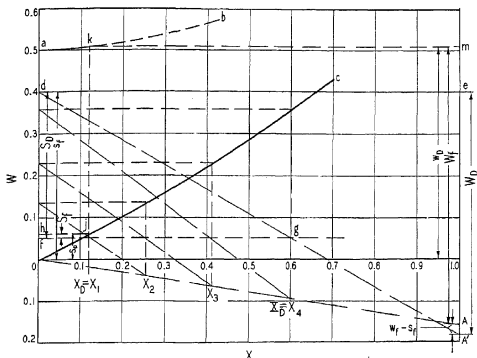



FIG. 200.—Solution of Example 37.

From the data given, these quantities are found·

$$\frac{800}{2,000} = 0.400$$

$$S_f \quad \frac{20}{2,000} = 0.010$$

$$s_D = \frac{120}{2,000} = 0.060$$

$$w_f \quad \frac{50 + 800}{2,000} \quad 0.425$$

$$W_f = \frac{1310 + 20}{2,000} = 0.665$$

From these quantities:

$$X_f = \frac{0.010}{0.665} = 0.01503$$

$$x_f = \frac{0.400}{0.425} = 0.941$$

$$w_f - s_f = 0.025$$

The value of S_D, the oil leaving in concentrated solution, can be found by plotting the known values of s_f, s_D, and S_f as shown in Fig. 200; or, also, by an oil balance over the entire unit.

$$S_D = s_f + S_f - s_D$$
$$= 0.400 + 0.010 - 0.060 \quad 0.350$$

The plotting of these quantities establishes lines fg and hj. The intersection of hj and oc establishes point j, the abscissa of which is 0.120, and this is the value of x_D, the concentration of oil in the solution leaving in the extracted solid. A vertical through point j to curve ab establishes point k, and horizontal line km is drawn. The ordinate of this point is 0.508, and this is the value of w_D, which is the weight of solution adhering to the extracted solids. The horizontal line through point k gives point m, on the $x = 1$ line, and W_f (0.665) establishes point A. By measuring a distance equal to $w_f - s_f$ (0.025) below point A, point A' is found. The distance eA' is equal to W_D, the weight of concentrated solution leaving the unit, and this is found to be 0.582. Line $A'd$ can now be drawn, and the point g found by intersection with the horizontal line through f. The abscissa of point g is 0.600, and this is the value of X_D, the concentration of the concentrated solution.

To determine the number of units required, the line oA is drawn, and the triangular construction of Fig. 199 is carried out. The number of equilibrium units is found to be four.

The answers to the problem are:

a. Composition of strong solution = 0.600 lb. oil per lb. solution.

b. Composition of solution adhering to extracted solids = 0.120 lb. oil per lb. solution.

c. Weight of solution leaving with extracted meal = (0.508)(2,000) = 1,016 lb. per hr.

d. Weight of strong solution = (0.582)(2,000) = 1,164 lb. per hr

e. The number of units is four.

Rate of Extraction.—The general principles of extraction rate correspond closely to those of other diffusion processes. Consider a particle of solid containing extractable material in contact with liquid that is being used for extracting. At any instant the concentration of the solute in the extract is c. At the same instant, the concentration of the material at the interface may be represented by c_o. This value of c_o may or may not be equal to the concentration of the material in the solid itself, depending upon the structure and nature of the solid. In any case the rate of diffusion from the interface into the liquid is proportional to $c_o - c$. A general calculation of the rate of extraction would require a knowledge of the rate of diffusion through the solid itself, as well as the film coefficient for the diffusion from the

interface into the bulk of the extract. Certain cases can be distinguished.

First, consider a solid entirely impervious and inert to the action of the solvent, with a film of strong solution on its surface. For such a case the process would involve simply the equalization of concentrations in the bulk of the extract and in the adhering film. Such a process is rapid and any reasonable time of contact will bring about substantial equilibrium. This is especially true if there is agitation and if the system is warm so that viscosity is lowered and the diffusion coefficient increased.

A second case is one where the soluble material is uniformly distributed through the interior of a permeable insoluble solid —for example, black ash or caliche. In this case, the rate of diffusion of the solute to the interface may be the controlling factor in the rate of extraction. For such cases little help is obtained by decreasing the thickness of the liquid film, but the process is greatly hastened if the solid is finely subdivided.

A third important case is that in which the solid being leached is impermeable and contains the soluble portion distributed through it in very fine particles. In such cases, the actual surface of the particles of solute may be an extremely small proportion of the total surface of the solid and the rate of extraction will be correspondingly small. Here the best method is to grind the solid to such a point that the individual particles of soluble material are released.

Still another case is that typified by such substances as sugar-beet chips and tanbark, where the soluble material is contained in cells and the process involves the diffusion of the solute through the cell walls, which act as semipermeable membranes. Such a process involves the phenomenon of osmosis and the diffusion of crystalloids through colloids. In the case of such materials the soluble constituent is, in effect, very finely subdivided throughout the mass of inert solid. It would seem, therefore, that the logical procedure would be to grind the material very fine. This would necessitate grinding the material so fine that each individual cell would be ruptured. When it is recalled that material ground to pass a 200-mesh screen represents almost the limit of fine grinding and that a particle that will pass a 200-mesh screen will contain hundreds of individual cells, it is seen that the suggestion is highly impractical. Further, in some

cases, especially that of the extraction of beet sugar, the cell contains other substances than sugar in solution. If the material were so finely subdivided that all the cells were broken, undesirable soluble impurities would pass into solution; whereas if extraction be carried out by direct diffusion, some of these impurities of high molecular weight are held back. For material of this sort, the degree of subdivision should not be so great that the chips will form a mass that cannot be penetrated by the solvent. At the same time the chips should have the largest pos ible surface to increase the area of contact between solvent and solid. The diffusional processes are accelerated by carrying out the operation at as high a temperature as possible, and any surface-film resistance is decreased by making the velocity of the solvent high, with correspondingly great turbulence and decrease in the thickness of the stagnant film. If the velocity of the solvent past the chips is high, the necessary time of contact for producing high-concentration solutions can be made sufficiently great only by having the path of the solvent correspondingly long. This is accomplished in diffusion batteries by placing a considerable number of cells in series. The extraction of tanbark proceeds rather rapidly, the velocity of the solvent need not be so high, and 4 to 6 cells are usually sufficient. In the extraction of beet sugar the diffusional processes are slow, the velocity of the solvent is high, and from 8 to 14 cells must be placed in series to give sufficient time of contact.

Quantitative data on the rate of extraction of different materials, which might form the basis of rational calculations for extraction systems, are practically non-existent. Consequently, only the above qualitative statements can be made and it is not possible to derive quantitative expressions. In the various industries employing extraction processes, the size of apparatus and time of contact are determined by empirical experience.

Nomenclature of Chapter XI

a = ratio of solvent retained to solvent drawn off
F = fraction of solute entering on the fresh solids that is unextracted
m = number of units in battery
S or s = solute in the stream
W or w = weight of the stream
X or x = composition of the stream
sub D = discharge from system

sub f = in feed
sub m = in last unit
sub n = nth unit from the solvent end

Problems

1. Barium carbonate is to be made by the interaction of sodium carbonate on barium sulfide solution in a countercurrent decantation process. There are fed to the reaction agitators, per 24 hr., 40,000 lb. of BaS dissolved in 60 tons of water, and the necessary amount of Na_2CO_3. Overflow from the second thickener also goes to the agitators. There are five thickeners in series. The overflow from the first thickener is to be a 10 per cent solution of Na_2S. Mud is discharged from the thickeners with a consistency of one part $BaCO_3$ to two parts water by weight. What is the concentration of Na_2S in each thickener, and how much Na_2S will remain in the dried $BaCO_3$ precipitate?

2. A plant makes caustic soda by causticizing soda ash with lime. The reaction is carried out in batches. A number of settling tanks are provided. These are vertical cylindrical tanks, 6 ft. in diameter, with a 60-deg. cone bottom. One batch contains 1 ton total NaOH and fills a tank to a height of 9 ft. on the straight section. After 10 hr. settling, 8 ft. of clear liquor may be drawn off and sent to the evaporators. The material remaining in the tank is stirred with enough fresh water to bring the batch to the original level, settled 10 hr., and 8 ft. of wash liquor are drawn off. In all, four such washes are used, each with fresh water, and these washes are combined. Part of this is used to make up the batches, and the rest is sent direct to the evaporators. Two complete batches are made per day, and the evaporator produces a 50 per cent NaOH solution. The specific gravity of all washes, both individually and combined, is 1.0. The specific gravity of the solution in a finished batch is 1.10, and that of the $CaCO_3$ precipitate is 2.20. What is the weight of water evaporated per ton of caustic made?

3. In the plant of Problem 2, it is proposed to replace the four washes with fresh water by four countercurrent washes. The total caustic made per day is to be the same as before. All specific gravities of solutions and settling rates may be assumed to be substantially as before. What effect will the change have on (*a*) caustic lost in the final mud and (*b*) water evaporated per ton of caustic?

4. The problem of Example 37 is to be modified as follows: The amount of meal fed and the concentration of oil and solvent in it are to be unchanged; the amount and composition of the solvent added are to be unchanged; the extraction battery is to contain five units instead of four. What will now be (*a*) the weight of solution produced, (*b*) the concentration of oil in this solution, and (*c*) the oil lost in the outgoing meal?

CHAPTER XII

CRYSTALLIZATION

The crystallization of substances on the commercial scale involves factors that do not often appear in work carried out on the laboratory scale. For purely scientific work practically the only requirements are that the yield be as high as possible and the crystals pure. These same requirements are important in commercial crystallization, but in addition there are certain requirements as to size, range of sizes, and shape. Oftentimes these demands are not intelligent. For example, large crystals of certain substances are demanded where smaller crystals would be as satisfactory as, or more so than, the type actually in demand. In some cases, definite crystal shapes are desired—for example, needles. In other cases it is difficult to market crystals that are badly agglomerated, that is, where the individual particle consists of several crystals cemented together. The purchaser often imposes specifications that limit the product to a narrow range of sizes. Consequently, commercial crystallization processes must be examined, not only from a theoretical point of view but also from the standpoint of these special demands.

Crystal Forms.—The only logical and accepted method for the classification of crystals is according to the angles between the faces, and this is the domain of the science of crystallography. In this system the types of crystal form have no relationship to the relative sizes of the faces, since the relative development of the faces is not a constant that is characteristic of a specific material. Any substance, however, always crystallizes in such a way that the angle between a given pair of faces is the same in all specimens and is characteristic of that particular substance. For example, the cubic system is characterized by the fact that its faces can be referred to three equal axes, each at right angles to the other two.

One simple set of faces that may be referred to this system is the set at right angles to the axes—the faces of an ordinary cube.

The actual crystal may be a symmetrical cube, or it may be a needle, a plate, or an aggregate of imperfect crystals. As long as the faces are at 90 deg. to each other, crystallographically it is a cube. The same holds true of any other set of faces that may be referred to the same axes, such as the octahedral faces, the dodecahedral faces, and so on. In any case, it is not relative extent of the faces or the superficial form of the crystal that forms the basis of the classification, but only the angles between the faces.

There are six classes of crystals, depending on the arrangement of the axes to which the angles are referred. These classes are:

Cubic. Three equal axes at right angles to each other.

Tetragonal. Three axes all at right angles, one longer than the other two.

Orthorhombic. Three axes all at right angles, but all of different lengths.

Hexagonal. Three equal axes in one plane at 60 deg. to each other, one at right angles to this plane but not necessarily the same length as the others.

Monoclinic. Two axes at right angles in one plane, and a third axis at some odd angle to this plane.

Triclinic. Three axes at odd angles to each other.

Crystal Habit.—The term crystal habit is used to denote the relative development of the different types of faces. For example, sodium chloride crystallizes from aqueous solutions with cubic faces only. On the other hand, if sodium chloride is crystallized from an aqueous solution containing a small amount of urea, the crystals obtained will have octahedral faces. Both types of crystals belong to the cubic system but differ in habit. The question of whether a material crystallizes in symmetrical crystals, in plates, or needles, or prisms is usually an accident resulting from the conditions under which it is grown and has no relation to either its crystallographic classification or its habit. The word *habit* is sometimes incorrectly used to designate these features of external form, but when properly used it refers to the type of faces developed and not to the shape of the resulting crystal.

CRYSTALLIZATION THEORY

The theory of crystallization involves not only the equilibrium relationships and the rate of crystal growth but also another process, namely, the formation of the crystal. The tiny crystal, at or soon after its formation, is called a *crystal nucleus*. In the

following there will be considered, first, the equilibrium relationships; second, the formation of crystalline nuclei; and, third, the growth of the crystal.

Equilibria and Calculation of Yields.—Any solid in contact with its solution will dissolve if the solution is not saturated or will grow at the expense of the solution if the solution is supersaturated. In any case, the equilibrium is represented by the solubility of the solid in the solvent. In general, solubility is

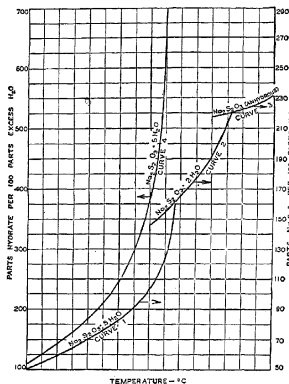



Fig. 201.—Solubility of sodium thiosulfate.

dependent mainly on temperature, although it is slightly dependent upon the size of the material and the pressure. For all ordinary cases, however, these last two factors are negligible and solubility data are given in the form of curves where solubilities in some convenient unit are plotted against temperature.

Some substances give a continuous solubility curve without breaks throughout the entire length of the curve in the range of temperature considered. The solubility curves of other substances, however, show sharp breaks. For example, the curve of sodium thiosulfate or "hypo" is shown in Fig. 201. The

branches of the curve represented by 1, 2, and 3 show the solubility of this material when the concentrations are expressed in parts of $Na_2S_2O_3$ per 100 parts of water. This method of expressing concentrations in terms of the anhydrous salt, even if the solid phase is hydrated, is usual. The breaks in the curve correspond to transitions in the composition of the stable solid phase. Between 0 and 48.2°C. the stable phase is the pentahydrate $Na_2S_2O_3 \cdot 5H_2O$; from 48.2 to 66.5°C. the dihydrate $Na_2S_2O_3 \cdot 2H_2O$ is the stable solid; while above 66.5° it is the anhydrous thiosulfate that exists in stable form in contact with the solution.

For example, if a hot solution containing 90 parts of $Na_2S_2O_3$ per 100 parts of water is cooled, the first crystal that is formed (assuming that the solution does not become appreciably supercooled) will appear at a temperature of 34.5°C., and this crystal will be a pentahydrate crystal. If a solution containing 190 parts of $Na_2S_2O_3$ per 100 parts of water is cooled to 60°C., crystals of the dihydrate will begin to form, while a solution containing 230 parts of $Na_2S_2O_3$ will precipitate crystals of the anhydrous salt at a temperature of 79°C.

Most crystallization processes are carried out so slowly, and the surface of solid material in contact with the solution is so great, that at the end of the process the mother liquor is saturated at the terminal temperature. It is true for some materials that crystallize very slowly, such as cane sugar, the solution can support a considerable supersaturation even when in contact with solid sugar for an appreciable length of time. In general, however, the final concentration of the mother liquor can be taken as that read from the solubility curve. The yield of a crystallization process can therefore be calculated from the solubility data, if the initial concentration and the final temperature of the solution are known.

The calculation of the yield in the case of a substance that comes out in the anhydrous form is simple. It is necessary only to take the difference between the initial composition of the solution and the solubility corresponding to the final temperature to get the yield. Both concentrations must be expressed in terms of the water content of the solution and not in per cent solids. A correction must be made for evaporation, if this takes place.

If the material precipitates as a hydrated salt, this simple method of calculation will not be correct, since the solid salt brings with it a definite amount of water that does not remain in the mother liquor and therefore cannot dissolve any of the soluble material. The key to calculations of such a process is to express all compositions in terms of hydrated salt and free or excess or solvent water, since it is this latter quantity that remains constant during the crystallization process, and compositions expressed on the basis of this free water can be subtracted to give a correct result. In Fig. 201, curve 4 is the curve for the pentahydrate calculated in this manner.

Example 38.—What will be the yield of hypo ($Na_2S_2O_3 \cdot 5H_2O_3$) if 100 lb. of a 60 per cent solution is cooled to 20°C.?

Solution.—The initial solution contains 60 parts $Na_2S_2O_3$ in 40 parts water. From Fig. 201, curve 1, the solubility at 20°C. is 70 parts $Na_2S_2O_3$ per 100 parts water. These figures must be converted to parts $Na_2S_2O_3 \cdot 5H_2O$ per 100 parts excess water, by using the ratio of the molecular weights of the anhydrous and hydrated salts

$$Na_2S_2O_3/Na_2S_2O_3 \cdot 5H_2O = {}^{158}/_{248}$$

	Total water, lb.	$Na_2S_2O_3$, lb.	$Na_2S_2O_3 \cdot 5H_2O$, lb.	Hydrate water, lb.	Excess water, lb.	Hydrate per 100 parts excess water, lb.
Initial solution.	40	60	94.2	34.2	5.8	1,625
Final solution..	100	70	109.9	39.9	60.1	183

In cooling from the initial to the final temperature, the excess water is not affected and is, therefore, a valid basis of comparison. The figures in the last column may be subtracted to obtain the yield

$$1{,}625 - 183 = 1{,}442 \text{ lb.}$$

This is obtained from sufficient 60 per cent solution to contain 100 lb. excess water or 1,625 + 100 = 1,725 lb. solution. The yield is 1,442 lb. crystals from 1,725 lb. solution, or 83.6 lb. crystals from 100 lb. 60 per cent solution.

Nucleus Formation.—If a solution is very carefully protected against dust, especially of the solute, it can be cooled to a definite temperature below that at which the solution is saturated, before new crystals will form. The degree of supersaturation obtainable varies with the solute. For sodium chloride, it is negligible; for a solution of $MgSO_4 \cdot 7H_2O$, a supercooling of about 17°C.

can be supported without nucleus formation; for cane sugar, very great, but unknown supersaturations can be maintained indefinitely. In order to obtain reproducible results in such experiments, however, great precaution must be taken to start with a clear solution. Under industrial conditions, the solution will be "seeded" with small dust crystals of the solute, and crystallization will start at much lower supersaturations than in the case of a protected solution. The effect of the presence of seeds on crystal formation is the most important commercial factor, especially in view of the fact that even if the original solution were carefully protected, it would be seeded from the moment of the formation of the first crystal.

It has been shown[1] that if seeded solutions are cooled at a definite rate at a definite concentration, in the presence of seeds of a definite size and number, and under definite stirring conditions, the supersaturation at the moment of the formation of new crystals is fixed and reproducible. This supersaturation varies, however, with all of the factors mentioned above. Also, it has been found that, if a solution of definite supersaturation is seeded with crystals, new nuclei will eventually form, even if the supersaturation is quite low. The higher the supersaturation, the more quickly will new crystals form, and the smaller they will be. In time, of course, the concentration of the solution drops to saturation, and equilibrium is reached.

These facts indicate that in a crystallization process, nucleus formation and crystal growth are parallel processes, and that it is difficult to obtain one without the other. The net result depends on the relative rates of the two processes—nucleation and growth. For low supersaturations, both are slow, but formation is relatively slower than growth, so that large crystals are obtained. For large supersaturations, both processes are accelerated but nucleation becomes faster relative to growth, and small crystals will be formed.

In practice, the supersaturation is largely determined by cooling rate. At high rates, the supersaturation is large. At low rates, the supersaturation is small. Quick cooling results in small crystals, and slow cooling gives large crystals.

There are undoubtedly a number of other factors involved in the relative rates of the two processes in question. Some

[1] TING AND MCCABE, *Ind. Eng. Chem.*, **26**, 1201–1207 (1934).

of these are: stirring rate, average distance between crystals, total surface available on existing crystals, and purity of the solution. The action of all of these factors is greatly modified by the nature of the solute. At present no quantitative treatment of these factors is possible.

Rate of Growth of Crystals

The information available on the rate of growth of a crystal after it has been formed is more definite than is that on the formation of the crystal itself. It can be seen that if a crystal is suspended in a solution and is growing at the expense of dissolved solute in the solution, the solute must be diffusing to the crystal faces. The mechanism of this diffusion process has been discussed in Chap. VI. Since the diffusion is taking place through a liquid film, the driving force of the diffusion process is the difference in concentration between the bulk of the solution and the concentration of the liquid at the crystal interface, while the resistance is the diffusional resistance of the film of solution adjacent to the crystal face. Considerable controversy has taken place over the question whether or not the only factor influencing the resistance is the diffusional resistance of the film itself, whether there is an additional phenomenon taking place at the interface, or whether the diffusional resistance of the film is of any importance or not. The bulk of the evidence seems to indicate that the diffusional resistance is followed in series by a process at the interface, and the net rate of crystallization is a function of the two processes acting in series. This is supported by the experimental fact that the rate of growth of the various faces of a single crystal is not the same for all faces. If the liquid film were the only factor, all faces of the same crystal should grow at the same rate under identical conditions, since all faces would be in contact with a film of the same characteristics. If, however, each face possesses a specific rate of growth, the overall rate will vary from face to face, and this has been confirmed by experiment. The ratio of these rates is constant, and therefore if a crystal is formed and grows under constant external conditions, it will remain substantially similar in shape, geometrically, as it grows.

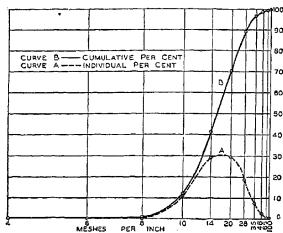
The specific rate-of-growth constants of the faces of crystals have been approximately determined for some substances under

some conditions. These specific constants are too little known, however, to permit of any extensive practical use. At any rate, the overall rate depends in some manner on the overall concentration difference between the bulk of the solution and the saturation concentration of the material at the temperature of the interface.

Screen Analyses.—Crystalline materials, in common with other solids of small size, are generally specified according to their screen analysis. A screen analysis of a material is carried out by placing a sample on the coarsest of a set of standard screens. The characteristics of these screens are given in Appendix VIII. Below this screen are arranged the remaining screens in the series in the order of decreasing size of mesh. The meshes are carefully standardized and the dimensions of the openings accurately known. The pile of screens with the sample on the top screen is shaken in a definite manner, either manually or mechanically, for a definite length of time, and the crystals collected on each screen are removed and weighed. If a plot is made of the percentage of the original sample retained on each screen against the size of that screen, and a curve drawn through the points, the result is known as the *differential screen analysis*. Curve A in Fig. 202 represents such a differential screen analysis. For example, 0.6 per cent of the sample was retained on a screen having 8 meshes to the inch, 11.3 more was retained on the next screen which had 10 meshes to the inch, 29.3 per cent was retained on the 14-mesh screen, 29.5 per cent was retained on the 20-mesh screen, and so on.

The second method of plotting a screen analysis is called the *cumulative screen analysis*. This curve is represented by curve B in Fig. 202. The points on this curve are determined by adding together all of the fractions that have been retained up to and including that particular size. Thus the cumulative point for the 14-mesh screen is given by adding 0.6, 11.3, and 29.3, giving a total of 41.2. The ordinate of any point on a cumulative curve represents the total per cent of original sample, the particles of which are larger than the size represented by the abscissa of that point. Every point on the cumulative curve can be so interpreted. On the differential curve, however, only the points corresponding to the actual mesh size have any significance.

Calculation of Screen Analysis of Product.—The problem[1] considered in this section may be stated as follows:

Given a saturated solution in which is suspended a known weight of seed crystals of known screen analysis, and assuming this solution to be cooled under known conditions, what will be the weight and screen analysis of the crystals at the end of the process if there is negligible formation of new nuclei?

TYPICAL SCREEN ANALYSIS

SCREEN SCALE RATIO 1.414		Per cent retained on screen	Per cent cumulative weights
Openings, inches	Meshes per inch		
0.131	6	0.0	0.0
0.093	8	0.6	0.6
0.065	10	11.3	11.9
0.046	14	29.3	41.2
0.0328	20	29.5	70.7
0.0232	28	18.5	89.2
0.0164	35	7.6	96.8
0.0116	48	2.4	99.2
0.0082	65	0.7	99.9
0.0058	100	0.1	100.0

Fig. 202.—Screen-analysis curves.

This problem is of importance because the more modern commercial crystallization processes are carried out by first forming nuclei and then allowing them to grow by slowly cooling the saturated solution in which they are suspended.

The solution of the problem proposed depends on the answers to two questions:

1. How much material is precipitated?

2. How does the precipitating material distribute itself on the seed crystals?

The answer to the first question will determine the yield and has been discussed on page 440. The answer to the second question will determine the size distribution and the screen analysis of the product.

[1] McCabe, *Ind. Eng. Chem.*, **21**, 30–33, 112–119 (1929).

Growth of a Single Crystal.—Consider a single crystal that is growing in a slowly cooled solution and kept in suspension by a suitable agitator. Let c denote the concentration of the bulk of the solution and c_o the saturation concentration corresponding to the temperature of the crystal. Then the supersaturation is $c - c_o$. Each face of the crystal is, at any instant, growing at a definite rate that can be measured in grams per square centimeter per hour. The rate of growth of a face is proportional to some function of the supersaturation. The form of the function is unknown, since the mechanism of the interfacial reaction is obscure. It will be shown that the form does not have to be known. A differential equation can be written for each face. The equations for various faces are

$$\begin{aligned} \frac{dW_1}{d\theta} &= k_1S_1[f(c - c_o)] \\ \frac{dW_2}{d\theta} &\quad k_2S_2[f(c - c_o)] \\ \frac{dW_n}{d\theta} &= k_nS_n[f(c - c_o)] \end{aligned} \qquad (290)$$

where

$W_1, W_2 \quad . \; W_n$ = weights of material depositing on the different faces.
$S_1, S_2 \quad . \; . \; S_n$ = areas of the faces.
$k_1, k_2 \quad . \; . \; k_n$ = overall reaction-rate constants of the faces.
θ = time.
$c - c_o$ = supersaturation of the solution surrounding the crystal.
n = total number of faces.

Since the crystal is growing under uniform external conditions, it is at all times geometrically similar to its original shape.

If S is the total area of the crystal at the time θ, then

$$\left.\begin{aligned} S_1 &= a_1S \\ S_2 &= a_2S \\ \\ S_n &= a_nS \end{aligned}\right\} \qquad (291)$$

where

$$a_1 + a_2 + \qquad a_n = 1.$$

Because of the constancy of the crystal shape, the fractions $a_1, a_2 \ldots a_n$ are constants during the growing process.

If the values of $S_1, S_2 \ldots S_n$ of Equation (291) are substituted into Equation (290) and the resulting equations added together, it is found that

$$\frac{dW_1 + dW_2 + \cdots dW_n}{d\theta} = \frac{dW}{d\theta} = (k_1a_1 + k_2a_2 + \cdots k_na_n)(S)[f(c - c_o)] \qquad (292)$$

where dW is the total increase in weight of the crystal.

If L is a linear distance between any two corners of a polyhedron, the weight and surface of that polyhedron are given by the expressions

$$W = \rho b L^3 \tag{293}$$
$$S = eL^2 \tag{294}$$

where

ρ = density of the polyhedron.
b and e = constants that are equal for all geometrically similar solids.
L = distance between corresponding points of the individual solids.

Since the crystal under consideration is growing under constant external conditions and the geometrical shape does not change, Equations (293) and (294) apply to all stages of the growth of the crystal and b and e are constants during the process.

From Equation (293),

$$dW = 3\rho b L^2 dL \tag{295}$$

Substituting values of S and dW from Equations (294) and (295) into Equation (292), the result is

$$\frac{dL}{d\theta} = \frac{(k_1a_1 + k_2a_2 + \cdot \cdot \cdot k_na_n)(e)[f(c - c_o)]}{3b\rho} \tag{296}$$

Since $k_1, k_2 \ . \ . \ . \ k_n, a_1, a_2 \ . \ . \ . \ a_n, \rho, b$, and e are all constants, Equation (296) can be written as

$$\frac{dL}{d\theta} = K[f(c - c_o)] \tag{297}$$

where K is a constant and is equal to $\frac{(k_1a_1 + k_1a_2 \ . \ . \ . \ k_na_n)(e)}{3b\rho}$.

The ΔL Law.—Consider, now, a second crystal of the same material, of the same geometric shape, and suspended in the same solution, but of different size from the first. If a differential equation is derived for this second crystal, it will be identical with Equation (297) if the L's of the two crystals connect geometrically corresponding corners. From this fact there follows an important principle:

All geometrically similar crystals of the same material suspended in the same solution grow at the same rate, if the growth is measured as the increase in length of geometrically corresponding distances on all of the crystals.

Two assumptions are implied in the above analysis. The first assumption is that c_o has the same value for all the crystals. The only possibility of variation of c_o from crystal to crystal is the change in solubility with particle size. This assumption is valid since, unless the crystals are very small, such solubility differences are negligible.

The second assumption is that it is possible to maintain a number of crystals under nearly enough identical conditions so that consistent results can be obtained. Although it is difficult to do this with only a few crystals, when there are a large number of crystals an averaging effect can be expected.

Since every crystal grows at the same rate, the total growth of all the crystals will be the same. If ΔL is the increase in a linear dimension of one crystal, it is at the same time equal to the increase in the corresponding dimension of each of the other crystals and is independent of the initial size of any of the original crystals.

This conclusion leads to a theoretical solution of the problem of predicting the screen analysis of the product if the screen analysis of the seeds and yield are known. The detailed method of solution of this problem follows:

Curve of Number of Crystals vs. Size.—Consider the cumulative analysis of 100 g. of seeds plotted as shown in Fig. 203. The significance of this plot is: The abscissa of any point on the curve represents the weight of crystals that will not pass through an opening of the size indicated by the

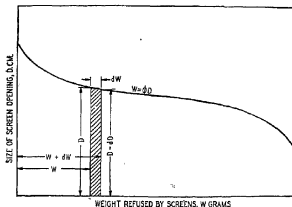



Fig. 203.—Plot of D vs. W.

ordinate of that point. If D is the size of the opening of a sieve and W is the weight of material that will not pass through this sieve, the plot of the cumulative screen analysis is the graph of the curve:

$$W = \phi(D) \tag{298}$$

where

$$\phi = \text{some unknown function}$$

There will be a definite size of crystal that will just pass through a given screen. This crystal will have a definite linear dimension L. If all the crystals are geometrically similar, there will be a definite ratio between the value of L of any particular crystal and the size of the screen opening that will barely allow that crystal to pass. Assume that this ratio is the same for all the sieves. Mathematically, this assumption states:

$$\alpha D = L \tag{299}$$

where α is a constant and is independent of D.

It will be desirable to obtain from the cumulative screen-analysis plot a cumulative plot of the number of crystals N against D. The abscissa of any point on this curve will represent the number of crystals that will not

pass through a sieve of size opening equal to the ordinate of that point. Assume that such curves have been drawn for the seeds and the product and that the same axes have been used for both plots. These curves are presented in Fig. 204. Consider seed crystal n. It will just pass through an opening D_1 cm. on a side. After growth it will just pass through an opening D_2 cm. on a side. Let ΔD represent this increase in D. Then, from Equation (299),

$$\alpha \Delta D = \alpha(D_2 - D_1) = \Delta L \tag{300}$$

But α is a constant, and ΔL is identical for all the crystals. The vertical distance between the two curves of Fig. 204 is therefore constant for all points on the curves, and if one curve and the ΔD of one crystal are known, the other curve can be constructed.

The construction of the N *vs.* D curves will now be described.

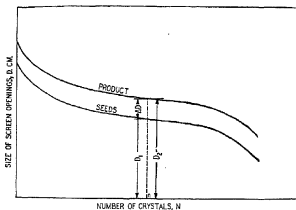



FIG. 204.—Plot of N *vs.* D.

Consider, first, any point on the cumulative screen plot of the seeds. This point has the coordinates W and D (Fig. 203). These coordinates mean that, of the 100 g. of seeds, W g. will not pass through a square opening measuring D cm. on a side.

Consider, next, a point very near to the point (W, D). The coordinates of this second point are $W + dW$ and $D - dD$. Then, dW g. of seeds can be trapped between two screens having opening sizes of D and $(D - dD)$ cm., respectively. The average linear dimension of the crystals in the fraction dW is L, where $L = \alpha D$, the weight of a single crystal is $\rho b L^3$, and the number of crystals in the fraction is

$$dN = \frac{dW}{\rho b L^3} = \frac{dW}{\rho \alpha^3 b D^3} \tag{301}$$

The total number of crystals too large to pass through a screen opening D cm. on a side is

$$N = \frac{1}{\rho \alpha^3 b} \int_0^W \frac{dW}{D^3} \tag{302}$$

In order to obtain the arithmetical value of N, the quantity $\alpha^3 \rho b$ must be evaluated. Although the density of the crystals ρ is usually known, the

determination of a and b is bothersome. For the purpose of predicting screen analyses, however, it is unnecessary to evaluate these constants. The plots are just as useful if N', defined by the equation

$$N' = a^3 bN \tag{303}$$

is plotted against D, and Equation (302) used in the form

$$N' = \frac{1}{\rho}\int_0^W \frac{dW}{D^3} \tag{304}$$

Coordinates of points on the N' *vs.* D curve corresponding to any given screen analysis can be determined by graphical integrations of Equation (304), since the cumulative screen-analysis curve gives the relationship between W and D.

Application of the ΔL *Law.*—Equation (304) can be written in the differential form for the seed crystals:

$$dN_s' = \frac{dW_s}{\rho D_s^3} \tag{305}$$

and for the product:

$$dN_p' = \frac{dW_p}{\rho(D_s + \Delta D)^3} \tag{306}$$

where the subscripts s and p refer to the seeds and product, respectively.

Since the same crystals form both seeds and product,

$$dN_s' = dN_p'$$

and, therefore,

$$\frac{dW_s}{D_s^3} = \frac{dW_p}{(D_s + \Delta D)^3} \tag{307}$$

it follows from Equation (307) that

$$W_p = \int_0^{W_s}\left(1 + \frac{\Delta D}{D_s}\right)^3 dW_s \tag{308}$$

Since ΔD is independent of D_s and W_s, the value of W_p corresponding to any value of W_s can be obtained by graphical integration of Equation (308), inasmuch as the screen-analysis curve of the seeds provides a relationship between D_s and W_s.

The steps in the calculation of the yield and screen analysis of the product of a crystal-growth process are:

1. Calculate the theoretical yield from the ratio of seeds to solution and the solubility change of the material during the process.

2. Assume a value of ΔD and calculate the weight of the product that corresponds to this assumed value by integrating Equation (308) over the range $W_s = 0$ to $W_s = 100$.

3. If the value of W_p calculated in step 2 does not check that calculated in step 1, adjust ΔD by trial and error until fair agreement is reached between the weights of product calculated by the two methods.

4. Using the correct value of ΔD, as determined in step 3, plot the integral curve of Equation (308).

5. Plot W_p against D_p. This plot can be constructed from the data at hand at this point; since the integral curve plotted in step 4 gives the relationship between W_p and W_s, the screen-analysis curve of the seeds exhibits D_s as a function of W_s, and $D_p = D_s + \Delta D$.

6. From the curve of W_p against D_p that was plotted in step 5, read off the values of W_p that correspond to the sizes of the various screen openings, and convert the values to percentages of the entire product. The result is the cumulative screen analysis of the product. The differential analysis is easily derived from the cumulative analysis by subtraction.

The above procedure is illustrated by the following example:

Example 39.—A Swenson-Walker crystallizer is cooling potassium chloride solution. It discharges a saturated solution at 60°C. The discharge consists of 1,750 lb. of saturated solution and 165 lb. of crystals per hr. The screen analysis of the crystals is as follows:

Meshes per inch	Size of screen opening, centimeter	Screen analysis, per cent	
		Differential	Cumulative
On 12	0.1397	0.0	0.0
14	0.1168	0.1	0.1
16	0.0991	2.9	3.0
20	0.0833	12.7	15.7
24	0.0701	13.0	28.7
28	0.0589	25.8	54.5
32	0.0495	19.6	74.1
35	0.0417	13.3	87.4
42	0.0351	6.3	93.7
48	0.0295	3.6	97.3
60	0.0246	1.0	98.3
65	0.0208	1.2	99.5
Through 65	(0.0175)[1]	0.5	100.0

[1] Estimated.

What would be (*a*) the weight and (*b*) the screen analysis of the product if additional sections were added to the crystallizer to cool the above product from 60 to 30°C.?

The solubility of potassium chloride at 60°C. is 45.0 parts per 100 parts water, and the solubility at 30°C. is 37.35 parts per 100 parts water.

Solution.—The answer to question (*a*), namely the new crop weight, is computed from the solubility data. Since the potassium chloride that precipitates is in the anhydrous form, and since the solubilities given above are based on weight of water, the increased yield due to the cooling from 60 to 30°C. is $45.0 - 37.35 = 7.65$ lb. per 100 lb. of water. Since the crystallizer handles $(100/145)(1{,}750) = 1{,}206$ lb. of water, the increase in yield is $(12.06)(7.65) = 92.3$ lb. The product from the new section will be, then, $92.3 + 165 = 257.3$ lb. of crystals per hr. This is the answer to question (*a*).

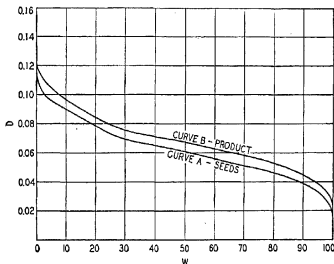



FIG. 205.—Data for Example 39.

In order to determine the screen analysis of the new product, the procedure of page 451 is applied. The screen analysis of the seed crystals is given in the data of the problem. The second and fourth columns of the table are D_s and W_s, respectively, based on 100 lb. of seed crystals. The product is $(257.3/165)(100) = 156$ lb. on this same basis. Step 1 of the procedure has been completed.

The second step is the trial-and-error determination of the correct value of ΔD that corresponds to the required weight increase of 56 lb. per 100 lb. of seeds. This is done by assigning a value to ΔD, evaluating the integral of Equation (308) between the limits $W_s = 100$ and $W_s = 0$, and thereby calculating a corresponding value of W_p. The required value of ΔD is that value that gives a value of W_p of 156.

Without reproducing the preliminary trials, it is found that if ΔD is 0.009 the yield requirement is closely met. This value will be shown to be correct in the next step.

The third step is to substitute $\Delta D = 0.009$ in Equation (308) and calculate the integral curve of the equation. This is done by plotting values of $\left(1 + \frac{0.009}{D_s}\right)^3$ as ordinates against W_s as abscissas. The relationship between D_s and W_s is obtained by plotting the second column of the screen-

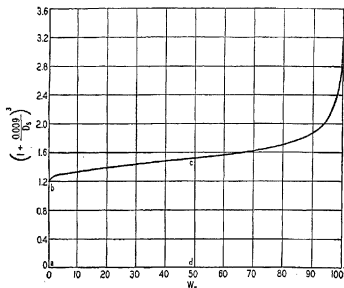



FIG. 206.—Plot of Equation (308) for Example 39.

analysis table against the fourth column. This curve is curve *A* of Fig. 205. The coordinates of the $\left(1 + \frac{0.009}{D_s}\right)^3$ *vs.* W_s curve are as follows, and the curve is shown in Fig. 206:

Coordinates of W_s *vs.* $\left(1 + \frac{0.009}{D_s}\right)^3$ cu

W_s	D_s	$\left(1+\frac{0.009}{D_s}\right)^3$	W_s	D_s	$\left(1+\frac{0.009}{D_s}\right)^3$
0.0	0.1397	1.21	50	0.0611	1.51
0.1	0.1168	1.25	60	0.0563	1.56
3.0	0.0991	1.30	70	0.0514	1.62
5.0	0.0960	1.31	80	0.0466	1.70
10.0	0.0901	1.33	90	0.0391	1.86
20.0	0.0789	1.38	95	0.0330	2.06
30.0	0.0695	1.44	98	0.0270	2.37
40.0	0.0653	1.47	100	0.0175	3.47

The integral curve of Equation (308) is obtained by determining the area bounded by the curve of Fig. 206, the *X*-axis, the ordinate $X = 0$, and the ordinate $X = W_s$, where W_s varies from 0 to 100. For example, the

point on the integral curve with an abscissa of $W_s = 50$ is equal to the area *abcda* of Fig. 206.

The coordinates of points on the integral curve, which are corresponding values of W_p and W_s, are

W_s	W_p	W_s	W_p
5	6.4	70	101.5
10	13.0	80	118.0
20	26.6	85	126.7
30	40.8	90	135.8
40	55.4	95	145.5
50	70.3	97.5	151.0
60	85.7	100	157.4

It will be seen that when $\Delta D = 0.009$, $W_p = 157.4$, which is a satisfactory check with the theoretical yield of 156.

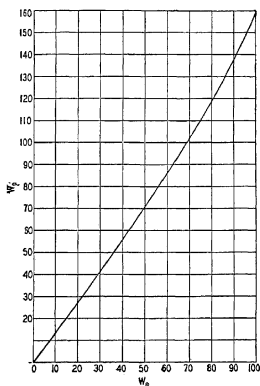



FIG. 207.—Integral curve, Example 39.

The plot of W_p *vs.* W_s is shown in Fig. 207.

The next step, the determination of W_p *vs.* D_p (the size distribution of the product), can now be carried out. It is convenient to assign values to D_p

equal to the actual sizes of openings of the screens, obtain the corresponding values of D_s by subtracting 0.009 from these D_p values, read the corresponding values of W_s from curve A of Fig. 205, and then read the values of W_p from Fig. 207. The cumulative and differential analyses are readily calculated from this. This procedure is shown in the following table:

Meshes per inch	D_p	$D_s = D_p - 0.009$	W_s	W_p	Screen analysis of product	
					Cumulative	Differential
On 12	0.1397	0.1307	0.0	0.0	0.0	0.0
14	0.1168	0.1078	0.6	1.0	0.6	0.6
16	0.0991	0.0901	10.0	13.0	8.3	7.7
20	0.0833	0.0743	23.7	32.4	20.8	12.5
24	0.0701	0.0611	49.9	70.3	44.6	23.8
28	0.0589	0.0499	73.2	107.3	68.2	23.6
32	0.0495	0.0405	88.5	134.0	85.1	16.9
35	0.0417	0.0327	95.1	145.7	92.5	7.4
42	0.0351	0.0261	96.2	152.5	96.8	4.3
48	0.0295	0.0205	99.5	156.2	99.2	2.4
60	0.0246	0.0156	100.0	157.4	100.0	0.8

The cumulative analysis of the product is shown as curve B, Fig. 205.

CRYSTALLIZATION APPARATUS

Crystallization is one of the unit operations that has been very little studied and in which the development of commercial apparatus has followed relatively few and simple lines. The apparatus may be classified as follows:

1. Batch or discontinuous apparatus.
   a. Atmospheric cooling, no stirring.
   b. Artificial cooling, with stirring.
2. Continuous apparatus.
   a. Swenson-Walker.
   b. Wulff-Bock.
3. Vacuum crystallizers.

Tank Crystallization.—For many years the common practice in producing crystals has been to prepare hot, nearly saturated solutions and run these solutions into open rectangular tanks in which the solution stood while it cooled and deposited crystals. No attempt is usually made to seed these tanks, to provide

for agitation, or to accelerate or control the crystallization in any way. Sometimes rods or strings are hung in the tanks to give the crystals additional surface on which to grow and to keep at least a part of the product out of the sediment that might collect in the bottom of the tank.

Under such conditions crystal growth is slow, the crystals formed are apt to be large and to be considerably interlocked. This interlocking results in the occlusion of mother liquor, thus introducing impurities. When the tanks have cooled sufficiently, which is usually a matter of several days, any remaining mother liquor is drained off and the crystals are removed by hand. This involves much labor and often results in the inclusion, with the crystals, of any impurities that have settled to the bottom of the tank. The floor space required and the amount of material tied up in the process are both large. The wide use of this method, however, has led many non-critical users to demand large crystals, because they have associated purity with size. This is probably due to the fact that the larger crystals produced by the above method are less apt to be contaminated with sediment from the bottoms of the tanks.

Agitated Batch Crystallizers.—It has long been recognized that the conventional method of growing crystals is wasteful of material, labor, and floor space and that artificial cooling was desirable. Figure 208 shows such a piece of equipment. Water is circulated through the cooling coils and the solution is agitated by the propellers on the central shaft. This agitation performs two functions: First, it increases the rate of heat transfer and keeps the temperature of the solution more nearly uniform; and, second, by keeping the fine crystals in suspension it gives them an opportunity to grow uniformly instead of forming large crystals or aggregates. Further, the agitation, combined with the more rapid cooling, results in the formation of a large number of nuclei as compared with the tank method, and therefore the product of this operation is not only more uniform but also very much finer than that from the older tanks. The difficulties with this apparatus are, first, that it is essentially a batch or discontinuous apparatus; and, second, that the solubility is least in the stagnant film on the surface of the cooling coils. Consequently crystal growth is most rapid at this point, and the coils rapidly build up with a mass of crystals which decreases the rate of heat transfer.

Attempts to get larger capacities and continuous operation led to the types of machines about to be described.

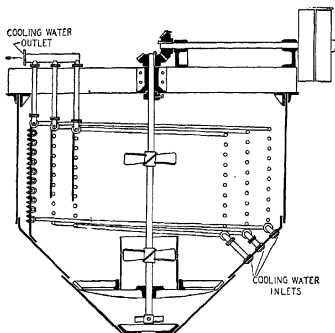



Fig. 208.—Agitated batch crystallizer.

Continuous Crystallizers

The Swenson-Walker Crystallizer.—The most successful continuous crystallizer used in the United States is the Swenson-

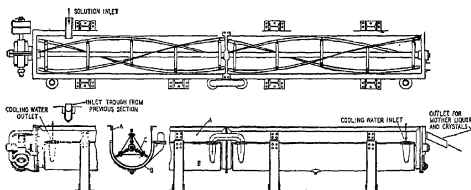



Fig. 209.—Swenson-Walker crystallizer. *A*. Trough. *B*. Jacket. *C*. Stirrer.

Walker crystallizer, shown in Fig. 209. It consists of an open trough *A*, 24 in. wide, with a semicylindrical bottom, a water jacket *B* welded to the outside of the trough, and a slow-speed,

long-pitch, spiral agitator *C* set as close to the bottom of the trough as possible. This apparatus is ordinarily built in units 10 ft. long, and a number of units may be joined together to give increased capacity. Forty feet is the maximum length usually driven from one shaft, and if lengths greater than this are desired, it is usual to arrange several such crystallizers, one above the other, and allow the solution to cascade from one bank to the other.

The hot concentrated solution to be crystallized is fed at one end of the trough, and cooling water usually flows through the jackets in countercurrent to the solution. In order to control crystal size, it is sometimes desirable to introduce an extra amount of water into certain sections. When conditions are properly adjusted, nuclei begin to form a short distance from the point where feed is introduced, and these nuclei grow regularly as the solution passes down the length of the crystallizer. The function of the spiral stirrer is not especially that of either agitation or conveying the crystals. Its purpose is, first, to prevent an accumulation of crystals on the cooling surface and, second, to lift the crystals that have already been formed and shower them down through the solution. In this manner the crystals grow while they are freely suspended in the liquid and therefore are usually fairly perfect individuals, reasonably uniform in size, and free from inclusions or aggregations.

At the end of the crystallizer there may be an overflow gate where crystals and mother liquor together overflow to a draining table or drain box, from which the mother liquor is returned to the process and the wet crystals are raked to a centrifuge. In other cases, a short section of inclined screw conveyor lifts the crystals out of the solution and delivers them to the centrifugal, while the mother liquor overflows at a convenient point. The advantages of this type over those previously mentioned are large saving in floor space and in material in process, but especially a saving in labor.

The Wulff-Bock Crystallizer.—The Wulff-Bock[1] type crystallizer has been widely used in Germany and in England but has not yet been used in the United States. It consists of a shallow, open trough set at a slight inclination and mounted on rollers so that it can be rocked from side to side. At frequent intervals

[1] Griffiths, *J. Soc. Chem. Ind.*, **44**, 7T (1925).

along its length are partitions extending part way across, so that the liquid, instead of flowing directly from one end to the other, flows in a zigzag path. Cooling is entirely by natural convection. Consequently, the capacity is lower than in the Swenson-Walker. The Bock-type crystallizer has the advantages of small labor cost, though this is no lower than for the Swenson-Walker, and because of its much lower rate of cooling can make larger crystals than the Swenson-Walker crystallizer. Its principal disadvantage is its small capacity.

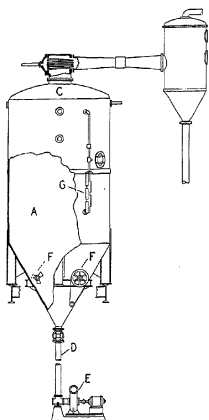



FIG. 210.—Vacuum crystallizer. *A*. Crystallizer body. *C*. Vapor outlet. *D*. Discharge pipe. *E*. Pump. *F*. Propeller stirrers. *G*. Sight glass.

Vacuum Crystallizers.—If a warm saturated solution be introduced into a vessel in which a vacuum is maintained that corresponds to a boiling point of the solution lower than the feed temperature, the solution so introduced must flash and be cooled by the resulting adiabatic evaporation. Not only will the resultant cooling cause crystallization, but also there is some evaporation taking place at the same time which thereby increases the yield.

A vacuum crystallizer is very simple and contains no moving parts. It can, therefore, be made of acid-resistant materials and even lead- or rubber-lined. Its capacity may be made as large as is desired. By the use of steam-jet ejectors of the type shown in Fig. 105, it is possible to produce very low temperatures and therefore obtain very large yields, at the same time returning the minimum amount of material to the process in the form of mother liquor. A typical example of this type is shown in Fig. 210.

In Fig. 210 the crystallizer proper is the cone-bottomed vessel *A*. The feed enters at any point that is convenient and

the vapors leave at C to go to the ejector or other vacuum-producing equipment. The flashing of the feed solution produces a considerable ebullition in the crystallizer, and the crystals are kept in suspension until they become large enough to fall into the discharge pipe D, from which they are removed as a slurry by the pump E. The discharge from the pump E goes to centrifugals or continuous vacuum filters, sometimes with and sometimes without the inclusion of an intermediate settling tank to thicken the slurry.

Due to the low absolute pressure ordinarily carried in the vapor space, a hydrostatic head of a very few inches is sufficient to prevent the liquid from flashing. There is sometimes a tendency for the feed to short-circuit to the discharge pipe without being flashed. For this reason two propellers F, F are often installed to keep the liquid in the crystallizer thoroughly stirred and to prevent feed solution from reaching the discharge pipe without flashing. The level of the liquid in the crystallizer is indicated by the sight glass G. An ordinary gage glass would be apt to freeze up, and the gage glass G consists of a rather large casting with windows to carry rectangular sight glasses on opposite sides.

Crystallizer Operation.—In the operation of the tank system with natural cooling, there is obviously no way to control either nucleus formation or rate of crystal growth. In any of the other types, however, some control may be exercised. It has been found, in general, that the more rapid the cooling at the temperature where the solution reaches saturation the larger will be the number of nuclei formed. It is possible by too great a temperature drop at this point to produce so many nuclei that all the rest of the material deposited will not suffice to grow them to the desired size. Consequently, cooling should be rather gradual at the point where the nuclei are formed. In the Swenson-Walker continuous crystallizer, for instance, if the temperature at the feed end of the crystallizer is below the saturation temperature of feed solution, the resultant sudden cooling through the saturation temperature will produce an extremely large number of nuclei and consequently result in a very fine product. It is desirable, therefore, to maintain the temperature at the feed end slightly above the saturation temperature so that there will be no shock at the point of feed.

After nuclei have been started, it is also desirable to keep the rate at which material is thrown out of solution such that, so far as possible, it is all deposited on existing nuclei without the formation of new nuclei. This is accomplished in the Wulff-Bock crystallizer with its slow atmospheric cooling, and therefore the Wulff-Bock crystallizer usually makes quite coarse crystals. In the Swenson-Walker, the rate of cooling should be as nearly uniform as possible, though it may be slower at the point where nuclei are formed. Many common salts liberate a considerable quantity of heat when they crystallize from solution. This heat of crystallization may be several times the sensible heat that must be removed from the solution. At the point of nucleus formation a larger amount of this is liberated than at any other point along the crystallizer, and, therefore, to produce uniform and steady temperature gradients it is sometimes desirable to add more cooling water in this section of the crystallizer. If the mass becomes too stiff, due to an undue proportion of crystals to mother liquor, if the mother liquor becomes too viscous due to too low a temperature, or if for any other reason a free movement of the crystals through the mother liquor is hindered, further cooling will result in new nuclei being formed instead of the material thrown out of solution diffusing to some crystal face and becoming attached to that face. For this reason the final concentration and the final temperature cannot be carried to extremes in any type of crystallizer without the formation of undue amounts of fine material. An additional reason for leaving a reasonable amount of mother liquor in the product is to prevent the crystallization of undesirable impurities.

CAKING OF CRYSTALS

A serious problem that is often met in handling crystalline products is their tendency to cake or bind together. This is often troublesome in bulk storage or in barreled products but is most serious in those cases where crystals are sold in small packages. The difficulty may exist in degrees, varying from loose aggregates that fall apart between the fingers to solid lumps that can be crushed only by considerable force. The demand of the average consumer that the material shall flow freely from the package makes the prevention of caking a serious problem for the manufacturer.

Critical Humidity.—Just as the vapor pressure of water is fixed by its temperature, so the vapor pressure of any solution is fixed by its temperature at an amount somewhat lower than the vapor pressure of water at that temperature. If a saturated solution is brought into contact with air in which the partial pressure of water is less than the vapor pressure of the solution, the solution will evaporate. On the other hand, if the air contains more moisture than this limiting amount, the solution will absorb water until it is so dilute that its vapor pressure is equal to the partial pressure of the moisture of the air with which it is in contact. If a crystal of a soluble salt is in contact with air that contains less water than would be in equilibrium with the saturated solution, the crystal must stay dry, because if it were surrounded with a film of solution, that solution would necessarily evaporate. On the other hand, if the crystal is brought into contact with air containing more moisture than would be in equilibrium with its saturated solution, then the crystal will become damp and in time will absorb water until it is completely dissolved and the solution is so dilute that it is in equilibrium with the air.

In the range of temperatures around ordinary room temperatures, the vapor pressure of a given solution varies with temperature in such a way that it is nearly a constant percentage of the vapor pressure of water at the same temperature. Saturated sodium chloride, for instance, has a vapor pressure approximately 80 per cent of that of water at the same temperature. If sodium chloride, therefore, is brought into contact with air of more than 80 per cent relative humidity it will absorb moisture, while if it is brought into contact with air of less than 80 per cent relative humidity it will stay dry. From this follows the conception of *critical humidity* of a solid salt. This is the humidity above which it will always become damp and below which it will always stay dry. If the crystal should be coated with impurities derived from the mother liquor from which it was separated (in the case of sodium chloride such impurities would be calcium and magnesium chlorides), this may result in a critical humidity higher or lower than that of the pure salt, according to whether the impurities give solutions having greater or less vapor pressures than that of the salt in question. Consequently, the critical humidity of a commercial grade of a crystalline material

may differ appreciably from the critical humidity of the pure substance.

Prevention of Caking.—Suppose a sample of sodium chloride be exposed for a short time to an atmosphere more moist than its critical humidity and then that it be removed to an atmosphere less moist than its critical humidity. During the first period it will absorb some moisture and during the second period it will lose this moisture. If the crystals are large, so that there are relatively few points of contact and there is a large free volume between the crystals, there will probably be no appreciable bonding of the crystals due to this solution and reevaporation, if the time of exposure were not too great. If, on the other hand, the crystals are fine, or have a small percentage of voids, or are in contact with a moist atmosphere for a long time, sufficient moisture may be absorbed to fill the voids entirely with saturated solution; and when this has been reevaporated the crystals will lock into a solid mass. Consequently, to prevent the caking of such salts the following conditions are desirable: first, the highest possible critical humidity; second, a product containing uniform grains with the maximum percentage of voids and the fewest possible points of contact; third, a coating of powdery inert material that can absorb reasonable amounts of moisture.

The first condition (maximum critical humidity) is often met by removing impurities, such as calcium chloride in the case of common salt, free acid where a salt is formed in acid solution, etc. It often happens that the impurities have a lower critical humidity than the product desired, although this is entirely accidental. To increase the per cent of voids it is not necessary to produce larger crystals but to produce a more uniform mixture. For a given crystal form, and for absolutely uniform crystals as to size, the per cent of voids is the same no matter what the size of the crystals. A variation in particle size, however, rapidly decreases the per cent voids. On the other hand, a fine product has more points of contact per unit volume than a coarse one, and hence a greater tendency to cake. The third remedy is not always applicable. Illustrations of its use are the dusting of table salt with magnesia or tricalcium phosphate and the coating of flake calcium chloride (25 per cent H_2O) with anhydrous calcium chloride.

Some hydrated salts have a melting point so near room temperature that they may sometimes be stored under conditions where fusion begins. Here, again, the same considerations hold, for if the percentage of voids is large or the points of contact between adjacent crystals few, the amount of fused material may not be sufficient to lock crystals together on resolidification. If, due to extremely fine crystal size or a mixture of sizes, the percentage of voids is too far reduced or the number of points of contact too greatly increased, then the crystals may be firmly locked on resolidification. In this case, also, caking may be partly prevented by dusting the crystals with powdery material. In the case of hydrated salts this powdery material may be produced from the salt itself by drying under such conditions that a very thin surface layer is dehydrated.

Nomenclature of Chapter XII

a = a constant during growing process
b = a constant
c = concentration
D = size of openings of a sieve in centimeters
e = a constant
K = a constant
k = overall reaction-rate constant
L = distance, or length
N = number of crystals
n = total number of faces
S = area
W = weight
sub o = at saturation
sub p refers to product.
sub s refers to seeds.
sub 1, 2, 3 $\cdots n$ = faces of a crystal
α = a constant
θ = time
ρ = density
ϕ = a function

Problems

1. Referring to Fig. 201, calculate the yield of hypo ($Na_2S_2O_3 \cdot 5H_2O$) if 1 ton of a 60 per cent solution of $Na_2S_2O_3$ is cooled to 30°C.

2. A 30 per cent solution of Na_2CO_3 weighing 10,000 lb. is cooled slowly to 20°C. The crystals formed are the decahydrate $Na_2CO_3 \cdot 10H_2O$. The solubility of Na_2CO_3 at 20°C. is 21.5 parts anhydrous salt per 100 parts water. During the cooling, 3 per cent of the weight of the solution is lost by evaporation. What is the weight of the decahydrate in the crop?

3. A crystallizer is fed with crystals having the following differential screen analysis, based on the Tyler standard screen scale (Appendix VIII):

Mesh	Per cent	Mesh	Per cent
14	0.0	32	15.3
16	2.1	35	11.8
20	12.6	42	3.5
24	28.2	48	0.7
28	25.8		

The weight ratio of seeds to mother liquor is 1.53 lb. of seeds to 100 lb. of mother liquor.

It is desired to increase the average crystal size twofold (the average ordinate on the N' *vs.* D curve is to be doubled). Assuming that a negligible weight of new nuclei form, calculate

a. The weight of product per 100 lb. of seed crystals.
b. The screen analysis of the product.

4. As a result of batch crystallization there is available a quantity of crystalline material with the following differential screen analysis:

Mesh	Per cent	Mesh	Per cent
12	0.0	24	29.2
14	2.1	28	12.4
16	10.8	32	6.1
20	39.4		

It is desired to grow these seed crystals in a continuous crystallizer under such conditions that as large a weight fraction as possible of the product will be of 16- and 20-mesh size. What will be this maximum weight fraction? Per 100 lb. of seeds, how many pounds of material must be precipitated to give this product?

CHAPTER XIII

FILTRATION

The general problem of separating solids from liquids may be solved by a wide variety of operations depending upon the character of the solids and the proportion of solid to liquid in the mixture to be separated. When the amount of solid is relatively small as compared to the liquid, the process is usually called filtration. As the percentage of solid in suspension becomes higher, the operation passes into either pressing or centrifuging. Curiously enough, the centrifuge is also used in certain separations where the amount of solid to be removed is almost infinitesimal. The operation properly known as filtration is far more important than any of the other operations for separating solids from liquids and will therefore be discussed in greater detail than centrifuging. A characteristic of this field is that the apparatus has been developed almost entirely from the standpoint of practical considerations and without any relation to theory. Consequently, the apparatus used for filtration consists in a wide variety of types dictated by the specific mechanical considerations in the various industries where filters have been used.

FILTRATION APPARATUS

Because of the wide variety in which filtration apparatus exists, it is not possible to make a simple classification that will include all known types of filters. The following outline is not complete but covers the most important types:

1. Sand filters.
   - *a.* Open.
   - *b.* Pressure.
2. Filter presses.

Chamber	washing	open delivery.
Plate and frame	non-washing	closed delivery.

3. Leaf filters.
   - *a.* Moore.
   - *b.* Kelly.

c. Sweetland.
d. Vallez.
4. Rotary continuous filters.

Sand Filters.—The simplest possible filter is a wooden box with a perforated bottom, filled with loose sand. Filters of

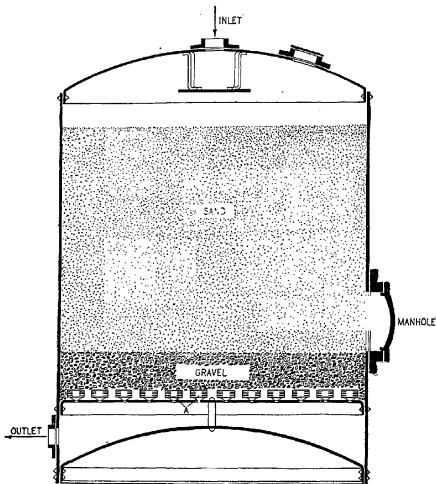



FIG. 211.—Pressure sand filter. *A.* Strainers.

exactly this type have often been used, but the sand filter of today is rather definite and quite completely standardized. It is useful mainly where relatively small amounts of solid are to be removed from the liquid and where very large volumes of liquid must be handled at the minimum cost.

Figure 211 shows a typical pressure sand filter as used for the filtration of boiler feed water or water for similar purposes.

In the bottom of the tank, either mounted on a false bottom or connected to a manifold embedded in concrete, are a number of strainers *A*. These are made of brass and have narrow slots sawed in them. Over the strainers is a layer of several inches of moderately coarse gravel, and on top of that is the sand that forms the actual filter medium. The layer of sand may be from 2 to 4 ft. deep. In operation the water to be filtered is introduced at the top on to a baffle which prevents disturbing the sand bed by a direct stream. The water is drawn off through the strainers at the bottom. When the precipitate has clogged the sand to such an extent that the flow is retarded, it is removed by back-washing. This operation consists of introducing water through the strainers so that it may flow up through the sand bed and out through the connection that is normally the inlet. This water is wasted. It is obvious that sand filters of this type are applicable only to the separation of precipitates that can be removed from the sand in this manner and that are to be discarded. Precipitates that are gelatinous, or which for any other reason coat the sand so that they cannot be removed by back-washing, or precipitates that must be recovered, cannot be handled in a sand filter. The arrangement of valves on a pressure sand filter is entirely standardized and is illustrated in Fig. 212.

The capacity of the average sand filter on such materials as boiler feed water is from 2 to 4 gal. per min. per sq. ft. of surface of filtering area. It is obvious that for very large quantities of water the number of these pressure filters that would be needed would become excessive. Consequently, another type has been developed that is called the *open* or *rapid sand filter*. The general operation of such a filter is exactly like the filter of Fig. 211 except that the sand, instead of being contained in a closed pressure tank, is contained in large open concrete boxes. This type of filter is mainly used in the filtration of municipal water supplies and is therefore not often within the province of the chemical engineer. On the other hand, paper mills, for instance, require enormous amounts of very clean water, and in such cases the open sand filter is the only practical way of clarifying this supply.

Use of Coagulants.—It frequently happens that the amount of material to be removed is so small, or this material is so finely subdivided, that the sand filter will not remove the precipitate.

In processes involving organic materials there may be danger of bacterial infection from an infected process-water supply, and the sand filter cannot remove bacteria as such. When these problems are met, it is customary to add a coagulant to the water before filtration. This coagulant is either ferrous sulfate or aluminum sulfate. Both of these salts are hydrolyzed by the

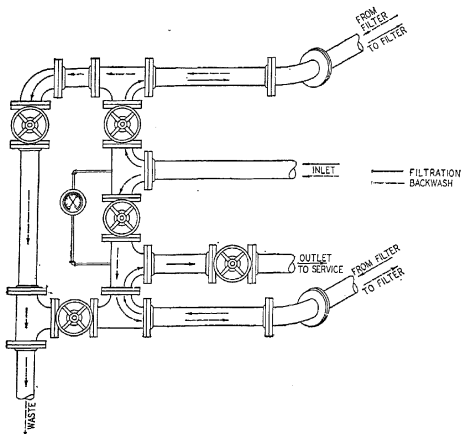



FIG. 212.—Piping for pressure sand filters.

alkalinity of most normal waters, with the production of a flocculent precipitate of iron or aluminum hydroxide. This precipitate adsorbs finely divided suspended matter and even bacteria, even if added to the water in exceedingly small amounts. The resulting flocs, though fine, are removed by the sand filter.

FILTER PRESSES

The traditional method of filtration is the use of an apparatus called the filter press. Although in many cases devices are

now available that are superior to the filter press, there are many other cases in which the filter press is still the most desirable device to use. Many thousands of them are in operation and they have been built in a bewildering variety of designs. These variations, however, affect only minor features.

Chamber Presses.—The simplest and cheapest type of filter press is known as the chamber press or the *recessed plate* press. A front elevation of a plate is shown in Fig. 213, and a typical section through such a press along the line *MM* of Fig. 213 is shown in Fig. 214. The main external features are the same as those of the press shown in Fig. 215 (page 473). A heavy fixed head of cast iron is mounted on a suitable frame and has the necessary pipe connections on it. Extending from this head are two horizontal bars, supported at the other end by the end frame.

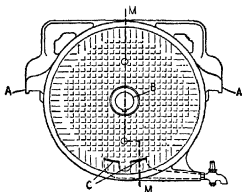



FIG. 213.—Plate of chamber filter press. *A*. Lugs. *B*. Feed channel. *C*. Outlet ports.

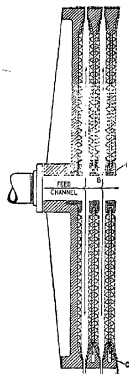



FIG. 214.—Section through chamber filter press. *B*. Grommets. *C*. Outlet ports.

These support the plates of the press by lugs *A* of Fig. 213. The plates are usually of cast iron, from 12 to 36 in. across, approximately ½ in. thick, and with a raised edge from ½ to 2 in. high around the outside. They may be either square or round. In the center of each plate is a hole, which is in line with a connection on the head of the press where the feed is introduced. Over each plate is thrown a sheet of filter cloth as shown in Fig. 214, with a hole cut in the center to register with the hole in the

plate. The filter cloth is fastened to the plate at this point by means of rings B called *grommets*, which either screw together or lock with a bayonet lock. This draws down the cloth so that it is shaped around the plate approximately as shown in Fig. 214.

When all the plates have been so dressed, a heavy follower plate is placed behind the whole assembly and the plates are pressed tightly together by means of a heavy screw or a hydraulic pressure device. The cloths serve as gaskets between the edges of adjacent plates. If, now, the material to be filtered is pumped through the connection at the center of the head of the press, it will fill all the openings between the cloths. As material continues to be pumped in, the filtrate passes through the cloths, runs down corrugations on the face of the plate, and escapes through holes C cored in the bottom of the plate connecting with an external outlet. This external outlet usually discharges into an open launder. It will be obvious that under pressure the cloths are forced back against the face of the plate as shown in Fig. 214. The space between the plates formed by the raised edges is the volume available for cake; therefore, the height of the raised edges on the plate is determined by the thickness of cake through which the filtrate can be forced with reasonable pressure. When sufficient cake has accumulated to fill the chambers, the closing screw is released, the follower is drawn back along the supporting rails, the plates are drawn back, and the cakes discharged, one at a time.

Many variations of the type described above may be found. The feed may be introduced from a channel along the side of the plates instead of through the center. The discharge connections may unite in a closed pipe instead of discharging into an open launder. None of these changes, however, affects the method of operation. The chamber press is the simplest and cheapest type of filter press, but it is not adapted for washing the cake. It is hard to get a clean discharge of cake; and, most important, the wear on the cloths is severe.

Plate-and-frame Presses.—A more satisfactory and more versatile type of filter is represented by the plate-and-frame (sometimes called the *flush-plate*) filter press. This press is made up of plates with very slightly raised edges, and hollow frames, assembled alternately in the same type of structure

that was employed for the chamber press. A side elevation of such a plate-and-frame press is shown in Fig. 215; a front view

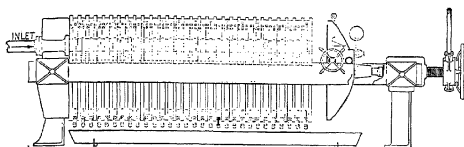



Fig. 215.—Plate-and-frame filter press.

of the plates and frames is shown in Fig. 216; and a longitudinal section through part of the assembled press along the line *BB* of Fig. 216 is shown in Fig. 217. In assembling such a press a

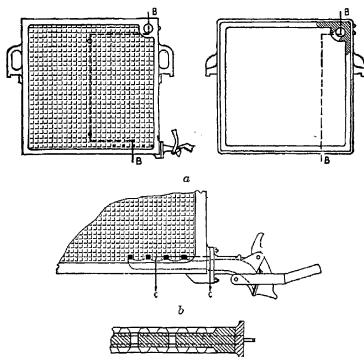



Fig. 216.—Plate and frame of open delivery, non-washing, plate-and-frame filter press. *a*. Elevation, *b*. Details of open delivery ports. *c*. Section along line C-C.

filter cloth is thrown over each plate, but not over the frame. This filter cloth, of course, has holes that register with the

connections on the plates and frames, so that when the press is assembled these openings form a continuous channel the whole length of the press and register with the corresponding connections on the fixed head. The channel shown opens only into the interior of the frames and has no opening on the plates; and at the bottom of the plates, holes are cored that connect the faces of the plates to the outlet cocks. The discharge is shown in detail in Fig. 216, *b* and *c*. As the material to be

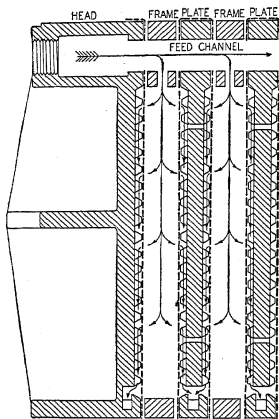



Fig. 217.—Section through non-washing plate-and-frame press.

filtered is pumped through the feed channel, it first fills all of the frames. As the feed pump continues to supply material and builds up pressure, the filtrate passes through the cloth, runs down the face of the plate, and passes out through the discharge cock. When the press is filled, it is opened and dumped exactly as described in connection with the chamber press. In such a press the cake cannot be washed and therefore is discharged containing a certain amount of the filtrate with whatever valuable or undesirable material it may contain.

The open discharge arrangement shown in Figs. 215, 216, and 217 is the usual one. Each plate discharges a visible stream

of filtrate into the collecting launder, and therefore if any cloth breaks or runs cloudy, that plate can be shut off without spoiling the whole batch. If the filtrate is hot or volatile, or if for any other reason the open discharge is not desired, a channel like the feed channel can be supplied to take this discharge as shown

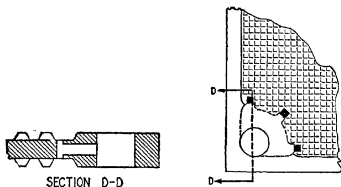



FIG. 218.—Details of closed delivery ports.

in Fig. 218. Here, if there is a cloudy filtrate, there is no way of knowing which cloth or cloths have broken, without opening the press.

Washing Presses.—The principal advantage of the plate-and-frame press over the chamber press is the relative ease with which the precipitate may be washed. The plates and frames of a washing press are shown in Fig. 219, and Fig. 220 shows a

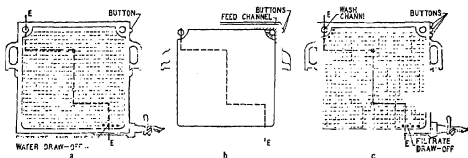



FIG. 219.—Plates and frames of washing, open delivery, plate-and-frame filter press. *a*. One-button plate. *b*. Two-button frame. *c*. Three-button plate.

section through part of the assembled press along the line *EE* of Fig. 219. It will be noted that there are two different kinds of plates in Figs. 219 and 220, distinguished by the connections of the channels. In order to differentiate these two kinds of plates and also the frame, it is universal practice in filter-press

manufacture to cast small buttons on the outside of the plates to guide the workmen in assembling them. The plate of Fig. 219*a* is known as the *one-button plate,* and the plate of Fig. 219*c* is the *three-button plate,* while the frames (Fig. 219*b*) always have two buttons. The press is so assembled that the order of buttons is 1-2-3-2-1-2-3, etc. The various channels lead to connections on the fixed head. Figure 215 shows such an

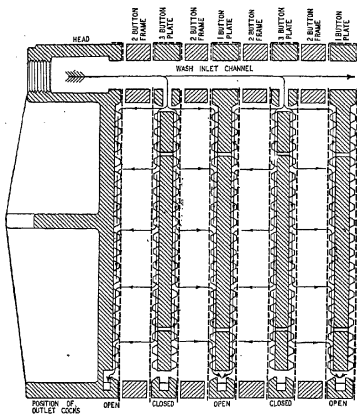



FIG. 220.—Section through washing plate-and-frame filter press along line *EE* of Fig. 219.

assembly. During filtration the wash channel is closed by a valve on the head of the press and the material to be filtered is pumped through the feed channel. This opens only into the frames. The frames fill, therefore, and filtration is carried out as in the non-washing plate-and-frame press described above. When filtration has reached the practical limit and a good compact cake formed, the feed connection is closed, the outlet cocks on all the three-button plates are closed, and water is turned into the wash channel. This wash channel has cored

openings connecting with both faces of the three-button plates. Water therefore enters between the plate and the cloth on all of these plates, but since the outlet from the three-button plates has been closed, the wash can escape only by passing through the cake, down the faces of the one-button plates, and out through the cocks that have been left open on the one-button plates. Obviously there can be no connection between the wash channel and the faces of the one-button plates. By comparing Figs. 217 and 220, it will be seen that the wash water passes through the whole thickness of the cake, whereas during filtration the filtrate passes through only half the thickness of the cake. Theoretically, this added resistance of the cake should cause the water to distribute itself uniformly over the faces of the three-

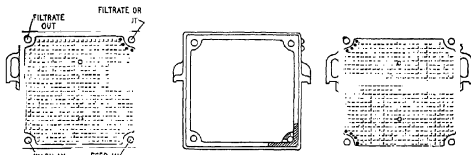



FIG. 221.—Plates and frames of washing, closed delivery, plate-and-frame filter press.

button plates and therefore to pass through the cake uniformly. As a matter of fact, however, washing is always best in the neighborhood of the wash inlet and poorest in the other corners. Consequently, many washing presses are provided with two wash channels in diagonally opposite corners of the plates. The press is washed first through one channel and then through the other.

If a washing plate-and-frame press is to be also a closed delivery press, two separate discharge channels must be provided, so that the three-button plates may be shut off during washing. Such a set of plates is shown in Fig. 221. Filter presses may also be obtained with channels so that the press may be heated or cooled.

A wide variety of details may be found in the presses of different manufacturers. These variations are all structural, and an understanding of filter-press operation will make the identifi-

cation of any press a simple matter. Figure 222 shows a variety of plates and frames for the reader to identify. The plates and frames with the same letters go together.

Materials of Construction.—The usual filter press is built of cast iron. It could conceivably be built in any metal that can be cast, but actually most such constructions are too expensive to be practical. Lead- or rubber-covered plates and frames may be obtained for filtering liquids that attack cast iron, and filter presses made entirely of wood are available. These latter require the greatest care to keep them tight and in serviceable condition, due to the tendency of the wood either to warp or to shrink in service.

The commonest filter medium is some form of cotton duck or twill. Various types of fabrics may be used, from light cotton sheeting to exceedingly heavy canton flannel or even burlap. Where the filter medium must be fine but where the pressures are high enough to burst it, a heavy coarse cloth of burlap or jute may be used, and then a finer cotton cloth laid over it. In the filtration of some oils, sheets of filter paper are used instead of cloth, but these will not withstand any but the lowest pressures and must always be backed with a heavy fabric for support. Wire cloth sufficiently fine for filtering purposes can be obtained but is rarely used on filter presses. It is more often used on the leaf-type filter to be described later. By far the commonest arrangement is a cast-iron press with steel bars or channels for the side rails, a capstan-operated closing screw, and a cotton duck filter cloth.

Leaf Filters

The filter press, although it is well adapted for many purposes, is not so effective as it might be for handling, without heavy labor charges, large quantities of sludge; or for efficient washing with a small amount of wash water, due to the channeling that always takes place in a washing press. The types of filters to be described in this section were first developed in the metallurgical industries, are more effective for the work mentioned above than is the ordinary filter press, and require far less labor. On the other hand, the first cost of these appliances is higher, and the filters themselves are in general somewhat more complicated.

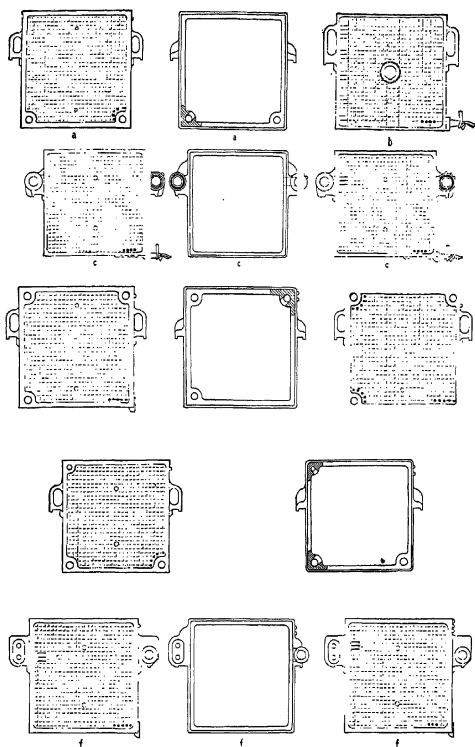



Fig. 222.—Filter-press plates and frames.

This type of filter was made possible by the development of the *filter leaf.*

At present the filling of the leaf is a coarse-mesh, double-crimp screen of rather heavy wire. Around the circumference of this disc of screen is a light piece of U-shaped metal that binds the edges. To this is attached the nipple which serves both to remove filtrate and to suspend the leaf. Over this leaf is then drawn a sack of filter cloth which is sewn tightly around the edge of the leaf and caught under a collar around the nipple. *Wire filter cloth* is frequently used in these modern filters. A simple woven-wire screen cannot be made fine enough to act as a filter medium unless it is made of exceedingly fine wires, and in that case its strength would be low. Metal filter cloth is woven of

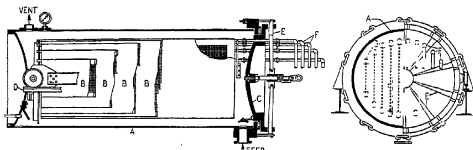



FIG. 223.—Kelly filter. *A.* Filter drum. *B.* Filter leaves. *C.* Removable head. *D.* Internal rails for carriage. *E.* Locking bars. *F.* Filtrate outlets.

loosely twisted strands of wire with as many wires to the inch as practical. After weaving, the cloth is rolled, which spreads out the strands of the meshes so that a fairly compact and yet strong filter medium results.

The Kelly Filter.—The Kelly filter is shown in Fig. 223. This filter consists of a drum *A* in which are filter leaves *B*, fixed to the removable head *C* of the drum. The back end of the leaves is supported by a frame running on tracks *D* fastened to the shell of the drum. The removable head is fastened to the drum by quick-opening levers *E* so that the filter leaves can easily be run out and back. The operation of the filter consists in pumping the slurry into the drum under pressure so that the filtrate is forced through the filter cloths and drains from the inside of each leaf through connections *F* provided with separate bibs on the outside of the removable head. For washing, water is simply pumped in behind the slurry, and the wash water passes through

the cake in the same direction as did the filtrate, so that the wash-

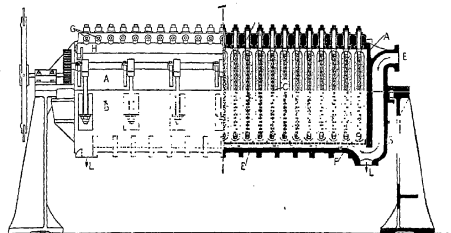



FIG. 224.—Elevation of Sweetland filter. *A*. Stationary half of casing. *B*. Hinged half of casing. *C*. Filter leaves. *D*. Filtrate outlet pipes. *E*. Feed channel. *F*. Distributing plate. *G*. Filtrate sight glasses. *H*. Filtrate manifold. *J*. Locking cams. *K*. Wash-water manifold. *L*. Discharge ports.

ing is by direct displacement. To discharge, the filter leaves are run out, and compressed air is blown into the leaves to remove the filter cake. The main disadvantage of the Kelly press is the fact that each filter leaf is of different size, due to their vertical arrangement in the horizontal, cylindrical shell, and non-uniformity of filtering and washing results.

FIG. 225.—Cross-section of Sweetland filter. (See legend for Fig. 224.)

Sweetland Press.—The Sweetland press is a later development of the pressure leaf filter. Diagrams of this press are shown in Figs. 224 and 225. It consists of a horizontal cylindrical shell, of which the top half *A* is stationary, while the bottom half *B* is hinged on one side and can be dropped. The filter leaves *C* are circular and are arranged crosswise in the shell. They are suspended from the top

half of the casing by means of the filtrate outlets *D*. The sludge to be filtered is pumped through a channel *E* in the bottom of the shell and is distributed by a perforated plate *F*. The filtrate passes through the sides of the leaves to the interior, and each leaf discharges through its own sight glass *G* into a common manifold *H*. When the filtration is complete, wash water is sent through exactly the same way as was the filtrate. The press can be emptied by loosening the eccentric *J* shown to the right of Fig. 225, dropping the lower part of the cylinder *B*, and sluicing the cake off with water pumped through the port *K* at the upper left-hand corner of Fig. 225, or the press may be discharged by sluicing the cake away without opening

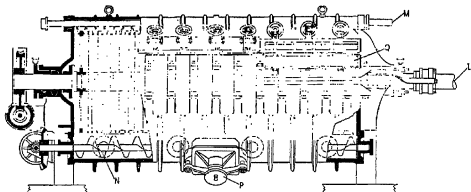



FIG. 226.—Side view of Vallez filter. *L*. Filtrate outlet. *M*. Wash-water manifold. *N*. Discharge screw. *P*. Outlet door. *Q*. Inspection door.

the press. The material sluiced away leaves through the channels *L* of Fig. 224. A Sweetland press has the advantage of having large filtering area per unit of floor space, good washing efficiency, low labor cost, and low dead space in the filter.

Vallez Filter.—The Vallez filter, shown in Figs. 226 and 227, consists of a horizontal cylindrical shell with circular leaves at right angles to the axis of the shell as in the Sweetland, but it differs from the Sweetland in that the leaves are assembled on a central shaft that rotates during filtration. In the illustrations the shell is shown with a casing split horizontally, but this is merely for purposes of assembly, and the shell is not opened during operation. The leaves are made of double-crimp wire screen *B* between perforated metal plates *A* and assembled on a hub *C*. Over the perforated metal plates are placed discs of

filter cloth or wire screen G held in place by clamps E and F. A spacing ring D gives the leaf the proper width and rigidity. The hubs C slide over the central shaft and are keyed to it through key seats K. Filtrate passes from the leaf through port H into corresponding perforations in the shaft and leaves at L.

In operation the feed enters through a number of ports along the bottom, shown dotted in Fig. 226. After the cake has been formed, wash water may follow the same path. When the cake is to be discharged it is either blown off the leaves with compressed air entering at L or washed off by sprays from the spray pipe M. In either case the screw conveyor N carries the mud to the discharge door P. Hinged inspection plates Q are provided on the shell, so that during filtration the pressure can be temporarily cut off and these doors opened to determine whether or not the cake is properly formed. In normal operation, however, it is unnecessary to inspect the cake. A *cake indicator*, consisting of a movable rod carrying a side arm and disc and operated from the outside, is used to determine the cake thickness during operation.

FIG. 227.—Construction of Vallez filter leaf. *A*. Perforated plates. *B*. Double-crimp wire screen. C. Hub. *D*. Spacing ring. E, F. Clamps. *G*. Filter medium. *H*. Filtrate ports. *K*. Key seats.

In all types of filters previously described, if the precipitate to be removed consists of a mixture of coarse and fine particles, the coarse particles are apt to settle to the bottom of the cake, and the fine particles are apt to be caught in the top of the cake. This not only gives uneven cake thickness but also results in most of the wash water passing through the coarser parts of the cake and leaving the fine parts unwashed. This segregation may go so far as to give such a coarse cake at the bottom that it will not adhere to the filter leaves. The principal advantage of the Vallez filter is that, due to the rotation of these leaves, such segregation is avoided and the cake is of nearly uniform density over the whole leaf.

Rotary Continuous Filters

It has always been recognized that the intermittent character of the operations carried out in all types of filtration apparatus so far described is a serious disadvantage. About 1906 the first rotary continuous filter, the Oliver, was built; and the rotary continuous filters now on the market are essentially modifications of the Oliver.

Oliver Filter.—The principal element of the Oliver filter (Fig. 228) is a horizontal cylindrical drum *A*. This drum may be from 1 ft. in diameter and 1 ft. long up to 15 ft. in diameter and 20 ft. long. Its exterior surface is divided into a number of segments *B* by means of raised strips *C* parallel to the axis. Each segment has one or more suction pipes *D*. On the surface of the drum between the dividing strips is a support for the filter medium; and this support may be wood strips, heavy double-crimp wire screen, or other suitable material. Over the whole of the drum is then wrapped the filter medium, which may be either cloth or screen according to the service. The filter medium is held in place with a tight wrapping of wire.

The drum is mounted on trunnions and rotates about half submerged in a tank containing the material to be filtered. All of the suction and discharge pipes lead to an automatic valve located on one of the trunnions. Each section operates as an independent filter. As the drum rotates and any given section approaches the surface of the liquid, suction is applied to that section by the automatic valve and maintained during nearly the whole revolution of the drum. The speed of the drum is so adjusted that during the period of submergence the desired layer of cake is accumulated. The half revolution of the drum during which the section in question is not submerged may be divided into four parts, in which the cake is first sucked free of filtrate, next washed, then sucked free of wash water, and finally discharged. During the first three of these operations the section is under suction. The wash is applied by one or more sets of sprays. As the section under consideration approaches the discharge point, the automatic valve cuts off the suction and applies compressed air to the interior of the section. A doctor blade removes the cake that has been loosened in this manner. The automatic valve also serves to separate filtrate from wash;

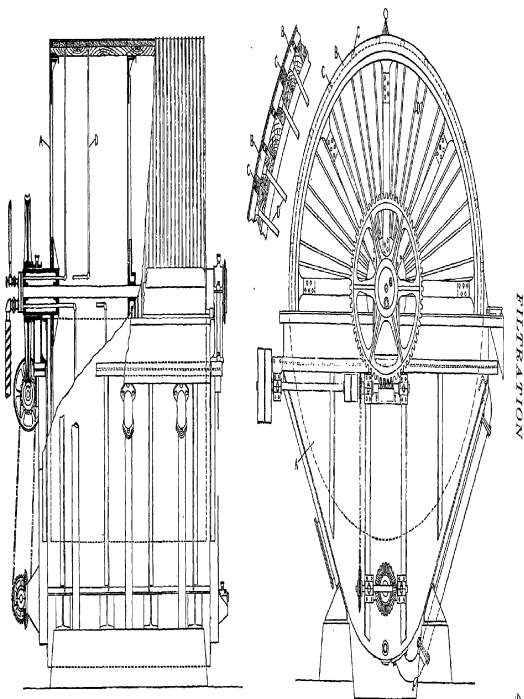



Fig. 228.—Oliver filter. *A*, Filter drum. *B*, Filter compartments. *C*, Separating strips. *D*, Filtrate and air pipes.

or if desired, more than one wash may be used and still kept as separate fractions by proper ports in the valve. Figure 228 shows a drum of wood construction, but metal construction is now much more common.

The construction of the valve is shown diagrammatically in Fig. 229. The valve seat is attached to the drum, rotates with it, and the various pipes that connect to the different sections are brought down to this valve seat. Each hole in the seat represents a section of the drum. The stationary plate is pressed tightly against this valve seat and does not rotate. It contains connections for filtrate and wash outlets, which are in turn connected to separate receivers, and these receivers are connected to a vacuum pump that supplies the suction. A channel around

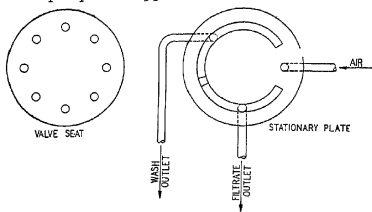



FIG. 229.—Diagram of Oliver filter operating valve.

the circumference of the stationary plate registers with the ring of holes on the valve seat. This channel is shown divided into two parts, one part of which is connected to the filtrate outlet, and one part to the wash outlet. The division between the two is made by means of a tightly fitting block whose position may be changed during operation, to regulate the separation between the filtrate and wash. Compressed air is connected to the small blow port on one side of the stationary plate. As the valve seat rotates, the holes to which are attached the pipes communicating with the various sections of the filter come successively under the filtrate channel, then under the wash channel, then finally under the blow port. The wash outlet channel may be divided into two or more sections, if successive washes of different strength or with different wash materials are to be collected separately.

The rotary continuous filter has been developed for all kinds of service; for handling fine or gelatinous precipitates that may be collected in cakes only an eighth to a quarter of an inch thick, up to filters designed for separating salt, which may work with cakes as thick as four inches. They may filter liquids nearly at the boiling point, although these filters are more difficult to operate as the solution is hotter because less suction can be applied before the filtrate begins to boil. For very porous cakes, such as salt, where large volumes of air are drawn through the cake with low pressure drops, valves with very large ports are provided so that on salt filters the diameter of the valve disc is almost as great as the diameter of the drum. The precipitate contained in the tank under the drum may have a tendency to settle, and consequently most filters of this type are provided with some sort of agitator to keep the precipitate in suspension. The action of these mechanical agitators may be supplemented, in the case of heavy materials, by jets of filtrate which help to keep the precipitate stirred. When filtering very coarse crystalline precipitates the feed may be on the top of the drum and the tank may be omitted. For producing cakes of low moisture content, part of the drum may be enclosed in a hood and hot air used for drying.

FILTER OPERATION

The types of sludges handled in commercial filtration vary widely from granular, incompressible, free-filtering materials to slimes and colloidal materials that are compressible, that tend to plug the filter cloth, and that are, in general, very difficult to separate completely from the liquid in which they are suspended. The first important characteristic of a sludge is its structure, for example, whether it is granular and open or whether it is colloidal and dense. Filtration of a precipitate of barium sulfate or calcium carbonate is different from filtration of ferric hydrate. A second property of a sludge is its compressibility. If a sludge is incompressible, the resistance of the cake will be substantially independent of the pressure, while if the sludge is compressible the resistance of a unit cube of cake may increase rapidly as the pressure increases.

Filter Aids.—For sludges that are difficult to filter, various filter aids are used. A filter aid is a solid material, finely divided

but consisting of hard, strong particles that are, *en masse*, incompressible. They can be used in either of two ways. The first method is the use of *precoat* of filter aid, or a thin layer of the material laid down on the filter before the sludge proper is pumped to the apparatus. A precoat prevents the colloidal particles of the sludge from becoming so entangled in the filter cloth that the resistance of the cloth itself becomes high. It also facilitates the removal of the cake at the end of filtration. With a precoat, the filtering medium is really the precoat material rather than the filter cloth itself.

The second method of using filter aid is the incorporation of a certain percentage of the material with the sludge before sending it to the press. The presence of the filter aid increases the porosity of the sludge, decreases its compressibility, and reduces the resistance of the cake during filtration. The most important filter aid sold commercially is that known as *filter-cel.* This material is also known as *diatomaceous earth* or *kieselguhr.* It consists of siliceous skeletons of very small marine organisms, known as *diatoms,* that have been deposited on prehistoric shore lines in California and other places. These skeletons are characterized, first, by the fact that they consist of practically pure silica and, second, by the fact that the individual skeletons, though of very small size, are exceedingly complex in their structures and therefore offer an enormous surface for the adsorption of colloids. The material is mined as a soft rock and is pulverized if it is to be used as a filter aid. Certain grades of filter-cel have been given a special treatment to increase their power of adsorbing colloids. Although this material, because of its enormous surface and rigid structure, is highly effective as a filter aid, any granular incompressible material may be added to a compressible sludge with helpful results. Precipitated calcium carbonate or any other granular crystalline precipitate may be so used.

Effect of Pressure.—It should be noted that, in general, the filter fabric itself is rarely, if ever, the true filtering medium. Photomicrographs of fabrics and of precipitates will usually show that the average particle is considerably finer than the average opening between fibers. The real filtering medium is a layer of the precipitate itself, which is entangled in the surface of the fabric. Evidently, therefore, the formation of this

first layer is of prime importance in securing satisfactory filter operation.

A filter press or a pressure leaf filter may be operated in any one of several ways. The simplest method is to apply the full pressure at the start of the filtration, and to maintain the pressure constant throughout the run. A serious disadvantage of this method is that, if the initial pressure be high, the first particles caught will be compacted into a tight mass that largely fills the pores of the cloth and results in a low rate of filtration throughout the rest of the cycle. If the sludge is non-homogeneous—that is, if it contains both crystalline and colloidal particles—high initial pressure tends to force the colloidal portions of the sludge into the interstices between the granular portions and greatly decreases the rate of filtration. On the other hand, if the initial pressure be low, the initial layer of precipitate will be more open so that rates of filtration will be higher, and this layer will not be crowded into the fibers of the fabric, with the result that the cake will separate more cleanly from the cloth. A low initial pressure may mean that the first runnings from the press will not be clear, but this is more than offset by the more rapid filtration rate and consequently larger capacity.

Another method is to maintain a nearly constant rate by starting at low pressure and continuously increasing the pressure to overcome the increasing resistance of the cake, until the maximum pressure is reached at the end of the run. The difficulty with this method is that the maximum pressure is reached only at the end of the run, and hence the whole cycle is at less than maximum capacity; but this has no advantage after the initial layer of cake is well formed. A common method of meeting the difficulties of constant-pressure filtration is to operate at constant rate during the first part of the filtration. As soon as the cloths are well coated with cake and the filtrate is clear, the pressure is increased to the maximum, and filtration is completed at constant pressure.

For quite compressible sludges, it is not necessarily true in all cases that the rate of flow increases with the pressure. At low pressures, the increase in rate of filtration accompanying a slight increase in pressure is greater than the decrease in rate due to the resistance brought in by the compression of the sludge. As the

pressure increases, however, these two factors tend to become equal, and at a certain pressure known as the *critical pressure*, the rate of flow is a maximum. For pressures higher than this critical pressure, the added resistance due to the compression of the sludge is greater than the added driving force represented by the increase in pressure, and the net effect is a decrease in the rate of flow with increase in pressure. Obviously, filter operation should be maintained at a pressure below the critical pressure. This phenomenon is absent when filtering non-compressible sludges.

Filter Auxiliaries.—For filter presses a very simple arrangement of auxiliaries is usually sufficient. The material to be filtered should be stored in tanks provided with agitators so that a uniform suspension is fed to the press, unless the press is fed from a thickener like the Dorr thickener, in which case the uniformity of supply is assured by the action of the thickener itself. The most desirable pumps for feeding filter presses are centrifugal pumps, because they give a uniform pressure, and as the result of their characteristic curves tend to build up higher pressures as the rate of flow decreases. Reciprocating pumps are undesirable because the pressure pulsations in their discharge tend to make an unduly compact cake.

The ordinary filter press of the open-discharge type usually discharges into a launder with drains at one end that lead to storage tanks for filtrate and wash. There may be one launder with two connections in the end, one leading to a filtrate tank and one to a wash tank, and these connections are usually opened or closed by wooden plugs. It is also possible to have two launders side by side, with wing ends on the bibs so that the discharge from any one plate can be directed into either launder.

With rotary continuous filters, such as the Oliver, the arrangement shown in Fig. 230 is common. A vacuum pump is supplied and is connected to two receivers, one for the filtrate *A* and one for the wash water *B*. These receivers are connected by pipes having flexible hose connections to the ports on the fixed plate of the filter valve. Centrifugal pumps *C* are provided to remove wash or filtrate from these receivers, but these pumps must be of a special design, since ordinary centrifugal pumps are not suited for pumping out of a vacuum. If the filtrate is warm so that the air removed by the vacuum pump contains appreciable

quantities of water vapor, a condenser *D* is placed between the receivers and the vacuum pump to condense this water vapor and decrease the volume that the pump must handle. Since dry vacuum pumps are designed with small clearances and usually run at fairly high speeds, it is imperative that no liquid water enter the vacuum pump. Consequently, the filtrate and wash receivers are provided with float-operated relief valves *E*. If, for any reason, the pump that is removing liquid from either receiver fails to function and the receiver fills, this float opens the relief valve. Opening the relief valve destroys the vacuum

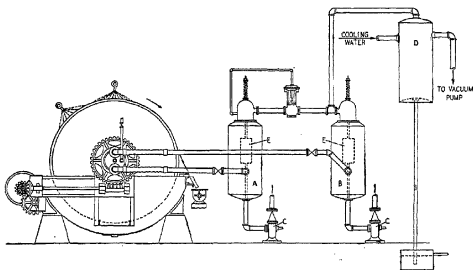



FIG. 230.—Rotary continuous filter and auxiliaries. *A*. Filtrate tank. *B*. Wash-water tank. *C*. Centrifugal pumps. *D*. Condenser. *E*. Floats operating relief valves.

on the receiver and permits the centrifugal pump to regain its suction. At the same time the operation of the filter ceases until suitable conditions have again been restored. Two or more separate washes may be given on the drum and a corresponding number of wash receivers placed between the filter and the vacuum pump if these washes are to be kept separate.

COMPARISON OF FILTER TYPES

A considerable variety of filter constructions has been described in this chapter, and the question naturally arises, How does one choose the type of filter to be used in a given case? This question is often answered by tradition or accepted practice in a particular industry. A logical analysis of the situation is difficult to make.

Field of the Filter Press.—The plate-and-frame filter press is the cheapest press per unit of filtering surface and requires the smallest floor space. The labor of opening and discharging such a press, especially in the larger sizes, is very great. Consequently, if the material to be filtered contains a large proportion of solid matter, especially if the solid matter is not the valuable constituent, the operation of the filter press becomes unduly expensive. The principal consideration, however, is not the cost of labor and maintenance per cycle but the cost of labor and maintenance for unit value of product. Consequently, in the manufacture of dyes, although the proportion of solid to liquid may be high, the filter press is usually used, partly because such materials are seldom made by a continuous process or in large enough batches to keep the automatic types of filters in operation, but more particularly because the value of the cake is high, the recovery of solid is complete, the solid is in the form of cakes suitable for processing in the trays of a shelf drier, and the cost of labor per unit of value in the product is quite low.

Field of Leaf-type Filters.—On the other hand, in such processes as the extraction of gold from ores by the cyanide process where enormous volumes of worthless solid must be filtered per unit of value recovered, the leaf filter or the rotary continuous filter are the only ones possible.

The choice between filters of the leaf type and of the filter-press type is partly made on the basis of labor costs and fixed charges but also, to a considerable extent, on the method of washing. This is an important distinction which is not always sufficiently appreciated. In the plate-and-frame press the material to be filtered is fed at the center of the frame and passes out in either direction toward the plates. During washing the wash water passes from one plate through the cake on to the other plate. The wash water follows channels quite different from those followed by the filtrate, and therefore the first portions of the wash water are more or less mixed with the filtrate. The result is that the first portions of wash water issuing from the press are more dilute than the liquid that was being filtered, and the concentration of this wash water falls off gradually according to such a curve as curve A in Fig. 231.

On the other hand, the leaf-type filter operates by displacement washing. The wash water follows exactly the same channels as

was followed by the filtrate. Instead of mixing with the filtrate to give a discharge that is considerably diluted even at the start, the wash water pushes the filtrate ahead of itself so that the first portions discharged during washing are fully as concentrated as was the filtrate. If no mixing of filtrate and wash water occurred, when the wash water proper appeared it should have zero concentration and the operation would be represented by curve *C*. In practice some mixing occurs, but when more dilute solution appears, its concentration falls off very rapidly, and it can be made to yield wash water of practically zero concentration after a reasonable amount of washing. Such a curve is shown as curve *B* in Fig. 231. This means that for a given degree of removal of soluble material from the cake, displacement washing

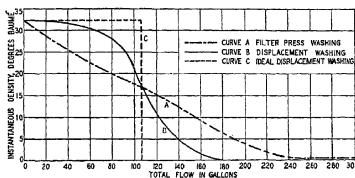



Fig. 231.—Typical filter washing curves.

not only yields a smaller volume of wash and hence a more concentrated wash, but it also yields part of the wash in a form so concentrated that it can be added to the filtrate without serious dilution. Where the filtrate is the valuable material, and especially where this filtrate must later be evaporated, this feature of the leaf filters has proved their most valuable characteristic.

Field of the Rotary Continuous Filter.—The rotary continuous filter is, in general, indicated for those operations in which the precipitate is large in volume, where the process is continuous, and where labor cost must be kept down. The field of the rotary vacuum filter is rapidly enlarging. The principal obstacle to its general success is its inability to handle satisfactorily precipitates that contain both coarse material that settles readily and fine material that does not settle. In such cases it is quite difficult to build a satisfactory cake on a rotary vacuum filter.

Where the precipitate is small in amount and gummy or colloidal, the rotary vacuum filter is not suited, nor in general is it suited for any process where large pressures are required or where the amount of precipitate is exceedingly small.

A feature that is being adopted in many filtration operations is the use of a Dorr thickener ahead of a filter of any type. The operation of the thickener is so much cheaper than that of the filter that if an overflow of satisfactory clarity can be obtained from a thickener, the cost of filtration is greatly decreased, since only a fraction of the material needs to be put through the filter. This feature alone makes it possible to use the rotary continuous filter in cases where otherwise a filter-press or a leaf-type filter would be necessary.

The limiting conditions for a filter of any type come when the precipitate is so coarse and granular, or the ratio of solid to liquid is so high, that the material either cannot be pumped as a suspension or cannot be built into a cake. When this condition occurs, the operation passes over into the field occupied by centrifugal machines. Within recent years, however, the rotary continuous filters have been so developed that they will handle many mixtures that previously were supposed to require a centrifuge. The most important of these cases is the separation of crystallized salts that have been formed in evaporators.

FILTRATION THEORY

The most practical result that might be obtained from any filtration theory would be the answer to these questions:

Given the filtration conditions and the design of the filter, what volume of filtrate can be obtained in a definite length of time? What volume of wash water can be passed through the cake in a definite length of time? What will be the relationship between concentration of recovered material in the wash water and the amount of wash water used?

In spite of much careful investigation, complete answers to these questions cannot be given. The following discussion will be based on the most recent work in this field.[1] The theoretical solution of these questions is retarded by the facts that different sludges vary greatly in characteristics and that the resistance to

[1] Ruth, Montillon and Montonna, *Ind. Eng. Chem.*, **25**, 76–82, 153–161 (1933); Ruth, *Ind. Eng. Chem.*, **27**, 708–723, 806–816 (1935).

flow of any one is extremely sensitive to temperature, method of preparation, and age.

Constant-pressure Filtration.—Consider a filter through which there is pumped a slurry consisting of W lb. of a filterable solid or precipitate suspended in a liquid. Let s be the weight fraction of the precipitate in the slurry. Let V be the cubic feet of filtrate obtained, let ρ be the density of the filtrate in pounds per cubic foot and let m be the weight ratio of wet cake to washed dry cake. The washed, dry cake is the precipitate of the original slurry. Then, the liquid retained by the cake is $(m - 1)W$; the total liquid in the slurry is $\frac{W}{s}(1 - s)$ and the difference must be the weight of filtrate, $V\rho$. Therefore,

$$V\rho = \frac{W}{s}(1 - s) - W(m - 1)$$

from which

$$W = \frac{V\rho s}{1 - sm} \tag{309}$$

The problem of developing a quantitative method of filtration calculation is essentially that of obtaining a differential equation for the rate of filtrate production at any time in terms of the operating variables, and integrating the equation over the time of filtration.

The rate of flow of filtrate through the fine capillary pores of the cake is probably subject to Poiseuille's law (page 33). It is not possible to apply this law directly, however, because neither the length nor the diameter of the capillaries is known. It can be expected, however, that the rate will be inversely proportional to the viscosity of the filtrate. The driving force is the pressure drop across the cake. Therefore:

$$\frac{dV}{d\theta} = \left(\frac{A}{\mu}\right)\left(\frac{P_c}{R}\right) \tag{310}$$

where

V = the volume of filtrate collected up to time θ.
θ = the time.
A = the total filtering area.

μ = the viscosity.
P_c = the pressure drop across the cake.
R = the resistance of the cake.

In practice the pressure drop that can be most easily measured is not the pressure drop across the cake itself, but across the whole press. If the final calculation is to involve this pressure drop, then the resistance of the whole press (and not simply of the cake itself) is also involved. The total resistance to the flow of filtrate is the sum of several terms. It can be written as:

$$R_T = R_1 + R + R_2 \tag{311}$$

where

R_T = the total resistance.
R_1 = the resistance of the filter medium
R = the resistance of the cake.
R_2 = the resistance of the filtrate and slurry leads and channels.

If the filter is properly designed, R_2 should be negligible. The resistance R_1 of the filter cloth or other medium is best considered to be equivalent to that offered by an imaginary weight of cake. This fictitious cake, equivalent in resistance to R_1, can be considered to be W_1 lb. and to have been left by V_c cu. ft. of filtrate. The actual cake is W lb. and the total cake (real and fictitious) is $W + W_1$. If the actual volume of filtrate is V, the total volume (real and fictitious) is $V + V_c$.

The resistance offered by the cake at any time depends upon the weight of cake (real and fictitious) per unit area at that time, or

$$R_T = R_1 + R = \frac{(W + W_1)\alpha}{A} \tag{312}$$

where α is the average specific resistance of the cake. The resistance varies throughout the cake, but the average resistance has been shown[1] to depend only on the slurry and on the pressure used in the filtration.

[1] RUTH, MONTILLON, and MONTONNA, *Ind. Eng. Chem.*, **25**, 161 (1933).

By Equation (309),

$$W + W_1 = \frac{\rho s}{1 - sm}(V + V_c) \tag{313}$$

and eliminating R_T and $W + W_1$ from Equations (310), (312) and (313) it follows that:

$$(V + V_c)dV = (V + V_c)d(V + V_c) = \frac{A^2P(1 - ms)}{\mu\rho s\alpha}d\theta \tag{314}$$

Equation (314) can be integrated after choice of integration limits. If the time of the run, θ, be taken as zero when the slurry fills the press, and if V is the volume of filtrate at time θ, then $(V + V_c)$ is zero when the time is $-\theta_c$, where θ_c is the time required to lay down the fictitious cake that accounts for the press resistance. If the filtrate present in the filtrate channels be neglected, the total volume at time θ is $V + V_c$, and, therefore:

$$\int_0^{(V+V_c)} (V + V_c)d(V + V_c) = \frac{A^2P(1 - ms)}{\mu\rho s\alpha}\int_{-\theta_c}^{\theta} d\theta \tag{315}$$

$$(V + V_c)^2 = \frac{2A^2P(1 - ms)}{\mu\rho s\alpha}(\theta + \theta_c) = K(\theta + \theta_c) \tag{316}$$

where

$$K = \frac{2A^2P(1 - ms)}{\mu\rho s\alpha} \tag{317}$$

Equation (316), if plotted as V *vs.* θ, is the equation of a parabola the vertex of which is at the point $(-\theta_c, -V_c)$. The actual course of the constant-pressure filtration is, therefore, a section of a parabola.

The constant α must be determined experimentally by means of constant-pressure filtrations on the slurry in question. It will vary with pressure, unless the sludge is non-compressible. The more α varies with pressure, the more compressible is the sludge. In many cases, if the pressure range is not too great, α varies linearly with pressure.

Often, at the end of the filtration, V and θ are large in comparison with V_c and θ_c respectively; and if the end condition is the

only one of interest, Equation (316) can be written in the simplified form:

$$V^2 = K\theta \qquad (316a)$$

The constants K, θ_c, and V_c in Equation (316) can be obtained from observations of V *vs.* θ for any constant-pressure filtration. The most convenient method of determining the constants V_c, K and θ_c from experimental work is to differentiate this equation, giving

$$\frac{d\theta}{dV} = \frac{2V}{K} + \frac{2V_c}{K} \qquad (318)$$

From this equation it follows that if the reciprocal of the rate, $d\theta/dV$, is plotted against V, a straight line is obtained, whose slope is $2/K$ and whose intercept is $2V_c/K$. This plot is most easily prepared by taking the differences of both V and θ, dividing the θ difference by the V difference and plotting the quotient as the height of a rectangle, using the ΔV value as the base. This is shown in Fig. 232. A straight line is drawn through the tops of these rectangles, as nearly as possible through their mid-points, in such a way that the areas of the triangles above the line equal the areas of those below the line. From the slope and intercept of this line, K and V_c are easily calculated. θ_c can then be found by calculating it from Equation (316) for several pairs of values of V and θ. If the various factors composing K are known, α can be calculated for the pressure at which the filtration has been carried out. If the results of several constant-pressure filtration experiments are available, values of α can be determined for each of the pressures and plotted against pressure.

FIG. 232.—Determination of constants in filtration equation.

Example 40.—The results of a laboratory test[1] of a slurry containing $Fe(OH)_3$, filter aid, a soluble salt, and water are as follows:

[1] RUTH, MONTILLON and MONTONNA, *op. cit.*, **25**, 157 (1933).

Time, Minutes	Filtrate Collected, cubic feet
0	None
0.167	0.01725
0.333	0.0273
0.500	0.0352
0.666	0.0417
0.833	0.0478
1.000	0.0530
1.500	0.0672
2.00	0.0792
2.50	0.0897
3.00	0.0991
3.50	0.1080
4.00	0.1163

Composition of slurry:

$Fe(OH)_3$ = 0.01180 lb. per lb. water
Filter aid = 0.03646 lb. per lb. water
Soluble salt = 0.00686 lb. per lb. water

Pressure: 15.5 lb. per sq. in.
Temperature: 21.8°C.
Area of filter: 0.467 sq. ft.
The cake obtained was 19.2 per cent solids

Calculate the constants in Equation (316).

Solution.—Determine corresponding values of ΔV and $\Delta\theta$, calculate values of the ratio $\Delta\theta/\Delta V$, and plot these values as the heights of rectangles, the bases of which are the values of ΔV. The calculations are as follows:

θ	$\Delta\theta$	ΔV	$\Delta\theta/\Delta V$	θ	$\Delta\theta$	ΔV	$\Delta\theta/\Delta V$
0.167	0.167	0.01725	9.68	1.50	0.500	0.0142	35.20
0.333	0.166	0.01005	16.51	2.00	0.500	0.0120	41.67
0.500	0.167	0.0081	21.22	2.50	0.500	0.0105	47.60
0.666	0.166	0.0065	25.53	3.00	0.500	0.0094	53.2
0.833	0.167	0.0061	27.38	3.50	0.500	0.0089	56.2
1.000	0.167	0.0052	32.10	4.00	0.500	0.0083	60.2

The plot of the rectangles is shown in Fig. 233. A straight line is drawn as nearly as possible through the mid-points of the tops of the rectangles. The slope of this line is 484.5; and this is, by Equation (318), equal to $2/K$. The value of K is, therefore, $2/484.5 = 0.00413$. The intercept on the $\Delta\theta/\Delta V$ axis is 5.9, and this is equal to $2V_c/K$. The value of V_c is, therefore, $(5.9)(0.00413/2) = 0.0122$. The value of θ_c is obtained by calculating several values by means of the equation:

$$\theta_c = \frac{(V + 0.0122)^2}{0.00413} - \theta$$

For this purpose, the values of V and θ for the early part of the run give the best precision. In this case $\theta_c = 0.041$.

The final equation is:

$$(V + 0.0122)^2 = 0.00413(\theta + 0.041)$$

Rate of Washing.—It is usually assumed that, if filtration is stopped and the flow of slurry superseded by a flow of wash water, the resistance of a unit cube of cake to the flow of wash water is the same as it was to the flow of filtrate. In the case of filters

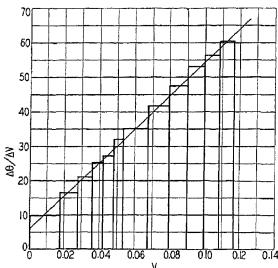



FIG. 233.—Solution of Example 40.

using true displacement washing (leaf-type filters), this is nearly correct, except for any changes due to the wash water having a lower viscosity than the filtrate. Hence, in such cases, the rate of washing is constant and equal to the rate of filtration at the end of the cycle.

Equation (318) may be written

$$\frac{dV}{d\theta} = \frac{K}{2(V + V_c)} \tag{319}$$

This gives the rate of filtration at any time during the run. By substituting in this equation the conditions that exist at the end of the filtration period, the rate of filtration at the end of the period is obtained. This may be considered the rate

of washing and may also be considered to hold constant throughout the washing period.

In the case of the plate-and-frame filter press, the same equation may be used to determine the rate of filtration at the end of the cycle. The surface available for washing is only half the surface used during the filtering period, however; and since the wash water passes through the whole thickness of the cake, while the filtrate passes through only half of it, it is usual to assume that the resistance per unit area of the cake is doubled. Hence the rate of flow during washing will be approximately one-fourth the final rate of filtration. This is not so precise as the assumptions made for leaf filters. In the plate-and-frame press the wash does not follow the path of the liquid. Although the cake is assumed to be uniform, in actual practice it is not. This new path may have a greater or a lower resistance than the channels followed by the filtrate, and hence the assumption that the rate of washing is one-fourth the final rate of filtration is only approximate.

Example 41.—The laboratory press of Example 40 is to be washed with an amount of wash water equal to the total amount of filtrate recovered. How long will this take?

Solution.—The data for substituting in Equation (319) are

$$K = 0.00413$$
$$V = 0.1163$$
$$V_c = 0.0122$$

From which $dV/d\theta = 0.01607$. This is the rate of filtration, in cubic feet per minute for the whole press, at the end of the run. This is the rate of washing if the press is a leaf-type press; or four times the rate of washing if it is a plate-and-frame press. The time to put 0.1163 cubic feet of wash through the press would be:

$$\text{If leaf-type,} \quad \frac{0.1163}{0.01607} = 7.23 \text{ minutes.}$$

$$\text{If plate-and-frame, } 7.23 \times 4 = 28.9 \text{ minutes.}$$

CENTRIFUGES

A centrifuge is an apparatus utilizing centrifugal force for the separation of liquid from solids. It is essentially a development of a gravity filter wherein the force acting on the liquid, instead of being restricted to gravity, is enormously increased by utilizing centrifugal force.

The force developed by centrifugal action is given by the equation

$$F \quad \frac{Wu^2}{gR} \tag{320}$$

where

W = the weight of the rotating mass in pounds.
u = the peripheral velocity of the basket in feet per second.
R = the radius of the basket in feet.
g = the acceleration of gravity.

If N be the speed in r.p.m., then u is $2\pi RN/60$. Substituting this for u and 32.2 ft. per sec. per sec. for g gives

$$F = 0.000341WRN^2 \tag{321}$$

Centrifuges can be classified as discontinuous, (which in turn may be classified as top suspension and bottom suspension, or as top discharge or bottom discharge), continuous, and the super-centrifuge.

Discontinuous Centrifuges.—A typical example of a top-suspension, bottom-discharge centrifuge is shown in Fig. 234. This machine consists of a rotating basket A suspended on a vertical shaft B and driven at a high speed by a belt C. The sides of the basket are perforated and are also covered with a screen D on the inside. External steel hoops E add stiffness to the basket. Surrounding the basket is a stationary casing F which collects the filtrate that passes through the perforations and discharges it at outlet G. Such a machine is operated as a batch apparatus. The charge is put into the stationary basket, the power applied, and the basket accelerated to a maximum speed. After a definite length of time, the power is turned off, a brake applied, and the basket brought to rest. The bronze valve H at the bottom of the basket is raised, and the charge of solid is cut from the sides of the basket into the opening, usually with the basket turning at a slow speed. Certain details of construction are dictated by the high speed of rotation employed. For example, the shaft and basket are supported at one point only. In the case of Fig. 234 the suspension is from the top. A short stationary shaft J is suspended from a ball-and-socket joint K, but its motion is

somewhat restrained by a rubber buffer L at the top. Outside this stationary shaft and suspended from it by a thrust-type bear-

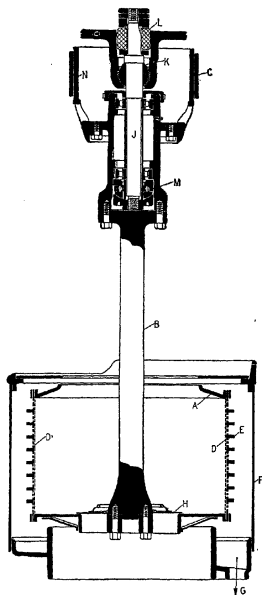



FIG. 234.—Weston top-suspended centrifuge. *A*. Basket. *B*. Main spindle. *C* Belt. *D*. Screen. *E*. Reinforcing hoops. *F*. Casing. *G*. Liquid outlet. *H*. Valve for discharge of solids. *J*. Stationary shaft. *K*. Universal joint. *L*. Rubber buffers. *M*. Thrust bearing. *N*. Pulley.

ing M is the rotating shaft B which carries the pulley N at the top and the basket at the bottom. This permits the whole machine to move more or less around the ball-and-socket joint as a center.

Such movement is quite necessary, because if the shaft were mounted between rigid bearings it would be impossible to make the construction heavy enough to withstand the strains resulting from a slight eccentricity in the loading. All centrifugal machines are supported by some type of universal joint or flexible bearing, although the details of construction may vary considerably from one machine to another. The different makes differ only in these mechanical details and not at all in principles of operation.

The machine shown in Fig. 234 with its bottom discharge is primarily suited for chemical work, because of the ease with which granular products can be discharged through the bottom. Centrifuges are widely used in the textile industry and these machines are usually made with a solid bottom so that the fabric may be removed through the top. Such machines are often underdriven, although the so-called *chemical type* or *bottom discharge* may also be underdriven. Such a machine is shown in Fig. 235. In this case the basket A is mounted on top of a short stiff shaft B which is carried by a thrust bearing C at the bottom and aligned between two sets of roller bearings in a bushing D. This bushing is maintained in its seat by stiff rubber buffers E. Consequently, although the shaft is rigidly aligned in the bushing, the bushing is free to move against the rubber buffer and therefore the necessary freedom of movement is obtained. The underdriven type can be made bottom discharge, but such constructions are more clumsy than top-driven machines, so that the underdriven machine is usually top discharge and its principal field is in the textile and laundry industries.

The stresses in tension developed in the shell of the basket are large, and therefore the basket shell itself must be fairly heavy and perforated with a relatively small number of holes. For instance, ⅛- or $\frac{3}{16}$-in. holes on ½- to ¾-in. centers is common. This perforation is far too coarse to accomplish the desired degree of separation. Consequently, the basket must be lined with either wire screen or fine perforated metal which performs the actual separation. If this screen or perforated sheet were simply laid inside the basket shell, only those portions of its area that came over the actual holes in the basket wall would be effective. Consequently, between the screen and the basket wall there is usually inserted a backing of coarse double-crimp screen or light corrugated metal perforated with rather large

holes. This keeps the fine screen away from the basket wall and at the same time gives space for the flow of liquid to the holes in the basket wall.

When the centrifuge is used for handling such materials as crystalline products, it is now customary to use automatic dis-

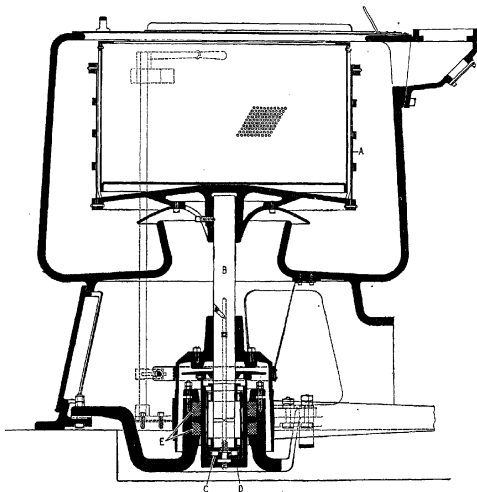



FIG. 235.—Underdriven centrifuge. *A*. Basket. *B*. Main spindle. *C*. Thrust bearing. *D*. Bushing. *E*. Rubber buffers.

chargers for removing the product. The automatic discharger consists of a scraper blade that is swung away from the basket during the centrifuging operation. At the end of this operation this scraper arm is swung over the basket, lowered by a rack and pinion arrangement, and then pressed against the basket wall by a hand lever. The basket is rotated slowly and at the same time the scraper blade is lowered slowly. This cuts the

entire charge from the wall and throws it down to the central bottom-discharge outlet.

Continuous Centrifuge.—The operation of the ordinary centrifuge is expensive, because of the labor necessary for its attendance and because of the power consumption. The inertia of the basket and charge is great, and the power necessary to bring the machine up to speed is many times the power required for maintaining speed once it is reached. For many years attempts have been made to devise continuous centrifuges but until very recently these attempts were all unsuccessful.

The *Laughlin continuous centrifuge* is shown in Fig. 236. A horizontal cylindrical cage *A* is rotated at a high speed. This

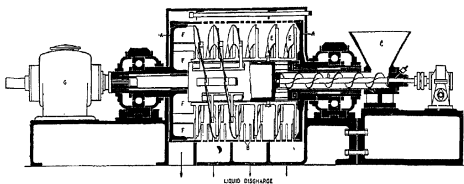



Fig. 236.—Laughlin continuous centrifuge. *A*. Rotating basket. *B*. Liquid-discharge slots. *C*. Feed hopper. *D*. Feed screw. *E*. Scrapers. *F*. Solid-discharge ports. *G*. Driving gears.

cage is perforated with slots *B* and lined with a screen just as in the case of the ordinary centrifuge. Material is fed from the hopper *C* by means of the screw conveyor *D* and is at once plastered against the outside of the basket by centrifugal force. Inside the basket is a spiral scraper *E*, which is driven at a rate slightly different from the rate of rotation of the basket. This results in a gradual scraping of the charge along the walls of the basket to the discharge end, where there are openings *F* in the basket wall through which the material escapes. Both the basket and the scraper are driven, and the necessary differential speed obtained, by the gear drive *G*.

The Supercentrifuge.—Referring to Equation (321), it will be seen that the centrifugal force exerted on a particle is inversely proportional to the radius of the basket but is proportional to the square of the speed. On the other hand, the strain in the

basket wall is proportional to its linear speed. If the radius of the basket be decreased by 50 per cent and the r.p.m. be doubled, then the linear speed of the basket, and hence the tensile stress in it, is unchanged but the centrifugal force has been doubled.

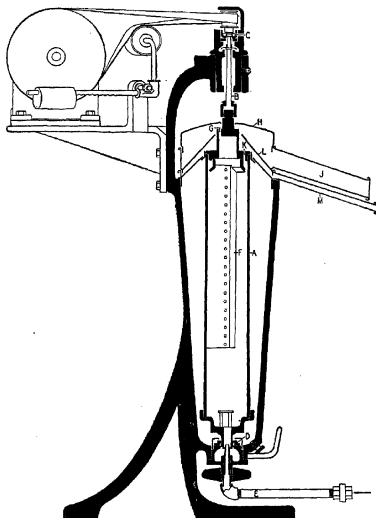



FIG. 237.—Sharples supercentrifuge. *A*. Basket. *B*. Drive spindle. *C*. Main bearing. *D*. Guide bushing. *E*. Feed pipe. *F*. Vanes. *G*. Liquid-discharge port. *H*. Discharge hood. *J*. Discharge spout. *K*. Heavy-liquid discharge port. *L*. Lower hood. *M*. Heavy-liquid discharge spout.

Consequently, where centrifugal separations are difficult to obtain, it is more practical to make the diameter of the basket small and the speed high, although this diminishes the capacity of the apparatus. Such a machine is shown in Fig. 237 which illustrates the Sharples supercentrifuge. The bowl *A* has now

become a relatively long vertical cylinder (4.25 in. is the maximum diameter used). This is hung on a flexible spindle *B* from ball bearings *C* at the upper end, and the bottom hangs free except as it is restrained from too great movement by a guide bushing *D*. The liquid to be treated is injected into the bottom of the bowl through a stationary nozzle *E*. Within the bowl are three baffles *F* to catch the liquid and force it to travel at the same speed of rotation as the perimeter of the bowl. The liquid is driven upward by centrifugal force and overflows at *G* near the top of the bowl, to be caught in a hood *H* and delivered by a spout *J*. This machine may be used to separate such small amounts of solid from the liquid that the solid may be allowed to accumulate on the inner surface of the bowl and be removed only at long intervals. In case the machine is to be used to separate two liquids from an emulsion, or to separate a clear liquid from a suspension of this same liquid with its sediment, a thin layer is discharged from the surface of the bowl at *K* into the second hood *L* and spout *M* below the first. This apparatus finds use in such processes as the filtration of varnish, of dry-cleaning liquids, of crude-oil emulsions, the removal of colloidal wax from lubricating oils, and many other similar cases where emulsions must be separated or very small amounts of solid removed. The bowl rotates at a speed of about 15,000 r.p.m.

Nomenclature of Chapter XIII

A = total filtering area
F = force
g = acceleration of gravity
K = a constant
m = weight ratio of wet cake to washed dry cake
N = speed in r.p.m.
P = total pressure drop across press and cake
P_c = pressure drop across the cake
R = resistance of the cake, or radius of the basket in feet
s = weight fraction of the precipitate in slurry
u = peripheral velocity of the basket in feet per second
V = volume of filtrate obtained up to time θ
W = weight
sub c = due to fictitious initial stage
sub T = total
sub 1 = of the filter medium
sub 2 = of the press channels
α = average specific resistance of cake

θ = the time
μ = the viscosity
ρ density of filtrate in pounds per cubic foot

Problems

1. A filter press has a filtering area of 300 sq. ft. It operates at a constant pressure of 20 lb. per sq. in. gage and gives 10,000 gal. of filtrate in 3 hr. Assume $\theta_c = 0.05$ hr. and $V_c = 50$ gal.

a. How much filtrate will be produced if the filter area is doubled, with other conditions maintained constant?
b. How much filtrate will be produced if the pressure is doubled, if doubling the pressure increases α by 25 per cent?
c. What will be the time of washing if the press is washed under the original conditions with 2,000 gal. of wash water after 3 hr. of filtering?

2. Laboratory filtration tests[1] on a precipitate of $CaCO_3$ gave the following data, where V is in liters and θ in seconds.

V	Test I 6.7 lb. per sq. in.	Test II 16.2 lb. per sq. in.	Test III 28.2 lb. per sq. in.	Test IV 36.3 lb. per sq. in.	Test V 49.1 lb. per sq. in.
	θ	θ	θ	θ	θ
0.5	17.3	6.8	6.3	5.0	4.4
1.0	41.3	19.0	14.0	11.5	9.5
1.5	72.0	34.6	24.2	19.8	16.3
2.0	108.3	53.4	37.0	30.1	24.6
2.5	152.1	76.0	51.7	42.5	34.7
3.0	201.7	102.0	69.0	56.8	46.1
3.5	.....	131.2	88.8	73.0	59.0
4.0	.....	163.0	110.0	91.2	73.6
4.5	.....	.....	134.0	111.0	89.4
5.0	.....	.....	160.0	133.0	107.3
5.5	.....	.....	.....	156.8	.....
6.0	.....	.....	.....	182.5	.....

In these experiments the following values of various constants were determined:

$$A = 440 \text{ sq. cm.}$$
$$s = 0.0230$$
$$m = 2.08$$

The filtrate was water and all tests were run at 25°C. Prepare a plot of α *vs.* pressure.

[1] B. F. Ruth, personal communication.

3. In the operation of a non-washing plate-and-frame filter press at constant pressure, show that, when the press is operated in such a manner that the average rate of filtration over the entire cycle is a maximum, the time of filtration is equal to the time lost in dumping, cleaning, etc. Assume that θ_c and V_c are negligible.

4. A filter press contains 24 frames, each 2 by 2 ft. inside dimensions. The frames are 1 in. thick. The press is equipped with 1- and 3-button plates for washing. The volume of wash water used is 10 per cent of the volume of filtrate per cycle. The time required for filtering is 2 hr. Washing is done at the same pressure as filtering. What is the time for washing? There are 0.05 cu. ft. of final cake per cu. ft. of filtrate.

5. A precipitate of the characteristics of the one in Problem 2 is to be filtered in a plate-and-frame press. The press will have 50 frames, each 1 in. thick and holding 0.93 cu. ft. The net effective filtering area of one side of one plate will be 12.30 sq. ft. The filtrate will have the same properties as water at 25°C. The final wet cake will weigh 90 lb. per cu. ft., the slurry will contain 1.75 per cent precipitate by weight, and m will be the same as in Problem 2. The pressure will be constant at 25 lb. per sq. in.

a. Make a plot of V *vs.* θ up to the time when the press is filled.

b. How long will it take to wash the cake with a volume of water equal to 15 per cent of the volume of the filtrate?

c. If it takes 2 hours to open, dump, and reassemble the press, what will be the capacity of the press in tons of dry precipitate per day of 24 hours, when each cycle is washed as in Part *b*?

CHAPTER XIV

MIXING

Any attempt to make a systematic study of mixing as now practiced meets with almost immediate failure for two reasons. The first is the tremendous multiplicity of types of mixing machinery used, which quite defies any attempt at systematic classification, and the second is the lack of any adequate theory that would enable one to understand the operation of these devices, make either quantitative or qualitative predictions as to their performance, or choose from different types the one that might be best suited to the process in hand. In general, one finds that the less definite knowledge there is regarding an operation the wider will be the variety of types of equipment used. On the other hand, in those operations whose theory is at least partly understood and where quantitative analysis can be made of the performance of a device, it will be found that the commonly used equipment falls into a small number of well-recognized types. This chapter, therefore, can do little more than collect a number of examples of mixing machinery and in a few cases indicate roughly the fields in which they are most widely used.

There is considerable confusion regarding the terms *mixing*, *agitating*, *stirring*, and *kneading*. With the exception of the last, which always means the manipulating of a plastic mass, these terms are not used technically with any precise significance and are to be considered almost interchangeable. Mixing, as possibly the most inclusive of them all, has been chosen as the title for this chapter, but no attempt is made to distinguish between mixing, agitating, and stirring.

In a very rough manner the whole field of mixing may be divided into three areas, namely, the mixing of liquids with liquids, the mixing of liquids with solids, and the mixing of solids with solids. Unfortunately these subdivisions have no lines of demarcation, so that apparatus that is suitable for mixing liquids with liquids is also suitable for mixing liquids with solids, up to

the point where the solid content begins to predominate. In the border line from here to the point where mixing is strictly a combination of solids with solids is the range in which kneading machines are usually found. There are very few devices for mixing solids with solids that could not be used equally well or at least with slight modification if the problem were one of mixing small quantities of liquids with large quantities of solids. Consequently, such a separation as was proposed at the beginning of this paragraph does not differentiate between types of equipment. The apparatus described in the following paragraphs is arranged very roughly in order, from the first types which are suitable for mixing liquids with liquids or for mixing large amounts of liquids with small amounts of solids, up to the last types which are suitable for mixing solids with solids.

Paddle Stirrers.—The simplest form of mixing device is a horizontal paddle in the bottom of a tank, driven by an upright post in the center. The paddle may be a square timber, a board set on edge, or a timber shaved off at an angle to give some lifting effect along with the rotation. There may be several paddles on a shaft at different heights in the tank, their ends may be connected by a strip to form the so-called *gate type* of agitator, or there may be fixed blades on the walls of the vessel in the intervals between the blades on the central shaft.

When this type is applied to a closed vessel under pressure, one paddle blade is usually set as close to the bottom of the vessel as possible and may be curved to conform to a dished bottom. In small pressure vessels the paddle stirrer is often mounted on a shaft sufficiently stiff and with sufficiently rigid bearings on the outside so that no internal step bearing is required. In open tanks, however, the construction is usually lighter and there is a step bearing at the bottom. Figure 238 shows various types of paddle stirrers as applied to different kinds of vessels.

In spite of the wide prevalence of the ordinary paddle type of stirrer, there are little or no published data on the performance of such equipment. The interesting variables are the time and power required to obtain a homogeneous mixture, expressed as functions of stirrer speed, for any definite piece of apparatus operating on any definite material. Figure 239 shows the type of characteristic curve that is obtained from a plain paddle

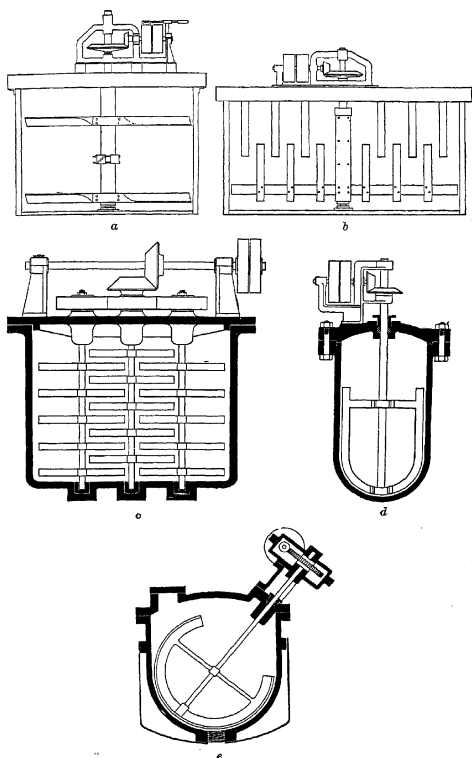



FIG. 238.—Various types of paddle stirrers.

stirrer.[1] These curves were determined experimentally on a 600-gal. vertical tank equipped with a paddle stirrer that consisted of a single 4 by 4-in. timber with one edge planed off, and mounted on a vertical shaft. The tank was 5 ft. in diameter by 5 ft. deep, and the propeller tips reached to about 6 in. from the wall of the tank.

The materials used in these experiments were water and strong brine. A layer of brine was introduced at the bottom of the tank at the beginning of the stirring operation, and the stirring was

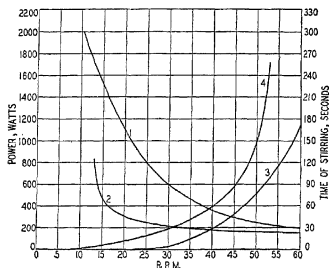



Fig. 239.—Characteristic curves for paddle stirrers. Curve 1, r.p.m. *vs.* time of stirring, plain paddle. Curve 2, r.p.m. *vs.* time of stirring, tank with baffles. Curve 3, r.p.m. *vs.* power consumption, plain paddle. Curve 4, r.p.m. *vs.* power consumption, tank with baffles.

considered to be complete when the mass in the tank had a constant electrical conductivity throughout. Curves 1 and 2 show the relationship between the time for complete mixing and the r.p.m. of the stirrer, while curves 3 and 4 represent the power as a function of the r.p.m. Curves 2 and 4 were obtained by installing four 1 by 4-in. baffles, vertically, 2 in. from the wall of the tank, while curves 1 and 3 show the results obtained without these baffles. It will be noted that as the r.p.m. is increased, a point is reached where the time for complete mixing is not appreciably shortened but the power required increases rapidly. Such an apparatus should therefore be operated at a lower r.p.m. than is represented by this point. It is also apparent from the

[1] Wood, Whittemore, and Badger, *Chem. Met. Eng.*, **27**, 1176 (1922).

curves that for a given r.p.m. the introduction of baffles will cut down the time of stirring but will correspondingly increase the power required.

It will also be noted from the curve that even such a crude apparatus gives surprisingly rapid stirring. Only a very few minutes are required to complete the stirring operation, with very moderate power requirements. It has been suggested that the curves for r.p.m. *vs.* time should really be expressed as functions of the linear velocity of the tip of the propeller rather than the r.p.m. of the stirrer. If this generalization is correct (there are no experimental data confirming it), the curves of Fig. 239 become of more general value by noting that 1 r.p.m. of this stirrer represents a tip speed of 0.21 ft. per sec.

Further work[1] has shown that the power consumed by a paddle stirrer may be represented by

$$Hp = 0.000129L^{2.72}\mu^{0.14}N^{2.86}\rho^{0.86}D^{1.1}W^{0.3}H^{0.6} \qquad (322)$$

where

Hp = horsepower consumed.
L = paddle length in feet.
μ = viscosity of liquid in pounds per foot-second.
N = speed of stirrer in revolutions per second.
ρ = density of liquid in pounds per cubic foot.
D = tank diameter in feet.
W = paddle width in feet.
H = depth of liquid in feet.

Propellers.—The use of high-speed propellers of exactly the same type as are used in ship propulsion is a method of stirring analogous to the paddle. The methods of utilizing propellers are practically infinite. Figure 240 shows three propeller arrangements. The effect of such variables as size of propeller with respect to the tank, pitch of the propeller, r.p.m. of the propeller, and position of the propeller in the tank, to say nothing of combinations of two or more propellers operating on the same or different shafts and in the same or different directions, has never been worked out experimentally. No general rule can be given for the most effective method of utilizing such propellers.

[1] White, Brenner, Phillips and Morrison, *Trans. Am. Inst. Chem. Eng.*, **30**, 570–584; White and Brenner, *Ibid.*, 585–597 (1933).

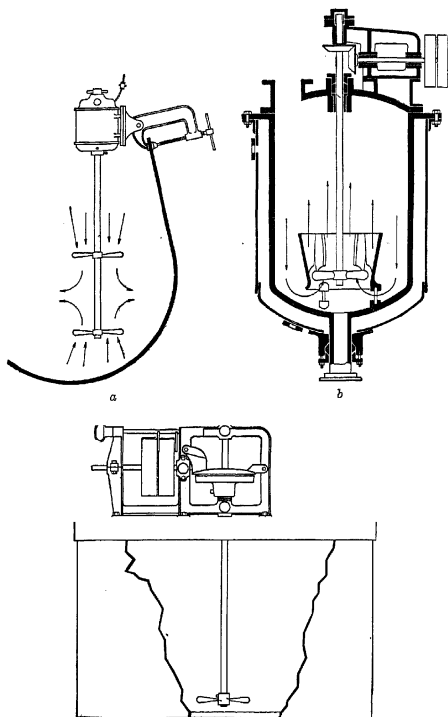



Fig. 240.—Typical propeller stirrers.

The only way that such a problem could be solved is by the laborious method of determining characteristic curves for all sorts of conditions and with widely different values for the above variables. Such work has not been carried out in any systematic way.

One set of curves, however, showing the performance of a typical propeller stirrer, is shown in Fig. 241. It will be seen that the qualitative shapes of the curves are the same as in Fig. 239, and it will also be seen that it is unnecessary to have a high

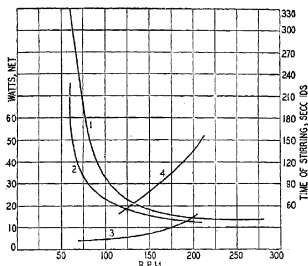



FIG. 241.—Characteristic curves of propeller stirrers. Curve 1, r.p.m. *vs.* time of stirring, 12-in. propeller, 5-ft. diameter tank, 12-in. from bottom of tank. Curve 2, r.p.m. *vs.* time of stirring, 12-in. propeller, 5-ft. diameter tank, 18 in. from bottom of tank. Curve 3, r.p.m. *vs.* power consumption, propeller 12 in. from bottom of tank. Curve 4, r.p.m. *vs.* power consumption, propeller 18 in. from bottom of tank.

speed in order to get rapid stirring. In fact, high speeds are uneconomical since they demand power inputs entirely out of reason in comparison with the gain in stirring rate. It must be understood, however, that the system of brine and water used in these experiments is much more easily mixed than would be a system of heavier or more viscous materials.

In order to get greater effectiveness, the propeller is sometimes located in a short open cylinder known as a *draft tube* (Fig. 240*b*). Various modifications of this are available, and this form gradually varies through paddle wheels with radial or spiral blades up to devices that are essentially the same as the impellers of centrifugal pumps. These draw liquid into the eye of the

impeller and discharge it at the periphery just as does a volute pump, except that there is no casing. This type probably displaces more liquid per unit time at a given speed than the propeller types.

Other Types of Stirrers.—Liquids may be agitated by bubbling compressed air or other compressed gases through them. In order to obtain satisfactory agitation, it is necessary that considerable volumes of air be used and that this air be very finely divided. This is usually accomplished by coils or manifolds located in the bottom of the tank and provided with a large number of very small holes, rather than a few large holes. The power consumed for agitation in this manner is apt to be much greater than would be used in a propeller or paddle type of stirrer.

Instead of distributing the air over the whole cross-section of the tank, it may be admitted under a central hollow shaft in which it acts as an air lift, and the discharge from this air lift distributed over the surface of the tank. Many agitators of this type have been used for keeping solids in suspension in liquids. The Dorr agitator (Fig. 194) and the Pachuca tank (Fig. 193) are typical of this class.

A method of agitation sometimes used is to locate a pump of any suitable type outside the vessel in which the materials are stored, pump continuously out of the bottom of the vessel, and discharge back into the top. The usual mistake that is made in this method of agitation is to use too small a pump. To be effective the pump should be able to displace the entire contents of the tank in a relatively short time. It is obviously less efficient to operate an impeller in an external pump casing than it would be to locate the impeller inside the tank as described in a previous paragraph. The method is often convenient when a relatively small amount of one material is to be mixed with a relatively large amount of another material, since the material that is to be added in the smaller amount can be added to the pump suction and thus be thoroughly incorporated in a stream of liquid in passing through the pump.

Beaters.—This term is used to cover a type of machinery, often used in the baking industry, in which liquids are whipped with moderate amounts of solids or in which two immiscible liquids are emulsified. This consists in a wire whipper driven

at a relatively high speed inside a container. The whipper not only rotates about its own axis, but this axis also revolves about the center of the container. This term is also applied to a device used in the paper industry; but in this case the apparatus subdivides fibers rather than performing a mixing operation.

Kneading Machines.—For mixing viscous masses, such as pastes, doughs, plastics, and all kinds of stiff mixtures, the kneading machine in general use has been fairly well standardized. It consists of an open trough with an approximately semicylin-

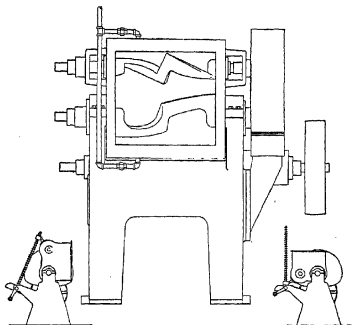



FIG. 242.—Kneading machine.

drical bottom. Within this trough two horizontal knives of roughly Z-shaped outline rotate, and these knives are so placed and so shaped that the material turned up by one knife is immediately turned under by the adjacent one. These machines are built in large sizes and may be designed to consume very large amounts of power. They may be jacketed for heating or cooling, and they may be closed to retain volatile solvents. They always operate on the batch principle and are therefore so mounted that they can be dumped by power-operated jacks. Figure 242 shows such a machine. The main illustration shows the machine in the dumping position to bring the knives into view.

Other than kneading machines, a considerable variety of devices are used in special industries. Some of these will be described in the chapter on crushing and grinding. For instance, the mixing of rubber and the incorporation into it of sulphur, filler, and accelerator is universally done on rolls, which for this purpose are about 3 diameters long and are internally water-cooled. The rear roll usually runs at a higher speed than the front roll. Putty and similar materials are mixed in devices called *chasers,* which are merely edge runners and will be described in Chap. XV. Many grinding operations involve as one of their important consequences a considerable degree of mixing, and in certain operations it is hardly possible to say whether the machine is a mixer or a grinder.

Dry Mixers.—The complete mixing of two dry powders is a difficult operation to perform and a still more difficult opera-

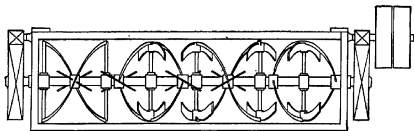



Fig. 243.—Dry mixer.

tion to control. It is usually quite difficult to say definitely that a certain batch is or is not uniformly mixed. Although dry mixing can be carried out continuously, it is often done in batches. Such batches are mixed in semicylindrical troughs, usually with covers to retain the dust, and provided with spiral ribbon stirrers. These are often so arranged that one set has a right-hand pitch and another set a left-hand pitch, thus moving the material continuously back and forth in the trough. Such a mixer is shown in Fig. 243. Various types of spiral ribbon stirrers and screw conveyors are used for this type of work. Possibly the best mixer for continuous operation consists in several lengths of screw conveyor with cut flights, as described in Chap. XVII.

Nomenclature of Chapter XIV

D = tank diameter in feet
H = depth of liquid in feet
Hp = horsepower

L = paddle length in feet
N = speed of stirrer in revolutions per second
W = paddle width in feet
μ = viscosity of liquid in pounds per foot-second
ρ = density of liquid in pounds per cubic foot

Problems

1. Based on Fig. 239, plot curves showing total energy consumed for complete mixing *vs.* r.p.m. of stirrer, both with and without baffles.

2. Repeat for the curves of Fig. 241.

3. What will be the power consumption of a paddle stirrer, consisting of a flat blade 4 in. wide and extending to within 2 in. of the tank wall, installed in a tank 6 ft. 4 in. in diameter, mixing two liquids whose density and viscosity are sensibly equal to that of distilled water, operating at 75 r.p.m.? The tank holds 2,000 gal. and is filled to within 1 ft. of the top.

CHAPTER XV

CRUSHING AND GRINDING

The general terms used to describe the operations that subdivide solids mechanically are seldom used with any very definite significance. The terms *crushing* and *grinding* are usually associated in this phrase to signify subdividing to greater or less extent, but neither of the terms is used alone with any precise meaning; although, in general, grinding means subdividing to a finer product than crushing.

In spite of the wide use of crushing machinery in hard-rock practice in the mining industry, little is really known of the basic theory that underlies processes for the mechanical subdivision of solids. As in certain other fields, this lack of theory and total reliance on empirical observation have led to an especially wide variety of types of equipment. By a process of natural selection rather than intelligent analysis certain devices have become preeminent for hard-rock crushing, and, as a result, the mining industry in recent years has practically standardized on certain types of machines for specific ranges of crushing.

In the fields in which the chemical engineer is interested outside hard-rock practice, there is absolutely no standardization. Consequently, this chapter will be largely a description of types of crushing machinery with an indication of the uses to which such apparatus is suited, although such usage arises more often from tradition and custom than from rational comparisons.

Because of the wide variety of devices used, it is extremely difficult to make a rigid classification of crushing machinery. The only classification in which definite limitations of the groups can be established is the division into coarse crushers, intermediate crushers, and fine grinders. Coarse crushers are defined as those types of machinery that can be developed to take, as feed, lumps as large as may be desired. Fine grinders are defined as those machines whose product will all pass a 200-mesh screen. Intermediate crushers are those machines that ordinarily

do not take indefinitely large feed, nor will they make a product that will pass a 200-mesh screen.

The different devices may be classified under these heads as follows:

1. Coarse crushers.
    a. Jaw crushers.
        (1) Blake.
        (2) Dodge.
    b. Gyratory.
2. Intermediates
    a. Rolls.
    b. Disc crushers.
    c. Edge runners.
    d. Coffee mills.
    e. Stamps.
    f. Disintegrators (cage).
    g. Hammer mills.
3. Fine grinders.
    a. Centrifugal.
        (1) Raymond.
        (2) Griffin.
        (3) Fuller-Lehigh.
        (4) Ring roll.
    b. Buhrstones.
    c. Roller mills.
    d. Ball mills and tube mills.

COARSE CRUSHERS

Machines in this class are ordinarily employed where the feed is from 1½ to 2 in. in diameter and larger. The largest devices of this class that have been made will take rocks up to 60 in. in diameter. No type of crusher except those listed in this class can be built in sizes that will take these very large pieces of feed.

Blake Jaw Crushers.—There are two distinct types of jaw crushers, the Blake and the Dodge. The Blake is by far the commoner, while the Dodge is rarely found. A typical Blake crusher is shown in Fig. 244. The Blake crusher is made by many concerns, and each maker has his own design. The one shown in Fig. 244 is not necessarily advocated as the best design but merely as one that includes most of the typical features. The crusher consists of an essentially rectangular frame *A* of either cast iron or steel. In one end of this frame is fastened the stationary jaw *B* which may be vertical or inclined. It is made

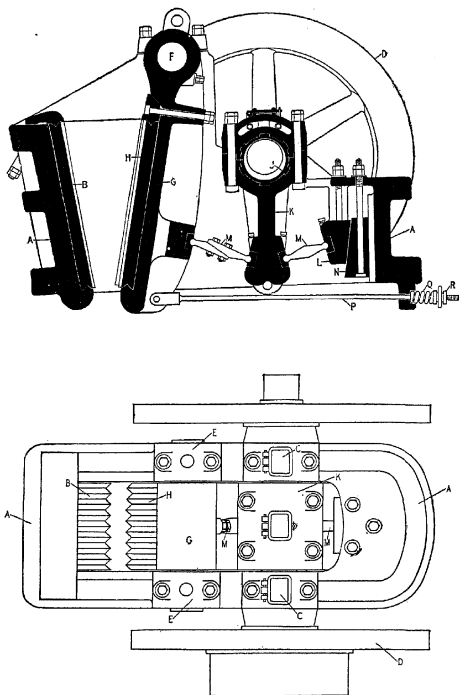



Fig. 244.—Blake crusher. *A*. Frame. *B*. Stationary jaw. *C*. Journal boxes. *D*. Pulley and flywheel. *E*. Movable jaw bearings. *F*. Movable jaw shaft. *G*. Movable jaw. *H*. Movable jaw wearing plate. *J*. Eccentric cam. *K*. Pitman. *L*. Fixed bearing. *M*. Toggle bars. *N*. Adjusting block. *P*. Tie rod. *Q*. Return spring. *R*. Adjusting nut.

of white cast iron, manganese steel, or some other material that will stand abrasion. The faces of the crushing jaws are usually corrugated to concentrate the pressure on relatively small areas. On the sides of the frame are the two journal boxes *C* between which runs a heavy shaft, and this shaft carries at one end the wheel *D* which serves both as a pulley and as a flywheel. Another pair of bearings *E* carry a shaft *F* from which is hung the movable jaw *G* with its wearing plate *H*. Most of the length of the shaft between the bearings *C* is developed into an eccentric cam *J* on which hangs the pitman *K*. Between the bottom of this pitman and the plate *G* on the one hand, and between the pitman and the fixed bearing *L* on the other hand, are two toggle bars *M*. As the main shaft rotates, the cam *J* causes the pitman to oscillate in a vertical direction, and the toggle bars transform this vertical motion of the pitman into a reciprocating motion of the movable jaw. There may or may not be an adjustable bearing, consisting of the two blocks *L* and *N*, in order to adjust the distance between the fixed and the movable jaws and thereby regulate the size of the product. The movable jaw is held back against the toggle by the link *P*, the spring *Q*, and the adjusting wheel *R*.

If accidental pieces of iron, such as hammer heads, stray bolts, etc., fall into the crusher, they will cause excessive strains unless some provision is made for relieving the crusher in this emergency. In the particular design shown, one of the toggle bars is made in two pieces and these two pieces are held together with bolts that are purposely made the weakest parts in the crusher. In case material that might otherwise cause destructive strains enters the jaws, these bolts shear through and allow the movable jaw to drop back far enough to discharge the obstacle. Thus the failure is made to take place at a predetermined point that can be easily and quickly repaired, instead of breaking some vital part of the equipment.

It will be noted that the maximum travel of the movable jaw is at the bottom. On the back stroke the material that has been crushed is permitted to drop freely from the jaws, thereby preventing any cushioning action from the accumulation of fine material around the coarser feed. This is the principal point of difference between the Dodge crusher and the Blake crusher.

The *Dodge crusher* is shown in Fig. 245. Its construction and its action are somewhat similar to the action of the Blake crusher, with the difference that in the Dodge crusher the movable jaw is hinged at the bottom. In the design shown in Fig. 245 the movable jaw A is prolonged into a lever arm B that rests on top of a pitman C, driven by an eccentric D, and there are no toggles involved. In the Dodge crusher the maximum travel (and therefore the minimum pressure) is exerted at the opening of the jaws, and the minimum travel (and therefore the maximum pressure) is exerted at the discharge point. This is just the reverse of the action of the Blake crusher and it would seem that the Blake

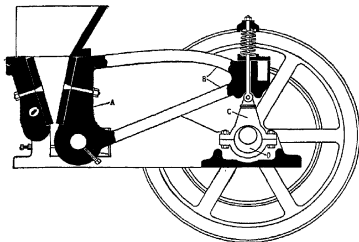



FIG. 245.—Dodge crusher. *A*. Movable jaw. *B*. Movable jaw lever arm. *C*. Pitman. *D*. Eccentric.

crusher is the more logical. The argument for the Dodge crusher is that, since it has the minimum movement at the discharge point, its discharge will be more uniform. This is not substantiated in practice; and the tendency of the Dodge crusher to become choked with fines is an additional disadvantage. Consequently, this type is rarely used and the Blake is by far the commoner.

Gyratory Crushers.—This type of crusher also is made by a large number of concerns, and there is much variation in the details of construction. Figure 246 shows a fairly typical design. A shaft A is hung by means of a bushing B and a lock nut C from a spider D. The bushing B does not fit tightly inside the bushing E, but there is some play allowed. On this shaft is fastened the conical crushing head F, and around this are the

concave crushing jaws *G*. The feed enters the hopper *H*, passes down between the arms of the spider *D*, and lodges between the head and the concaves. The lower end of the shaft *A* rides in a bearing in an eccentric bushing *J*, which is caused to rotate by the gear *K*, the pinion *L*, and the countershaft *M*. The rotation

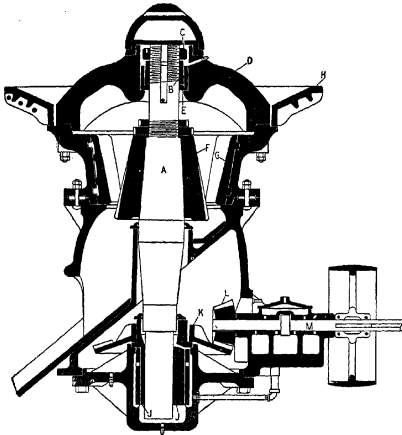



FIG. 246.—Gyratory crusher. *A*. Main shaft. *B*. Rotating bushing. *C*. Lock nut. *D*. Spider. *E*. Stationary bushing. *F*. Grinding head. *G*. Crushing jaws. *H*. Feed hopper. *J*. Eccentric bushing. *K*. Driving gear. *L*. Driving pinion. *M*. Driving shaft.

of this eccentric bearing causes the lower part of the shaft to wobble, not to rotate. The result is that the action between the head and the concaves in any one plane is exactly that of the Blake crusher, but this point of advance travels all the way around the head at each rotation of the bushing. The shaft *A* and the head *E* are free to rotate, but since the friction between the crushing head and the material is so much greater than the

friction between the shaft and the babbitted eccentric bushing, there is ordinarily little or no rotation of the shaft.

Because the gyratory is a rotary machine rather than a reciprocating one, the strains in it are more uniform, its power consumption is more steady, and it has a larger capacity per unit of discharge area than the reciprocating jaw crusher. For these reasons, the gyratory is becoming more and more widely used for the preliminary breaking of hard rock, and the jaw crushers are being gradually restricted to smaller installations where the first cost is of importance.

INTERMEDIATE CRUSHERS

It will be remembered that the class of intermediate crushers covers those machines that will not take indefinitely coarse feed or produce material that will pass a 200-mesh screen. The devices in this class vary widely among themselves as to the type of materials to which they are suited, the size feed they will take, and the size product they will make. Some will take feed as coarse as 1½ to 2 in., others must be fed with 40- to 60-mesh material. Some will make a product only ⅛ to ¼ in. in diameter, others will make a product that will approximate 100 mesh. These machines show a wide variation in construction, and it is not possible to say exactly what the advantages or disadvantages of the different types may be.

Rolls.—Crushing rolls have been adopted as the standard device to follow the gyratory crusher. They are made in a wide range of sizes, because they are not suited for a large ratio of reduction in one pass. Figure 247 shows a construction that is fairly typical. A heavy box casting A carries the fixed bearings B in which runs a shaft C on which one of the rolls D is mounted. This shaft carries the main drive pulley E. The other end of the main casting is finished somewhat like a lathe bed and on this ride the movable bearings F carrying the movable roll D'. These bearings are held in place by heavy springs G bearing against a nut on a rod H that passes entirely through the frame and has another bearing J. The purpose of this movable bearing held in place by springs is to permit the rolls to separate slightly in case tramp iron gets into the feed. The rolls are held in their normal position by shims K. These shims regulate the distance between the rolls and therefore the size of the product. The

pressure of the springs *G* is great enough so that in ordinary operation the movable roll bears tightly against the shims and

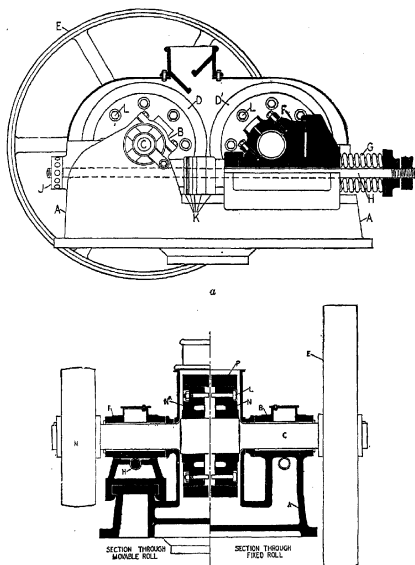



Fig. 247.—Crushing rolls. *a*. Elevation, *b*. Section. *A*. Frame. *B*. Fixed bearing. *C*. Fixed-roll shaft. *D*. Fixed roll. *D'*. Movable roll. *E*. Main drive-pulley. *F*. Movable bearing. *G*. Spring. *H*. Tie rod. *J*. Adjusting nut. *K*. Shims. *L*. Tie bolts. *M*. Movable-roll drive-pulley. *N*. *N'*. Main roll castings. *P*. Roll tire.

therefore the machine has a fixed discharge opening. The movable roll is driven by the pulley *M*, but the main power input

is through the pulley E, and the movable roll is driven largely by friction of the material being crushed.

The construction of the rolls themselves is shown in the cross-section. The body of the roll consists of two castings, one of which N is tightly pressed to the roll and the other of which N' is movable along the roll. A tire of high-carbon or manganese steel P is forged into shape with a slight taper on the inside from either edge toward the center. By drawing the two halves of the core together with the bolts L, the tire is firmly wedged into place.

Theory of Crushing Rolls.—In Fig. 248 let A_1 and A_2 be the centers of the two rolls of a pair, and let B be a spherical particle of material that has just been caught between the two rolls.

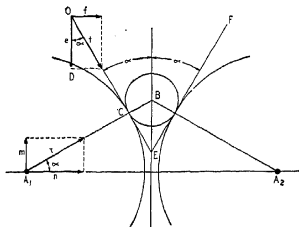



FIG. 248.—Theory of crushing rolls.

There is a certain force r acting on the particle, and this force makes an angle α with the line A_1-A_2. This force r may be resolved into two components m and n. There is also a force t tending to draw the particle between the rolls. This force t depends on the force r, and on the coefficient of friction between the material to be crushed and the roll surface. This force t may be resolved into two forces e and f. Since the line OC is perpendicular to the direction of the force r, it follows that the angle COD is also equal to α. If μ is the coefficient of friction, then

$$t = \mu r \qquad (323)$$

From the above statements the following equations may be written:

$$m = r \sin \alpha \tag{324}$$

$$e = t \cos \alpha = \mu r \cos \alpha \tag{325}$$

The forces e and m are opposed. Force e tends to draw the material between the rolls, while force m tends to eject it from the rolls. In order that the particle shall be drawn between the rolls and crushed, it follows that e must be greater than m, or

$$\mu r \cos \alpha > r \sin \alpha \tag{326}$$

$$\mu > \tan \alpha \tag{327}$$

In other words the tangent of the angle α must be less than the coefficient of friction. The coefficient of friction varies with different materials, but it has been found that an average value for the angle α taken from practice is about 16 deg. The angle *OEF*, which is twice the angle α, is called the *angle of nip.*

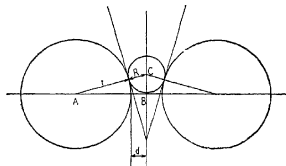



FIG. 249.—Capacity of crushing rolls.

There is a definite relation between the diameters of rolls, feed, and product. In Fig. 249 let R be the radius of the feed particle, r the radius of the roll, and d the radius of the largest possible particle in the product (half the minimum distance between the rolls). Then in the triangle ABC, angle CAB is α, AB is $r + d$, and AC is $r + R$. Then

$$\cos \alpha = \frac{AB}{AC} = \frac{r + d}{r + R} \tag{328}$$

Since for average conditions α is 16 deg. and $\cos \alpha$ is 0.961, the roll diameter is determined from the size of the feed and of the product by the equation

$$0.961 = \frac{r + d}{r + R} \tag{329}$$

Example 42.—What should be the diameter of a set of rolls to take feed of a size equivalent to 1.5 in. spheres and crush to 0.5 in., if the coefficient of friction is 0.35?

Solution.—Since $\mu > \tan \alpha$ (Equation 327), α must be less than $\tan^{-1}$ 0.35, or 19 deg. 17 min. Suppose α be taken as 18 deg. to allow some margin of safety. Then, substituting in Equation (328),

$$0.951 \quad \frac{r + 0.25}{r + 0.75}$$

Whence $r = 9.4$ in., or 18.8-in. diameter rolls. These odd sizes are not made, so 18-in. rolls should be used.

The theoretical capacity of a roll should be a continuous ribbon whose width is the width of the roll and whose thickness is the clear opening between the rolls. From this it follows that

$$C = \frac{(3{,}600)(12u)(w)(2d)}{1{,}728} = 50uwd \tag{330}$$

where

C = theoretical capacity in cubic feet per hour.
u = peripheral speed of rolls in feet per second.
w = width of roll face in inches.
d = half the roll clearance in inches (see Fig. 249).

If N is the speed of the rolls in r.p.m., and D the roll diameter in inches, u is $\pi ND/(12)(60)$. If Q is the capacity in tons per hour of material of sp. gr. s, then

$$\frac{62.3Cs}{2{,}000} \quad 0.0312Cs = 0.00680sNDwd \tag{331}$$

Actually the hourly output of a pair of rolls is from one-third to one-tenth of the amount so calculated.

Theoretically the speed of crushing rolls may be as high as desired provided the rolls have the correct angle of nip. Actually, if the speed is too high, the material rides on the rolls and is not drawn between them. On the other hand, if the speed of the rolls is too low, then the capacity is reduced unduly. The usual peripheral speed limit is from 6 to 10 ft. per sec., although the absolute upper limit may be as high as 20 ft. per sec.

Symons Disc Crusher.—This device is shown in Fig. 250. A horizontal hollow shaft C runs between two fixed bearings

and carries on its outer end a cage lined with conical grinding plates A. This shaft is rotated by the pulley D. A second shaft G is carried at one end in a spherical bearing B on the fixed shaft, and the other end is mounted in the eccentric bushing E, driven by the pulley F, which rotates at the same speed as pulley D. This shaft carries the grinding plates H. The movement of the eccentric bearing E, therefore, causes the grinding discs to approach at one part of their periphery and to separate at another part. This results in a crushing action very similar to that of the gyratory, except that this device is not suited

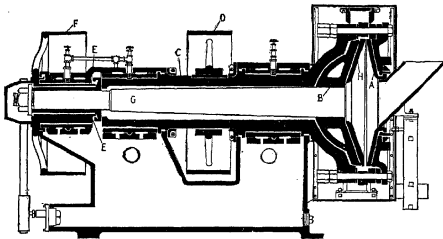



Fig. 250.—Symons disc crusher. *A*, *H*. Grinding plates. *B*. Spherical bearing. *C*. Hollow shaft. *D*. Drive pulley, hollow shaft. *E*. Eccentric bushing. *F*. Drive pulley, solid shaft. *G*. Solid shaft.

for such coarse feed as is the gyratory. The machine is used only on hard rock.

Edge Runners.—A considerable variety of mills and mixing machines fall in this general class. Probably the oldest type is the *arastra*. This consisted in a circular floor of roughly laid stones, and a vertical post was erected in the middle of this floor. Horizontal arms extended from this post, and to them were attached heavy stones by means of chains. A mule or an ox hitched to a long sweep caused the central post and its arms to rotate and dragged the stones over the material lying on the floor. This primitive device has been widely used and is, of course, obsolete at present, but several types of commonly used devices have grown out of it.

The first and most obvious development was to drive the central post by power and to replace the stones that were dragged from the side arms by heavy wheels. This type of mill was known as the *Chilean mill* and was at one time widely used in hard-rock crushing. Its only surviving representative is the *putty chaser*, which is really more of a mixing machine than a grinder. It consists of a stationary pan that will hold several hundred pounds of charge. Rising from the center of this pan is a vertical shaft that is rotated by means of a bevel gear and pinion, usually carried on heavy cast A-frames located at either side of the pan. To this vertical shaft is fastened a short horizontal shaft on which there is carried a heavy steel or granite wheel. This wheel is free to rotate about its own axis as well as to be rotated about the vertical axis. To the vertical shaft there is usually attached some sort of blade or plow that scrapes material from the outer edge of the pan back into the path of the wheel. If the motions above described be analyzed, it will be noted that the wheel, in addition to its crushing action, also has a rubbing action; and consequently this device is used as a mixer for such stiff masses as putty.

The only commonly used machine of this general type is the one in which the pan rotates and the horizontal axis of the grinding wheels is stationary. This machine is widely used in the clay industry, but little anywhere else. It is usually known as a *dry pan* or a *wet pan* according to whether the clay is crushed with or without the addition of water. It is not suited for hard-rock crushing.

A dry pan is shown in Fig. 251. Two heavy A-frames *A* are connected at the top by a yoke *B* that carries a bearing for a central vertical shaft *C* and the bearings for the horizontal jack shaft *D* that carries the pulley and the drive pinion of the bevel gear. The central shaft stands in a step bearing *E* at the bottom and carries the pan *F*, which may be from 4 to 12 ft. in diameter and from 6 to 12 in. deep. Near the center of this pan is a wearing ring *G* of steel or white cast-iron plates on which run the heavy cast-iron wheels or *mullers H*. These mullers are mounted on short shafts that rest in bearing boxes *J* at either end. These bearing boxes are not fixed but are free to move up and down in vertical slots so that the mullers may adjust themselves to the load in the pan. There is usually some form of scraper to

work the material in front of the mullers continually as the pan rotates.

The entire bottom of the pan may be solid, in which case the apparatus works in batches, and when a batch is completed it is shoveled out by hand. On the other hand, if the space between the wearing ring and the outside of the pan be covered with perforated plates, any material that becomes fine enough to

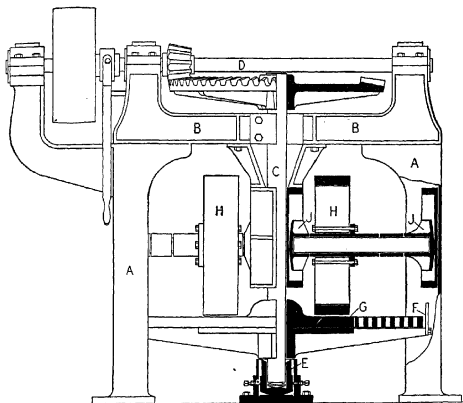



FIG. 251.—Dry pan. *A*. Frames. *B*. Yoke. *C*. Main shaft. *D*. Drive shaft. *E*. Step bearing. *F*. Pan. *G*. Wearing ring. *H*. Muller. *J*. Bearing boxes.

pass through these perforations is removed, and only the material that is not yet fine enough remains to be scraped back under the mullers. Such a pan operates continuously and may operate either wet or dry; in other words, the material may sift through the perforations or it may be washed through with a stream of water.

Coffee Mill.—The coffee-mill grinder for commercial operations is similar to the ordinary small-scale coffee grinder. Figure 252 shows an example of this type of crusher. A vertical shaft

A, driven from the bottom by bevel gears, carries conical grinding members B with the apex up. The part of the shaft that carries the grinding sections is square, so that the sections can be easily replaced and yet rotate with the shaft. The casing of the grinder consists of two sections C, each in the form of a truncated cone. The action of the machine is self-evident from the illustration. An interesting feature shown in this illustration is the adjustment

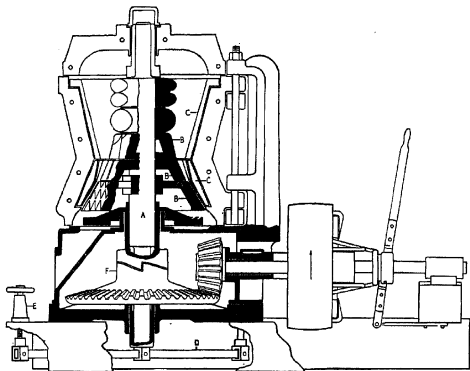



FIG. 252.—Coffee mill. *A*. Main shaft. *B*. Rotating grinding members. *C*. Stationary grinding members. *D*. Adjusting lever. *E*. Adjusting screw. *F*. Spiral jaw clutch.

to control fineness. The bearing of the vertical shaft is carried on a lever D, one end of which is pivoted under one side of the casing, and the other end of which is adjustable by means of a hand wheel and screw E. By turning the hand wheel, the shaft can be raised, the clearance between the revolving cone and the bottom cone of the casing can be decreased, and a finer product made. Conversely, if the shaft is lowered, the product is coarser, but the capacity is increased. The spiral step clutch drive F makes it possible to adjust the grinding member in a vertical direction and still permit the gears to run on the pitch line. This

mill is suitable for friable products such as limestone. It is not suitable for hard rock or for sticky materials.

Stamps.—A type of grinding equipment, now practically obsolete, but of great historical importance in the development of mining engineering, is the stamp mill. An example of this apparatus is given in Fig. 253. It consists of a frame in which are mounted a number of vertical stems *A*. Each stem carries a boss *B*, a shoe *C*, and a tappet *D*. A horizontal shaft *E* passing across the frame carries a cam *F* for each stem. The rotation of

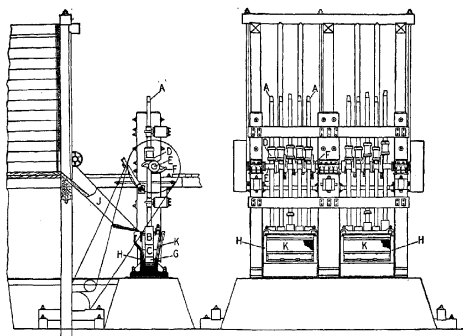



Fig. 253.—Stamp mill. *A*. Stems. *B*. Boss. *C*. Shoe. *D*. Tappet. *E*. Drive shaft. *F*. Cams. *G*. Die. *H*. Mortar box. *J*. Feed chute. *K*. Screen.

the shaft, transmitted through the cams to the tappets, lifts each stem and lets it fall. The assembly of stem, shoe, and boss weighs from 1,000 to 2,000 lb. The shoe, falling on the die *G*, crushes the feed by impact. The die is in the bottom of a trough *H* called the mortar box, which is fed from the rear through the chute *J*, and which is equipped in front with screens *K* through which the ground material must pass to escape from the mill. Stamp mills are usually operated wet; *i.e.*, water or cyanide solution is fed with the ore and washes the product through the screens. These mills have been almost entirely replaced by more modern types of equipment that are more efficient mechanically.

Squirrel-cage Disintegrator.—A type of grinder that is often called a disintegrator, because it is able to disintegrate or tear apart fibrous materials that are not too hard, is the squirrel-cage disintegrator. An example is shown in Fig. 254. The grinding elements are two or more cages, which consist of discs A on which are mounted rods B parallel to the axis of the mill. The example shown in Fig. 254 consists of four such cages. Two of them are mounted on disc A which is rotated by pulley C through the shaft D. The other two are carried on disc A' which is rotated in the opposite direction by shaft D' and pulley C'. The rotating cages are surrounded by a casing, and the feed is introduced into the

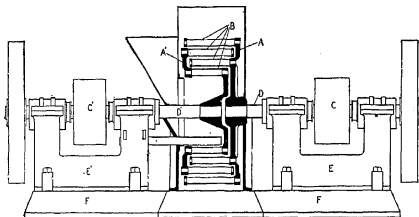



FIG. 254.—Squirrel-cage disintegrator. A, A'. Revolving discs. B. Disintegrator rods. C, C'. Drive pulleys. D, D'. Drive shafts. E, E'. Bearing supports. F. Base.

center of the inner cage. The centrifugal force imparted by the rapidly rotating cages throws the material from one cage to another. It is subdivided almost entirely by impact of the bars and is shredded or disintegrated by the time it reaches the outer casing. The machine is so constructed that either of the bearing supports E or E' can be backed away through slots in the base F so that the interior of the grinding cage is made accessible. This grinder is suitable for such friable materials as coal and limestone and also for such fibrous materials as packing-house tankage, bones, etc. It is mainly used in the fertilizer industry.

Hammer Mills.—This general name covers a wide variety of crushing and shredding devices that operate rather by impact than by positive pressure. One type of such device is shown in Fig. 255. In this machine a number of discs A are assembled on a

central shaft *B*. Between these discs are hinged hammers *C*, in the form of plain rectangular steel bars, which may be from one-eighth to one-half inch thick. On one side of the casing are breaker plates *D* of white cast iron or manganese steel, and around the bottom is a cage containing hardened screen bars *E*. The shaft is rotated at a high speed, and centrifugal force causes the hammers to swing out radially. Brittle or friable material like coal, pitch, limestone, or similar substances is beaten around inside the mill and by impact against the breaker plates or against the screen bars is crushed until it falls through the screen. By

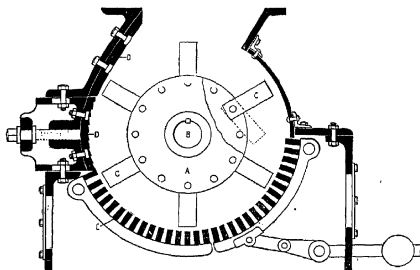



Fig. 255.—Hammer mill. *A*. Rotating discs. *B*. Shaft. *C*. Hammers. *D*. Breaker plates. *E*. Screen bars.

using hammers of different weights and screen bars of different cross-sections, the machine can be adapted to materials ranging from brittle materials like coal on the one hand to fibrous materials like tanbark on the other hand. The construction is such that the hammers can be easily replaced when they have worn. In the mill shown in Fig. 255, part of the screen cage is hinged so that it can be lowered to remove from the mill any hard material that cannot be pulverized. For brittle materials like coal or limestone the cross-section of the screen bars is usually rectangular as shown in Fig. 255. For shredding fibrous materials, the screen bars can be given a cutting edge as shown in Fig. 256. There are many types of these mills, differing in the details of

construction and in the shape of the hammer bars, but the action of all the mills of this type is essentially the same.

There are a number of modifications of this type in which the beaters are smaller and only a single ring of them is used. In such cases both the screen bars and the sides of the housing act

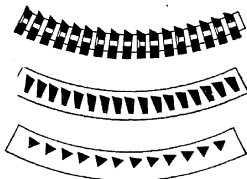



FIG. 256.—Types of screen bars for hammer mills.

as attrition surfaces. Such mills are used for grinding resin, pitch, drugs, cork, and similar soft or fibrous materials.

Single-roll Crusher.—Another type of machine that falls in this class because it depends mainly on impact, but which is not ordinarily classed as a hammer mill, is the single-roll crusher. Such a mill is shown in Fig. 257. The single roll is usually pro-

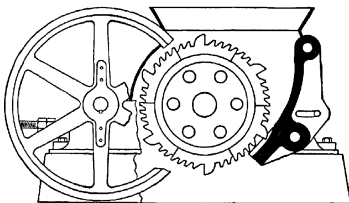



FIG. 257.—Single-roll crusher.

vided with corrugations or teeth of different sizes and rotates at a relatively high speed. Its action is similar to the hammer mill in that its crushing effect is produced by the teeth driving the material to be crushed against the breaker plate, thus crushing it by impact rather than by positive pressure. These machines

are made in a variety of designs and are quite generally used for crushing coal.

FINE GRINDERS

The machines in this class are all characterized by the fact that they will make a product most of which will pass a 200-mesh screen. This is the criterion of fine grinding, not because a 200-mesh particle is the smallest that can be produced, but because a 200-mesh screen is the finest screen that is ordinarily used for testing the product. Finer screens are made, but the wires are so fine, and the difficulty of weaving such fine cloth with a uniform mesh is so great, that such screens cannot be made with either the accuracy or the life that must be demanded of testing screens.

Buhrstone Mills.—The buhrstone mill is probably the oldest type of grinding machine still in use. Until relatively recently it was used for making flour, and some grain is still ground by this means. Many small buhrstones are still used for grinding paints, printers' inks, cosmetics, and pharmaceutical preparations.

Figure 258 shows an underdriven buhrstone mill. Buhrstones are distinguished as underdriven or overdriven not according to the location of the drive mechanism but according to whether the upper or the lower stone is the moving one. In Fig. 258 the upper stone *A* is held in the casing by a metal band *B* around the top, suspended from studs *C*. The lower stone *D* is carried on a spider *E*, which is driven by the shaft *F* resting in a step bearing *G*. By means of the handwheel *H* and the worm gear *J* the whole driving mechanism can be adjusted up or down to regulate the fineness of the product. Material is fed from the hopper *K* and is distributed to the surface between the two stones. The material gradually works out between the stones and ultimately leaves through the connection indicated. The stones used are a particular grade of sandstone that is mined in only a few localities. Buhrstones from France are often supposed to be superior.

One particular feature of the buhrstone mill is the method of dressing the faces. This method of dressing has been fixed for many years and its origin is uncertain. Figure 259 shows how the grooves are laid out. When the top stone is turned over so that the grinding faces of both top and bottom stones are uppermost,

the grooves are the same on both stones. When the top stone is in place, it follows that as the lower stone rotates, the grooves in the upper and lower stone will cross each other at an acute angle, and the point of intersection of two such grooves begins near the center of the stones and moves outward toward the

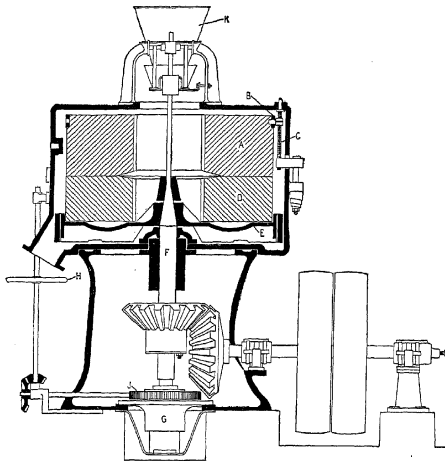



FIG. 258.—Buhrstone mill. *A*. Upper stone. *B*. Band. *C*. Studs. *D*. Lower stone. *E*. Lower-stone spider. *F*. Main shaft. *G*. Step bearing. *H*. Adjusting wheel. *J*. Adjusting gear. *K*. Feeder.

circumference as the stones rotate. The material to be ground is largely contained in the grooves, and therefore the action of the buhrstone mill is more nearly a shearing action than a rubbing action. The use of the buhrstone mill for grinding grain is rapidly diminishing and it is being replaced by the more modern roller mill.

Roller Mills.—The roller mill is at present used exclusively for the grinding of grain in the manufacture of flour. It is, however,

also suitable for grinding any moderately tough material that must be reduced to a very fine powder. One style of roller mill is shown in Fig. 260. It contains two pairs of rolls, and the rolls in each pair rotate toward each other. The rolls are corrugated, and one roll of each pair turns faster than the other one. This results in a shearing action instead of the direct pressure that is brought about in the ordinary crushing rolls. Figure 260 shows the drive side of the mill, half in elevation and half in section. Pulleys are attached to the fast roll *A* of either pair. The driving belt takes a turn around these pulleys and around the idler pulley *C*. This pulley drives an idler shaft *D* which passes through to the opposite side of the roll stand. From the

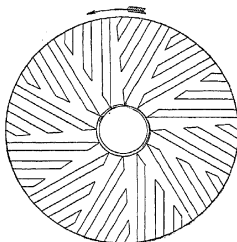



FIG. 259.—Method of dressing buhrstone.

other end of this idler shaft belts go to the slow roll *B* of each pair. Above the rolls proper is an oscillating feeder *E* that delivers the material to be crushed equally to both pairs of rolls.

One roll of each pair runs in fixed bearings. The other roll of each pair runs in bearings that are mounted on adjustable bell cranks *F*. These bell cranks are pivoted at the bottom and are adjustable by a hand wheel and screw at the top. In this way the distance between the rolls, and therefore the amount of reduction accomplished in each pass, may be regulated.

CENTRIFUGAL GRINDERS

The classification *centrifugal grinders* refers to those mills in which the grinding is done by one or more rotating mullers that exert a pressure on the material to be ground partly or entirely

by centrifugal force. This type of mill is widely used for the fine grinding of materials ranging from coal to hard rock and cement clinker. Only a few of the many types will be described here.

Raymond Mill.—The Raymond mill (Fig. 261) combines in one unit a grinding machine and an air separator. The main

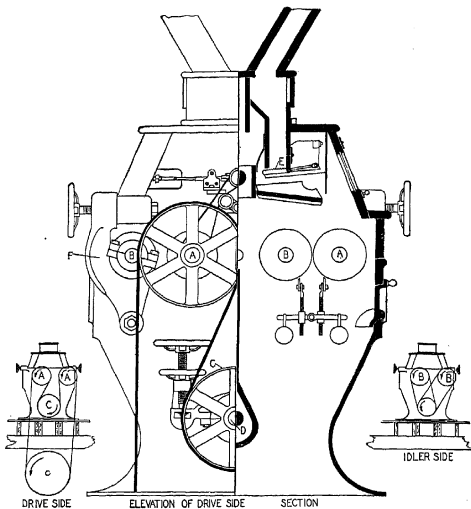



FIG. 260.—Roller mill. *A*. Fast rolls. *B*. Slow rolls. *C*. Idler pulley. *D*. Idler shaft. *E*. Feeder: *F*. Bell cranks.

base casting *A*, which rests on the foundations, carries a central vertical sleeve *B* through which passes the vertical driving shaft *C*. This shaft is supported on a step bearing below and is driven by a pair of bevel gears and a countershaft. To the top of this shaft there is attached a spider having two or more arms *D*, and each arm ends in a yoke. Within each of these yokes is sus-

pended a vertical shaft *E* which is pivoted around a short horizontal shaft *F* that crosses the opening of the yoke. The vertical shaft carries at its bottom a grinding head or muller *G* of manganese steel or chilled cast iron. The grinding head is free to rotate about the vertical shaft *E*, and the whole assembly is also free to rotate about the short horizontal shaft *F* at the end of the yoke As the main spindle with its spider is rotated, the grinding heads

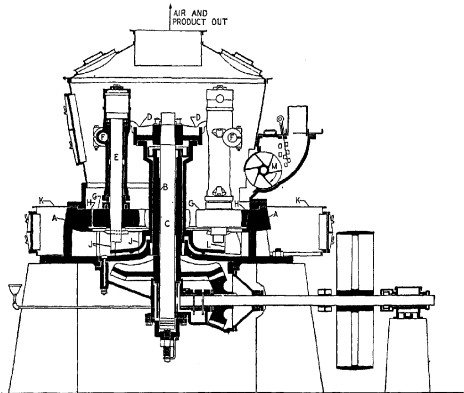



FIG. 261.—Raymond mill. *A*. Base casting. *B*. Central vertical sleeve. *C*. Main shaft. *D*. Spider. *E*. Muller shafts. *F*. Horizontal shaft. *G*. Muller. *H*. Bull ring. *J*. Fins. *K*. Casing. *L*. Plows. *M*. Feeder.

are thus pressed out against the bull ring *H*, which is a removable forging carried in the main base casting. The grinding heads crush by a rolling action rather than by a rubbing action. All of the above parts must be designed very carefully to insure that all bearing surfaces are kept free from dust.

Around the base casting below the bull ring are a series of openings provided with inwardly directed fins *J*, and around these openings is a light sheet-metal casing *K* into which air is blown under moderate pressure. This air passes up through the

mill, lifts any material that is fine enough to be suspended, and carries it out through the top of the casing to a separator. Material that has not been pulverized falls to the bottom, is picked up by plows L, and thrown on to the bull ring again. In this way the feed remains in the mill until it is ground sufficiently fine to be carried out in the air stream. By regulating the amount of air, the fineness of the material can also be regulated. Since the casing of the mill is under a pressure greater than atmospheric, there must be some positive mechanism for introducing the feed. This is accomplished as indicated by a feed hopper and a rotating toothed wheel M, which gives positive and regulated feed yet does not permit the blast to escape through the feed opening. The separation of the fine product from the stream of air is accomplished by devices that will be described in the next chapter.

The Griffin Mill.—The Griffin mill works on a principle very similar to that of the Raymond mill, except that there is only a single shaft and a single grinding head. The separation of the product is accomplished by blowing the material against a screen.

The Fuller-Lehigh Mill. The Fuller-Lehigh mill (Fig. 262) has a vertical shaft A driven by a pulley B in the lower part of the mill. On this shaft is built a cage C that carries pushers, and the pushers roll large cast-iron balls D around in a heavy forging or chilled casting E which acts as a ball race. Material to be ground is fed into the center of the mill and falls into this race. The cage that carries the pushers also carries fan blades F that blow the pulverized material against a vertical screen G. The material that passes through this screen falls from passages H in the casing and is blown out through a spout J by another series of fan blades K. These blades also induce the main draft through the mill. This mill has a positive feed device in the form of a short section of screw conveyor M. It has been especially used for grinding coal and cement clinker. Other modifications contain a more elaborate air-separating device in the upper part.

Ring-roll Pulverizer.—The ring-roll pulverizer is not strictly a centrifugal mill, but it is included in this classification because its method of operation and its field of usefulness are so similar to that of the other mills in this class. As shown in Fig. 263, the rolls are not driven, but the bull ring A is driven by a horizontal shaft B. Inside the head project three stationary shafts

C on which run the three grinding heads D. These shafts are mounted in bell-crank levers E, the fixed fulcrums F of which are attached to the head of the mill. The movable points of all three bell cranks are near the center of the head and are pressed

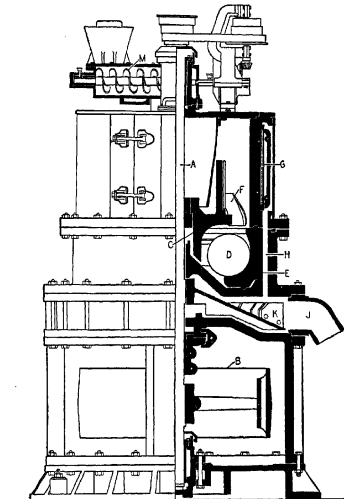



Fig. 262.—Fuller-Lehigh mill. *A*. Main shaft. *B*. Drive pulley. *C*. Cage. *D*. Grinding balls. *E*. Ball race. *F*. Fan blades. *G*. Screen. *H*. Discharge passage. *J*. Discharge spout. *K*. Main fan blades. *M*. Feed conveyor.

inward by a set screw G passing through a heavy yoke H which is held to the head of the mill by springs J. The bearing between the end of the set screw and the movable points of the bell-crank lever is formed by ball contacts so that the levers can move independently of each other. Material to be ground is fed through a spout at the top, and the rotating motion of the bull

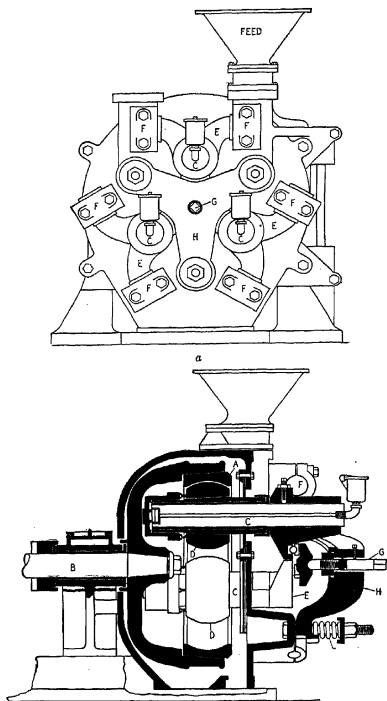



FIG. 263.—Ring-roll pulverizer. *a*. End view. *b*. Section. *A*. Bull ring. *B*. Drive shaft. *C*. Stationary shafts. *D*. Grinding heads. *E*. Bell-crank levers. *F*. Bell-crank lever fulcrums. *G*. Adjusting screw. *H*. Yoke. *J*. Tension springs.

ring throws it out by centrifugal force. The product escapes through an opening in the bottom of the casing.

Ball and Tube Mills

A very important class of fine-grinding machinery is that including ball mills and tube mills. The distinction between the two types at present is largely one of ratio of length to diameter, size of feed, and type of balls. Both consist essentially

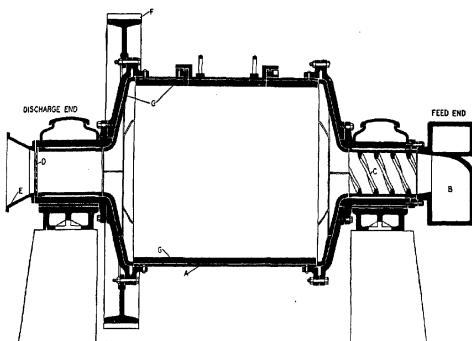



Fig. 264.—Ball mill. *A*. Casing. *B*. Feed scoop. *C*. Feed liner. *D*. Discharge screen. *E*. Discharge funnel. *F*. Drive gear. *G*. Liner plates.

of a horizontal cylinder, containing balls of flint, steel, or other materials, and rotated slowly about its axis. The feed is introduced into one end, and the impact of the balls on the material causes the fine pulverizing.

Ball Mills.—Figure 264 shows a section of a ball mill. It consists of a horizontal cylinder *A* whose diameter is roughly equal to its length, lined with heavy liner plates *G*. The feed end, which is at the right of the picture, consists of a helical feed scoop *B* which lifts feed into the spiral feed liner *C* from which it enters the mill. The load of balls, the size of the balls, the speed of revolution, and the rate of feed are all factors that are con-

trolled so that the discharge leaving the discharge screen *D* and the discharge funnel *E* is of the desired size. The discharge screen is a coarse-mesh screen, not to screen out particles that have not been ground to the correct size but to keep back any of the balls that may have entered the discharge end of the mill. The mill is rotated by the gear *F*. The mill in operation will contain balls of various sizes. They are not put in in various sizes but since the balls are being continually worn away by attrition, new balls must be supplied from time to time.

FIG. 265.—Tube-mill linings.

If the mill is rotated at a higher speed, more power will be required, but the fineness for a given capacity will be increased or the capacity for a definite fineness can be raised. Also, the smaller the balls the finer is the product. Finally, the faster the material is fed into the mill the faster will the discharge come out at the other end and the coarser will be the product.

The shell liners of a ball mill may be simply smooth tubular liners or they may be stepped liners. These mills may be operated either wet or dry.

Tube Mill.—A tube mill operates in the same general way as a ball mill except that its length is usually considerably greater than its diameter. Also, it is usually charged with flint pebbles rather than metal balls, and the average size of the balls is less than in a ball mill. Everything that has been said about the ball mill applies to the tube mill, except that the tube mill, in general, will deliver a finer product, other things being equal, than will a ball mill. Tube mills are very widely used in grinding hard rock and Portland cement clinker, because of their simplicity and because they operate at low speeds.

The liners of tube mills can be obtained in a wide variety of forms. Three examples are shown in Fig. 265. In general the linings are replaced as they wear out, while the outside shell is

permanent. In design *c*, pebbles wedge into the slots *A* and form the actual wearing surface, automatically renewed if a pebble cracks and drops out.

A modification of the tube mill is the *rod mill*, wherein the grinding agents, instead of balls or pebbles, are rods parallel to the axis of the tube.

Hardinge Mill.—A variation on the ball or tube mill is the Hardinge conical mill. An example is shown in Fig. 266. The principle of this mill is that, since the mill ordinarily contains balls of different sizes due to progressive wear, it would be advantageous to segregate the smaller balls near the discharge outlet where they can perform the finest grinding. The Hardinge

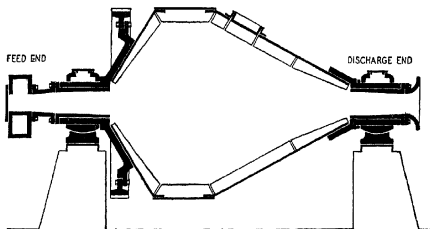



FIG. 266.—Hardinge mill.

mill accomplishes this by making the discharge end conical, which automatically sorts out the smallest balls and segregates them in the conical discharge end, while the largest balls remain in the cylindrical feed end.

THEORY OF CRUSHING

In spite of the wide use of crushing and grinding machinery, all attempts in the past to develop a theory of crushing or to coordinate crushing machinery by mathematical formulas have, in general, been unsuccessful. Two main theories have been advanced, known, respectively, as Rittinger's law and Kick's law. Neither of these laws exactly expresses the performance of any grinding machine, and the actual performance in many cases falls approximately midway between that calculated by the two expressions.

Rittinger's Law.—The law of crushing proposed by Rittinger is based on the assumption that the energy required for the crushing process is proportional to the surface sheared. To apply this assumption, consider the cube of material represented in Fig. 267. Each edge of this cube has a length D. Suppose that this cube is to be crushed into smaller cubes, each of whose edges has a dimension of d. In Fig. 267 the ratio of D to d has been taken as 1:4. This ratio $D:d$ may be represented by n. From Fig. 267, it will be seen that there are $(n - 1)$ planes of fracture in each of three directions. In other words, to crush this cube into smaller cubes there must be sheared a surface amounting to $3(n - 1)D^2$ sq. units. Suppose that it takes A ft.-lb. of work to produce 1 sq. in. of new surface. The work necessary to crush a cube whose dimension is D in. to cubes of a dimension d in. is, therefore,

FIG. 267.—Development of Rittinger's law.

$$\text{Work} = 3AD^2(n - 1) \tag{332}$$

In 1 cu. in. of material to be crushed there are $1/D^3$ of the larger units. It may be assumed that if the units are not perfect cubes, the ratio of the area of an actual unit to the area of a cube of the same major dimension will be the same for all sizes. This ratio will be represented by K. Inserting these values in Equation (332) and remembering that $n = D/d$, Equation (333) results:

$$\begin{aligned}\text{Work per cubic inch} &= \frac{1}{D^3}(3AD^2K)(n - 1)\\ &= 3AK\left(\frac{1}{d} - \frac{1}{D}\right)\end{aligned} \tag{333}$$

which is Rittinger's law.

One ton of a substance whose specific gravity is unity contains 55,320 cu. in., and, therefore, 1 ton of material whose specific

gravity is s will contain $55{,}320/s$ cu. in. Since 1 hp. is 33,000 ft.-lb. per min., it follows that the power necessary to crush 1 ton of material per hr. is given by

$$\text{Hp.} = \frac{\left(\frac{55{,}320}{s}\right)(3AK)\left(\frac{1}{d} - \frac{1}{D}\right)}{33{,}000 \times 60}$$

$$= \frac{0.0838AK}{s}\left(\frac{1}{d} - \frac{1}{D}\right) \tag{334}$$

It is not possible to determine A or K directly; and variations in specific gravity cannot be included safely in the use of the formula, for a change in specific gravity is almost certain to involve a change in the nature of the material that would affect A and K. It is best to write Rittinger's law as

$$\text{Hp.} = C\left(\frac{1}{d} - \frac{1}{D}\right) \tag{335}$$

where C is an arbitrary constant, to be determined from a test of a machine of the type to be considered, on the material to be considered, and with feed and discharge sizes not too greatly different from those in view.

Example 43.—A certain crusher takes rock whose average particle diameter is 0.87 in. and crushes it to a product whose average particle diameter is 0.31 in. at the rate of 15 tons per hr. At this rate the mill takes 8.0 hp., and 0.50 hp. are required to run it empty. What would be the power consumption for the same capacity if the average particle diameter in the product were 0.10 in.?

Solution.—The mill is using 7.5 hp. for actual crushing. The new net power consumption is given by

$$\frac{7.5}{\text{Net hp.}} = \frac{\left(\frac{1}{0.31} - \frac{1}{0.87}\right)}{\left(\frac{1}{0.10} - \frac{1}{0.87}\right)}$$

$$\frac{7.5}{\text{Net hp.}} = \frac{3.23 - 1.15}{10.0 - 1.15} = \frac{2.08}{8.85}$$

$$\text{Net hp.} = 31.9$$

$$\text{Total hp.} = 31.9 + 0.50 = 32.4$$

Kick's Law.—Kick's law assumes that the energy necessary for crushing material is proportional to the logarithm of the ratio between the initial and final diameters. This means that

if a definite amount of energy is required to subdivide a given weight of 1- to ½-in. cubes, the same amount of energy will reduce the ½-in. to ¼-in. cubes, or ¼-in. to ⅛-in. cubes, and so on. For example, according to this law, it should require twice as much energy to effect a ninefold as it would a threefold reduction. Kick's law can be expressed by the equation

$$\text{Hp.} = K \log \frac{D}{d} \tag{336}$$

where

K = constant.
D and d = the initial and final sizes, respectively.

Example 44.—How much power would be required under the conditions of Example 43 by Kick's law?

Solution.—Under present conditions Equation (336) is

$$7.5 = K \log \frac{0.87}{0.31}$$

from which

$$K = 16.75$$

For a final particle size of 0.10 in. the net power requirement should be

$$\text{Net power} = 16.75 \log \frac{0.87}{0.10} = 15.7 \text{ hp. or a total power of } 15.7 + 0.5 = 16.2.$$

It will be noted the Rittinger's and Kick's laws give widely different results for the same problem.

CRUSHER OPERATION

Crusher Feed.—Whether crushing and grinding follow Rittinger's or Kick's law, the power is largely consumed in crushing the finer particles. Consequently, if it is desired to reduce a given material to, say, 10 mesh, it is obviously a waste of power if any appreciable portion of the product is, say, 20 or 50 mesh. To insure a minimum of undersized material in the product, it is generally desirable that the feed to any crushing device be so limited that the product drops freely away from the crushing surface and there is no undue clogging of the surface or packing of fine material around the coarser lumps. This is known as *free crushing* and the opposite is known as *choke feeding.* In practice it is desirable to avoid choke feeding.

Many machines, such as the jaw crusher, gyratory, crushing rolls, and similar machines, will free themselves of the product if not fed too heavily and if the material handled is not damp or sticky. With other machines, and in fact with all machines when crushing sticky materials, some accessory device may be desirable to prevent choke feeding. Various automatic feeding devices have been developed, applicable to many types of machines. A grinding machine may be operated with a stream of water or solution flowing over the material to be crushed. The purpose of this is to carry away the fine particles and leave only the coarse ones for the action of the crushing surfaces. This is also the reason for the air-separation methods used in the Raymond, the Fuller-Lehigh, and similar mills. Still another method for preventing choke feeding, for reducing power consumption, and for preventing excessive formation of undersize material is described in the next paragraph.

Closed-circuit Grinding.—This term has come to mean the connection of a crushing or grinding machine with some type of device for carrying out size separation, in such a way that the entire product of the grinding machine goes to the size-separation unit. The undersize is the product and the oversize is returned to be reground.

The size-separation unit could conceivably be a screen of any of the common types. Since power consumption does not increase rapidly until the finer sizes are produced, it follows that closed-circuit grinding, though useful for any crusher, has its most important applications to those machines that normally make a fine product. It is precisely in these latter cases that the various types of screens are least effective, and consequently closed-circuit grinding is most often carried out with a stream of liquid flowing through the crushing unit and combining the grinding unit with one of the various wet separating devices (see pages 579 to 585).

One of the commonest applications of this system is in connection with ball mills or tube mills that are run wet. In this case the Dorr classifier (see page 580) furnishes a simple method for separating oversize and undersize. At the same time it can be designed to cause the oversize to travel a sufficient distance so that it is delivered from the classifier almost at the feed end of the tube mill, and, therefore, conveying equipment is reduced

to the minimum. Figure 268 gives a diagrammatic illustration of closed-circuit grinding applied to coarse and intermediate crushers as well as to a combination of a tube mill and a Dorr classifier.

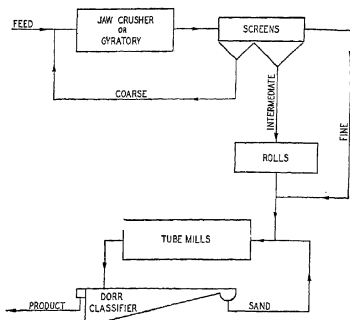



FIG. 268.—Flow sheet for closed-circuit grinding.

Nomenclature of Chapter XV

A = foot-pounds of work per square inch of new surface
C = theoretical capacity in cubic feet per hour, or a constant
D = roll diameter in inches, or length of edge of cube, or initial size
d = half the roll clearance in inches, or dimension of edge of small cube, or final size
K = ratio between area of an actual unit and area of cube of same major dimension, or a constant
N = speed of rolls in r.p.m.
n = ratio
Q = capacity in tons per hour of material of specific gravity s
R = radius of feed particle
r = force, or radius of the roll
s = specific gravity
u = peripheral speed of rolls in feet per second
w = width of roll face in inches
e, f, t, m, n, r = forces
α = angle
μ = coefficient of friction

Problems

1. A certain set of crushing rolls has rolls 40 in. diameter by 15 in. width of face. They are set so that the crushing surfaces are 0.5 in. apart at the narrowest point. The manufacturer recommends that they be run 50 to 100 r.p.m. They are to crush a rock having a specific gravity of 2.35, and the angle of nip is 30 deg. What are the maximum permissible size of feed and the maximum actual capacity in tons per hour, if the actual capacity is 12 per cent of the theoretical?

2. After long use, the tires on the rolls of the mill in Problem 1 have become roughened so that the angle of nip is 32 deg. 30 min. What will now be the maximum permissible size of feed, and the capacity?

3. A set of 36- by 16-in. rolls takes feed whose average diameter is 1.03 in. and crushes it to a product averaging 0.184 in. Under these conditions the power consumption is 9 hp. and the output is 100 tons per 24 hr. If the rolls are set up till the average size of the product is 0.118 in., what will be the power consumption according to Rittinger's law, neglecting power necessary to drive the rolls empty?

4. What will be the power consumption for the new conditions of Problem 3 according to Kick's law?

CHAPTER XVI

SIZE SEPARATION

This chapter will consider a number of operations that are used for the separation of solids on the basis of size. Some of the devices described can also be used for separating a mixture of solids by taking advantage of specific-gravity differences or variations in some property other than dimensions alone. Since this equipment is so similar to that which makes strictly size separations, however, it will be included in this chapter. The technique of such operations has been used far more widely in ore dressing than in industries ordinarily considered chemical, but since such operations illustrate important methods of separating solids, they will be discussed in this chapter.

SCREENS

The performance of size-separation equipment as well as the performance of crushing and grinding machinery is best shown by comparing the screen analysis of the feed with that of the product. Such analyses have been discussed in Chap. XII, and typical screen-analysis curves are shown in Fig. 202.

Testing Screens.—It has been the custom in the past to specify the screens used in testing merely by the number of meshes per linear inch. Thus, a screen analysis may show the weight-percentage of the material that passes through 10 mesh and on 20 mesh, through 20 and on 30, through 30 and on 40, etc. Such a report is quite meaningless and should never be employed unless the screens themselves are specified. The reason for this statement is that wire cloth with any given number of meshes per inch is made with a wide variety of wire diameters, and as the wire diameter varies the clear aperture of the screen varies. This is shown in the table on page 559.

This illustrates how meaningless such a specification as "30-mesh screen" may be, and it also shows how, by choice of wire diameter, screens from 20 to 35 meshes per inch are made with the same clear openings.

To remedy this situation various standard screen scales have been proposed, in which both the diameter of the wire and the number of meshes per inch are specified so as to give a definite ratio between the openings in one screen and the next succeeding screen in the series. The commonest set of standard screens is the *Tyler standard screen scale.* This is based on a 200-mesh screen with wire 0.0021 in. in diameter giving a clear opening 0.0029 in. square. Succeeding coarser screens have their mesh and wire diameter so adjusted that the area of the opening in one screen is approximately twice the area of the opening in the next-finer screen. This means that the linear sizes of the openings in any two successive screens have the ratio of $1:\sqrt{2}$. The full tabulation of these screen sizes is given in Appendix VIII.

Mesh	Wire diameter, inch	Clear opening, inch	Mesh	Wire diameter, inch	Clear opening, inch
30	0.017	0.0163	20	0.032	0.0180
30	0.014	0.0193	22	0.028	0.0175
30	0.012	0.0213	26	0.020	0.0185
30	0.010	0.0233	28	0.018	0.0177
30	0.008	0.0253	30	0.015	0.0183
			35	0.011	0.0176

Wire Screen.—Screen may be obtained in a variety of meshes and, as indicated above, in a variety of weights for any given mesh. In most screen the wire is given a double crimp which helps to preserve the alignment of the wires. The ordinary screen usually has the same number of meshes per inch in both directions, but special weaves are obtainable in which this is not the case. For example, a type of screen quite often used is the so-called *ton-cap* screen, in which the number of the meshes per inch in one direction is approximately twice that in the other direction. In general, if the opening in the screen is not rectangular, the size of particle that will pass through that opening is determined by the smaller dimension of the opening rather than the larger one. Screen can be obtained in all the common metals, although iron, brass, and copper are the most common. For very fine separations *silk bolting cloth* is sometimes used.

For special purposes punched metal is sometimes used instead of wire screen. The holes may be round or rectangular and the amount of solid metal between the holes is subject to considerable variation. For instance, a plate with a large number of holes and a small amount of residual metal will have a large capacity but will wear through rapidly, and *vice versa*. In general, for openings much over an inch in diameter a plate with round perforations is used rather than wire screen.

Screen Types

Since screens may be called upon to pass grains ranging from several inches in diameter down to 200 mesh, various types of screening equipment have been developed, differing largely in ruggedness, method of moving of the material across them, and materials of construction. A classification based largely on size of material is as follows:

1. Grizzlies are used for coarse screening of large lumps and are of rugged construction.
2. Trommels are rotating screens used for fairly large particles.
3. Shaking screens are used for fine sizing.
4. Vibrating screens are also used in fine sizing.

Grizzlies.—A grizzly is a simple device consisting of a grating made up of bars, usually built on a slope, and across which material is passed. The slope, and hence the path of the material, is parallel to the length of the bars. The bar is usually so shaped that the top is wider than the bottom, so that the bar can be made fairly deep for strength without being choked by particles passing part way through. The grizzly is often constructed in the form of a short endless belt so that the oversize is dumped over the end while the sized material passes through. The grizzly is used for only the coarsest and roughest separations.

Trommels.—A trommel consists of a rotating cylinder of perforated sheet metal or wire screen. It is open at one or both ends, and the axis of the cylinder is horizontal or slightly inclined, so that the material is advanced by the rotation of the cylinder. It is best suited for relatively coarse material (one-half inch or over). There is a considerable variation in trommel construction and arrangement. One type of construction is shown in Fig. 269. The discharge end is formed of a casting *A* carrying a stub shaft *B* resting on a bearing box *C*. This end also carries the driving

gear D which rotates the trommel. The other end consists of a heavy ring E resting on rolls F. Between the two ends extend heavy bars G or angle irons, to which the perforated plates are attached by steel straps. The oversize escapes through an annu-

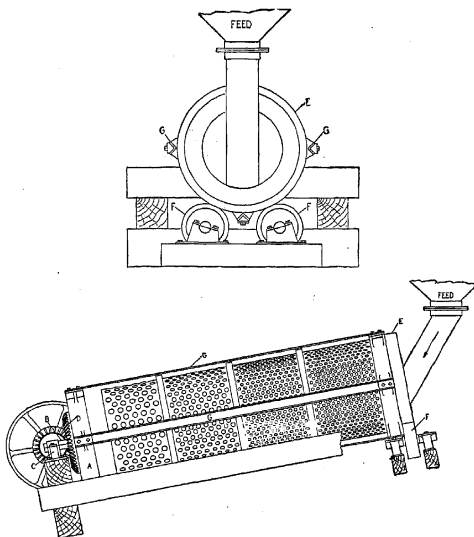



Fig. 269.—Trommel. *A*. Discharge end casting. *B*. Stub shaft. *C*. Bearing box. *D*. Drive gear. *E*. Tire. *F*. Supporting rolls. *G*. Angle-iron braces.

lar space between the end of the plate and the discharge end casting.

Several different arrangements of trommels are shown diagrammatically in Fig. 270. The simplest case is when the perforations are uniform over the whole length of the cylinder. The oversize

passes out the lower end into a hopper or chute. If a given material is to be separated into several size fractions, several trommels are operated in series. The first may have the coarsest perforations, in which case the undersize is delivered to the next trommel and it is most convenient to place the trommels one above the other as in Fig. 270*a*. If the first trommel has the smallest perforations, the oversize passes to the next trommel, and it is most convenient to put the successive screens in line end to end as in Fig. 270*b*. Still another variation (for smaller

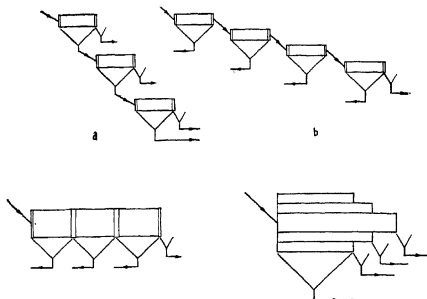



FIG. 270.—Trommel arrangements. *a*. One size screen to each trommel, coarsest trommel first. *b*. One size screen to each trommel, finest trommel first. *c*. Several size screens on one trommel. *d*. Different size screens on concentric trommels.

capacities) is to have a single cylinder with perforations ranging from the finest desired at the feed end to the coarsest at the discharge end. The arrangement is shown in Fig. 270*c*. In this case a separate hopper is placed under each belt of sizes. This has the disadvantage that the plate or screen with the finest openings is the weakest and at the same time is subjected to the heaviest wear. The trommel shown in Fig. 269 is of this type. A fourth arrangement (Fig. 270*d*) is several concentric cylinders. The innermost one is the longest and has the coarsest perforations. The outer ones are successively shorter and have finer

perforations. This has the advantage of putting the greatest load on the strongest screen but calls for more complicated and expensive construction.

When finer separations are to be made in a device of this type, the cylinder may be covered with fine wire or silk cloth instead of punched plate or coarse wire screen. Such an apparatus is usually called a *reel*. Reels are often used for very fine separations and in such cases some device must be employed to prevent *blinding*. This term means the wedging of particles not quite large enough to pass the screen, so that an appreciable fraction of the screen surface becomes inactive. This tendency

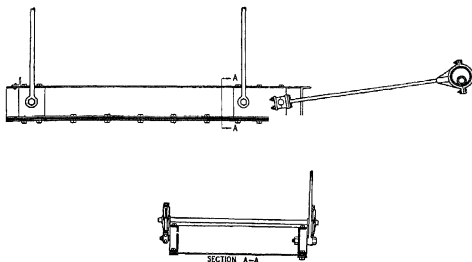



FIG. 271.—Shaking screen.

is more pronounced on fine screens than on coarse ones, and all fine-screening devices must involve some means to prevent blinding. In reels, this usually takes the form of rotating brushes mounted outside the reel cylinder. Since such fine separations are apt to produce considerable dust, fine reels are almost invariably enclosed in a housing with a screw conveyor to remove the product.

Shaking Screens.—Many size separations, in which the product may be from one-half inch down to almost the finest sizes that can be handled by screens, may be performed by means of flat or slightly inclined screens that are given a reciprocating motion. A wide variety of constructions is possible, but most such screens are quite simple. Figure 271 shows such a screen

made of simple mechanical elements. The frame is of channel irons and is suspended by hanger rods so that it can move freely. It is shaken by means of an ordinary eccentric on a rotating shaft. The screen cloth may be riveted directly to the frame, or it may be soldered over a light removable frame bolted into place. Another method of attachment is to provide a light angle that can be bolted to the inside of the frame. The edge of the screen cloth is drawn up between this angle and the main structure. A wide variety of such constructions is found, depending on the ingenuity of the designer.

Vibrating Screens.—To increase the capacity of flat screens, they are often made inclined instead of horizontal. In such cases, instead of giving the screen a shaking or a reciprocating motion, the screen is vibrated to keep the particles moving and to prevent blinding. This vibration may be accomplished by attaching to the screen cloth pins that pass through the screen casing. Rotating shafts on the outside of the casing carry hinged hammers that strike these pins. Another method is to place one or two light channels or other form of bearing surface on the underside of the screen frame. These channels rest on cams attached to rotating shafts. One of the well-known types of vibrating screen is the *Hum-mer*. This is shown in Fig. 272. The wire cloth is stretched between guides as shown and can be drawn up with considerable tension. Over the screen casing is mounted an alternating-current magnet whose armature is attached to the screen frame. This makes it possible to vibrate the screen very rapidly, and the result is both high capacity and freedom from blinding.

Another widely used screen is the *Rotex* (Fig. 273). In this device the screen *A* is nearly horizontal, and underneath the screen proper is a much coarser supporting screen *B*. On this supporting screen at intervals are wooden blocks *C* with sloping sides. Between the two screens are a number of rubber balls *D*. The shaking motion of the screen causes the rubber balls to fly about, and as they strike the sides of the wooden separating blocks the inclined sides deflect the balls upward against the underside of the screen cloth. The whole screen mechanism is supported at the lower end on sliding contacts *E* and at the upper end on a single bearing *F*, which is rotated by an eccentric pin on a heavy flywheel.

All of the flat or inclined shaking or vibrating screens can be made to perform one or more separations in a single piece of

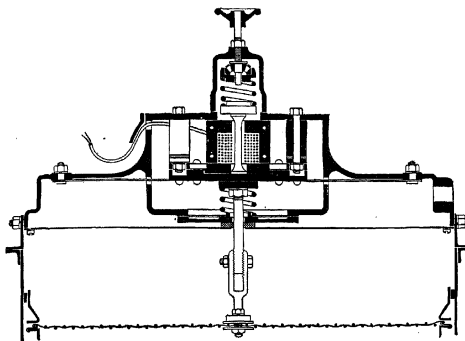



Fig. 272.—Hum-mer screen.

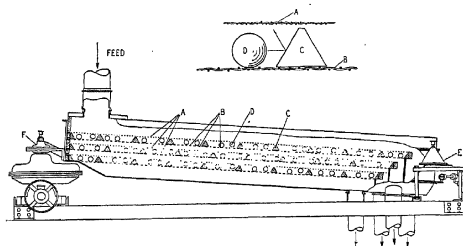



Fig. 273.—Rotex screen. *A*. Sizing screens. *B*. Coarse supporting screens. *C*. Wood separating strip. *D*. Rubber balls. *E*. Sliding bearing. *F*. Eccentric bearing.

equipment. If there is but one screen cloth, then only two products are made—oversize and undersize. It is possible, however, to mount two or more cloths, one over the other in

the same frame as shown in Fig. 273. The coarser cloths are above and the finer ones are below. Sufficient space is left between the screens, and each screen is made sufficiently shorter than the one above, so that a discharge chute can be attached to the end of each. In such an apparatus, if there are n cloths, there will be $n + 1$ fractions made.

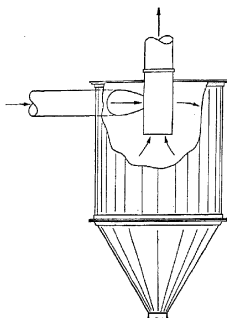



FIG. 274.—Cyclone separator.

AIR FILTERS

The word *filtration* generally brings to mind the separation of solids from liquids, but the problem of separation of solids from gases logically belongs in the same category. It is included in this chapter because it is the only method for obtaining a separation of solids finer than the finest commercial screens (approximately 200 mesh). Finely divided solids may be separated from gases by the following methods:

1. Cyclone separators.
2. Bag filters.
3. Cottrell precipitator.
4. Special air filters.

Cyclone Separators.—If a moderately complete degree of separation is satisfactory, and if the solid to be removed is not too finely divided, the separation can be made by utilizing centrifugal force. The apparatus used is universally called a *cyclone* and is illustrated in Fig. 274. The apparatus consists essentially of a short vertical cylinder, closed by a flat plate on top and by a conical bottom. The air with its load of solid is introduced tangentially at the top of the cylindrical portion. Centrifugal force throws the solid particles out against the wall and they drop into the hopper. The outlet for the air is usually in the center of the top, and is also usually provided with a nipple

that extends inwardly into the separator to prevent the air short-circuiting directly from the inlet to the outlet. Such separators are widely used for the collecting of wood chips, heavy and coarse dusts, and all manner of separations in which the material to be removed is not too fine.

Bag Filters.—Where a more complete separation must be accomplished than can be obtained in a cyclone, the bag filter is the next resort. In this case the air to be filtered is passed into long cylindrical bags of cotton or wool fabric, so that the dust that is removed stays inside the bag. Some provision is then made for cutting off the air at intervals and shaking the bags in such a way that the dust will be discharged into a collecting hopper. The ordinary household vacuum cleaner is a very simple bag filter.

One design of bag filter is shown in Fig. 275. A number of cylindrical fabric bags *A* are suspended in a sheet-metal container *B*. During ordinary operation, or filtering period, the gas to be filtered enters the hopper at *C*, passes up inside the bags, through the fabric, and out of the top of the casing to a main *D* leading to a suction fan. The suction fan is so designed that the whole apparatus is under less than atmospheric pressure. During the filtering operation the bags are suspended so that they are drawn taut. The shaft *E* rotates very slowly, so that at intervals of a few minutes the cam *F* presses against the bell-crank lever *G*, rotating it about its fulcrum *H*, and changing the position of the damper *J* to that shown in Fig. 275 as "shaking period." Since the unit shown is only one of several, and since all the units are under diminished pressure, this causes air to enter the casing and to pass into the bags, assisting in the displacement of the dust. Such dust-laden air passes into the hopper *K* and hence to the other units to be filtered. At the same time, the depression of the horizontal arm of the bell-crank lever *G* brings the bar *L* against the cams *M* on the rapidly rotating shaft *N*, and this results in violently jerking the bags so that they are freed from dust. The greater portion of the dust falls into the hopper *K*, from which it is removed at intervals by a gate attached at *P*. Such devices are entirely automatic in their action and can be designed so as to afford very large filtering surfaces per unit of floor space.

Figure 276 shows the *Perfection dust collector*, which is widely used in flour mills for removing explosive dust from the air.

A large number of fabric bags *A* are arranged around a central drum *B*. A tie bar *C* across the end of the bags in each row is

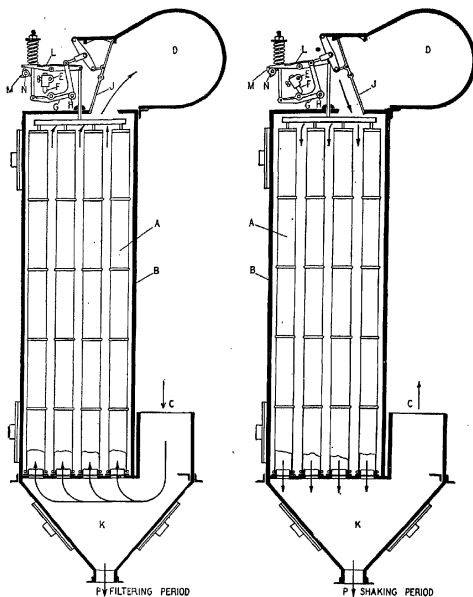



FIG. 275.—Bag filter. *A*. Filter bags. *B*. Casing. *C*. Inlet connection. *D*. Discharge manifold. *E*. Slow-speed shaft. *F*. Cam. *G*. Bell-crank lever. *H*. Bell-crank lever pivot. *J*. Damper. *K*. Dust hopper. *L*. Shaking lever. *M*. Shaking cams. *N*. Cam shaft. *P*. Product discharge.

held out by a spring *D* so that under normal operating conditions the bags are tight. Dust-laden air is admitted to the drum *B* at one end and passes through all the bags in parallel except

the row that is in the vertical position. A ratchet arrangement *E* slowly rotates the drum *B* by an amount equal to the spacing between the rows of bags at each stroke of the ratchet. The bags in the topmost row come over a discharge compartment *F* provided with a screw conveyor *G* for removing the dust. While they are in this position, a rapidly rotating shaft *H* provided with cams raps the bags violently with hammers *J* so as to shake the dust out of them into the discharge compartment *F*. This filter also operates continuously and automatically.

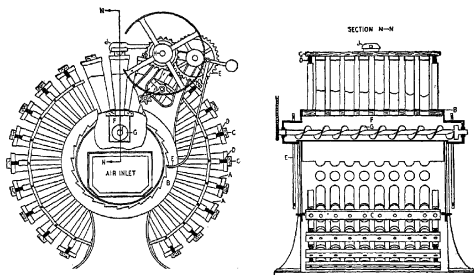



FIG. 276.—Dust collector. *A*. Filter bags. *B*. Inlet drum. *C*. Bag-supporting bars. *D*. Springs. *E*. Ratchet mechanism. *F*. Dust-discharge chamber. *G*. Discharge screw. *H*. Cam shaft. *J*. Hammers.

Cottrell Precipitator.—If a gas containing suspended solid or liquid particles is subjected to a strong unidirectional electrostatic field, the suspended particles are electrified by induction and will therefore be attracted to one or the other of the poles. If one of the electrodes has sharp points or edges while the other one is smooth, it is possible to ionize the carrier gas itself and therefore produce a drift of the gas toward the smooth electrode, which assists in the deposition.

The unidirectional high-voltage field (usually 50,000 to 60,000 volts) cannot conveniently be obtained by stepping up direct current. Alternating current is not suitable, as the drift must always be toward one electrode only. In practice the unidirectional high voltage is made by stepping up alternating

current in an ordinary transformer and connecting the two sides of the high-tension circuit to the contact points of a special switch. A disc is rotated at the same speed as the voltage alternations by using a synchronous motor. On this disc are contact shoes so arranged that they connect with the fixed contact points at the peak of each wave. This results in a pulsating but unidirectional electrostatic field.

The simplest method of applying the Cottrell process is to pass the gas through a number of vertical pipes, the inner surface of which serves as a smooth electrode. Down through the center of each pipe is a wire, which is usually composed of more than one strand twisted together with mica, fabric, or fine wire to provide the necessary discharge points. Because of the high voltages employed, the details of construction must be quite elaborate to furnish the necessary insulation and prevent power losses through leaks and arcs.

The Cottrell process has been successfully used for the removal of fine dusts from all kinds of waste gases. Examples are the precipitation of fine ore, lead oxide, or arsenic from smelter gases; the precipitation of potash-bearing dust from cement-kiln gases; the recovery of phosphorus pentoxide from phosphorus furnaces; and similar uses. Since drops of suspended liquid are also electrified by a static field, they may be removed by this method. The Cottrell process has been successfully used for the removal of acid mists in many processes. Permanent gases cannot be separated by this means, but if such a gas as sulfur trioxide or hydrogen chloride is present, which forms a liquid mist on exposure to water vapor, the gases may be first moistened and then passed through the Cottrell separator.

Special Air Filters.—In many processes it is necessary to have a supply of air as nearly as possible free from all suspended dust. Such processes are the drying of food products and other material susceptible to bacterial infection, the preparation of photographic films, and similar operations. For such purposes the air may be quite satisfactorily cleaned by passing it through a mat of steel or glass wool or similar closely packed fibrous material that has been wet with a heavy oil. The turbulence occasioned during the passage of the air through such tortuous passages sooner or later brings every particle of air into contact with an oily surface, so that quite high percentages of removal can be obtained. The

washing action of water sprays used for air conditioning is also quite effective in removing dust.

THEORY OF SETTLING PROCESS

When size separations are to be carried out on particles too small to screen effectively, or where very large tonnages are to be handled, methods involving differences in the rates of settling of particles of different sizes and of different materials are used. Suppose, for example, that two particles of different settling rates in water are placed in an upward-flowing water stream. If the velocity of the water is adjusted so that it lies between the settling rates of the two particles, the slower particle will be carried upward, the faster particle will move downward against the water stream, and a separation is thereby attained.

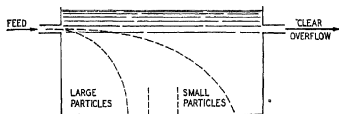



FIG. 277.—Simple hydraulic separation.

Another method would be to send a slow stream of water horizontally through a tank of water, as shown diagrammatically in Fig. 277. If the two particles are carried into the box by the entering water, they will both start to settle. Each will be carried horizontally at substantially the same rate, but the faster-settling particle will reach the bottom of the tank before the slower-settling particle and will be found nearer the entrance end of the tank than its slower companion.

It will be seen that the design and control of such settling devices depend on the laws of settling of solid particles in fluid streams.

GENERAL LAW OF SETTLING

Consider a particle of definite shape, of density ρ', of length D, starting from rest and settling in a stagnant fluid of density ρ and of viscosity μ. The particle will accelerate under the influence of gravity. As it accelerates, the fluid offers a greater and greater frictional resistance. A time will be reached when

the resisting force is exactly equal to the force of gravity, the acceleration will become zero, and the particle will settle at a definite constant velocity from this time on. Let this velocity be u.

It was shown in Chap. II (page 35) that the resistance offered by a fluid to a solid body in motion relative to the fluid is given by Equation (24):

$$\frac{F}{A} = \frac{\rho u^2}{g} \cdot \phi'\left(\frac{Du\rho}{\mu}\right) \tag{24}$$

where

F = the total resisting force.
A = the area of the solid in contact with the fluid.
u = the velocity of the fluid past the body.
ρ = the density of the fluid.
μ = the viscosity of the fluid.
ϕ' = a function the form of which is determined experimentally.

Since the area of contact is proportional to the square of the particle size,

$$F = \frac{\alpha D^2 \rho u^2}{g} \phi'\left(\frac{Du\rho}{\mu}\right) \tag{337}$$

where

α = a constant.

When the velocity u is a constant, the resisting force F must equal the force of gravity. Since the volume of the particle is proportional to the cube of the diameter,

$$F = BD^3(\rho' - \rho) \tag{338}$$

where

B = a constant.

Equating the force of gravity to the resisting force:

$$BD^3(\rho' - \rho) = \frac{\alpha D^2 \rho u^2}{g} \phi'\left(\frac{Du\rho}{\mu}\right) \tag{339}$$

Viscous Resistance. Stokes' Law.—It was shown in Chap. II (pages 35 to 39) that, in the application of Equation (24) to the resistance to fluid flow in pipes, if the velocity is below a certain critical point, the function $\phi'(Du\rho/\mu)$ becomes equal to

$$\phi'\left(\frac{Du\rho}{\mu}\right) = \frac{\mu}{Du\rho} \tag{340}$$

and that under these conditions the resistance is due to viscous friction alone, and not at all to turbulence. Above this critical velocity, however, there is turbulence, which adds to the resistance, and the form of the function $\phi'(Du\rho/\mu)$ is no longer simple. For velocities well above the critical, the influence of viscosity becomes small, the resistance is due almost entirely to eddies, and the function $\phi'(Du\rho/\mu)$ becomes substantially constant.

Due to the generality of Equation (24) it can be expected that the same sort of thing might happen in the process of settling; for low velocities the resistance should be viscous, and for higher velocities the resistance should be eddying. This is found to be the case. For low velocities, Equation (339) becomes, through the substitution of $\mu/Du\rho$ for $\phi'(Du\rho/\mu)$,

$$BD^3(\rho' - \rho) \quad \frac{\alpha D^2\rho u^2\mu}{gDu\rho} \tag{341}$$

or, solving for the velocity u,

$$u = \frac{BD^2(\rho' - \rho)g}{\alpha\mu} \tag{342}$$

The constant B/α has been determined by Stokes for spherical particles by means of a theoretical analysis, and Equation (342) is known as *Stokes' law*. If

u = the velocity of fall in feet per second,
D = the diameter in feet,
ρ' = the density of the solid in pounds per cubic foot,
ρ = the density of the fluid in pounds per cubic foot,
μ = the viscosity in foot-pound-second units,

Stokes' law is

$$u = \frac{D^2(\rho' - \rho)g}{18\mu} \tag{343}$$

This equation applies only to a spherical particle falling under viscous resistance in an infinitely large body of fluid, so that there are no effects of other particles or of the walls of the container.

If the particle is not spherical, the constant in Equation (343) is not $\frac{1}{18}$; and, in general, Equation (342) is most useful if written as

$$u = k_v \frac{D^2(\rho' - \rho)}{\mu} \tag{344}$$

where α, B, and g are combined in the viscous resistance constant k_v.

Eddying Resistance.—Under conditions of eddying resistance, which are associated with velocities above a certain critical point, $\phi'(Du\rho/\mu)$ becomes substantially constant, and Equation (339) becomes

$$u = k_e \sqrt{\frac{D(\rho' - \rho)}{\rho}} \tag{345}$$

where k_e is a constant.

Equations (344) and (345) show that for a given substance and a given fluid, in viscous resistance, u is proportional to D^2, and in eddying resistance, u is proportional to $\sqrt{D}$. Hence a plot of u *vs.* D on log-log paper should result in a straight line with a slope of 2 in the viscous range and a straight line with a slope of 0.5 in the turbulent range. Figure 278[1] shows the results of experiments with both quartz and galena that confirm the theory to a striking degree.

Just as in the case of fluid flow, velocities above the critical are of more technical importance than velocities below the critical and in the viscous range. In general, the settling velocity is so low and the particles so small in the viscous range that it is impractical to effect size separation in this manner. The process of sedimentation, however, or the separation of fine solid particles from liquids by settling, usually involves settling velocities below the critical.

Most hydraulic classification processes are described by Equation (345) rather than Equation (344), since such processes utilize

[1] RICHARDS, "Ore Dressing," McGraw-Hill Book Company, Inc., 1st ed., **3**, 1424 (1909).

settling under eddying resistance conditions, where the velocities are high enough to be practical. The process may involve particles of one material but of different size; of two or more materials of different specific gravity but of the same size; or of different materials, both materials appearing over a considerable range of sizes. Differences in shape may also be of importance, since the constant k_e varies with particle shape.

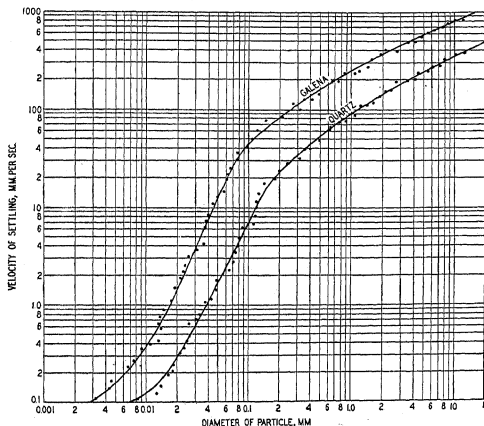



FIG. 278.—Relation between diameter of particle and velocity of settling (*Richards*).

Free Settling and Hindered Settling.—In all cases, Equation (345) can be used only if the particles are settling independently of one another. This is known as *free settling*. If the particles are so close together as to interfere with one another's movements, other factors are introduced and Equation (345) does not apply as written. A process carried out under such conditions is called *hindered settling*.

The simplest case of free settling, where the particles are of different sizes but of the same material and shape, is easily handled by Equation (345). Since k_e, ρ', and ρ are constants in such a case, Equation (345) becomes

$$u \quad K\sqrt{D} \tag{346}$$

and a plot of the velocity *vs.* diameter is a square root curve. If the particle sizes are known, and if the velocity of one of them is found, the velocity of the others can be calculated. The velocity of the separating fluid can then be fixed at some point between these two velocities, and a size separation affected.

Separation of Materials by Differences in Density.—If there are particles of two different materials, but of the same size and shape, Equation (345) can be written for each material. Thus if ρ_a' is the density of one material, and if ρ_b' is the density of the other, and if u_a and u_b are the respective velocities,

$$u_a = k_e\sqrt{\frac{D(\rho_a' - \rho)}{\rho}} \tag{347}$$

$$u_b = k_e\sqrt{\frac{D(\rho_b' - \rho)}{\rho}} \tag{348}$$

where k_e is a constant. It will be seen that by dividing Equation (347) by Equation (348),

$$\frac{u_a}{u_b} \tag{349}$$

and the velocities of the particles are proportional to the square roots of the differences in density between the materials and the separating fluid.

If, now, there are two different materials, and if both materials are of various sizes, it will be apparent that a large particle of the lighter material may have the same velocity of settling as a small particle of the heavy material. This is shown graphically in Fig. 279 where the velocity-diameter curves for the two substances are shown. Substance *a* is assumed to have the higher specific gravity. Suppose a mixture of *a* and *b* particles of the size range indicated by *MN* is considered. It is seen that the slowest (and smallest) particle of *a* is faster than the fastest (and largest) particle of *b*, and hence all of material *a* will settle faster than all of material *b*, and a free settling separation is possible. On the other hand, consider a mixture of *a* and *b* particles of size range *RS*. Here it is apparent that the slowest *a* particle (of diameter *R*) is considerably slower than the fastest *b* particle (of diameter *S*), and every particle of *a* in the size

range RS' has the same velocity as a larger particle of b in the size range $R'S$. Thus, there are three fractions possible: a fraction consisting entirely of b, every particle of which is in the size range RR'; a fraction consisting entirely of a, every particle of which is in the size range $S'S$; and an intermediate mixed fraction of a and b particles, the a particles of which are in the size range RS', and the b particles the range $R'S$.

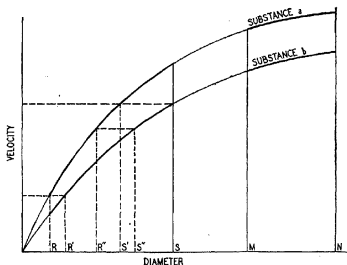



FIG. 279.—Separation of two materials by differences in rate of settling.

Equal Settling Velocities.—The relation between the particle sizes having equal settling velocities is obtained by writing Equation (345) twice, once for a and once for b, and equating u_a to u_b:

$$u_a = k_e\sqrt{\frac{D_a(\rho_a' - \rho)}{\rho}} = k_e\sqrt{\frac{D_b(\rho_b' - \rho)}{\rho}}$$

or

$$\frac{D_a}{D_b} \quad \frac{\rho_b' - \rho}{\rho_a' - \rho} \tag{350}$$

Equation (350) can be used to calculate R' and S' from R, S, and the densities of the two materials. It will be noted that the ratio of the diameters of particles of a and b settling at equal rates is a constant, depending only on the densities of the materials and the separating fluid. Thus, the ratio of S to S' is the same as that of R' to R or of R'' to S''.

Example 45.—It is desired to separate quartz particles from galena particles by taking advantage of their different specific gravities. A hydraulic classifier is employed under free settling conditions. The specific gravity of quartz is 2.65, and that of galena 7.5. The original mixture of particles has a size range from 0.00052 to 0.00250 cm. It is found that three fractions are obtained, one of quartz only, one of galena only, and one of a mixture of quartz and galena. What are the size ranges of the two substances in this third fraction?

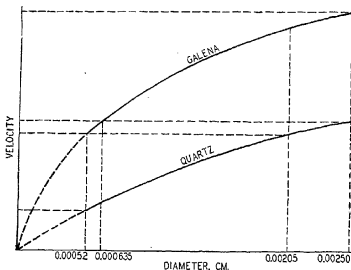



Fig. 280.—Solution of Example 45.

Solution.—It is obvious that specific gravities can be used in Equation (350) rather than densities.

The diameter of the largest galena particle in the combined fraction is

$$D_g = (0.00250)\left(\frac{2.65 - 1.0}{7.5 - 1.0}\right) = 0.000635 \text{ cm.}$$

and that of the smallest quartz particle in the combined fraction is

$$D_q = (0.00052)\left(\frac{7.5 - 1.0}{2.65 - 1.0}\right) = 0.00205 \text{ cm.}$$

The size ranges in the three fractions are shown in Fig. 280. The first fraction consists entirely of galena particles, 0.00250 to 0.000635 cm. in diameter; the second consists of a mixture of quartz particles, 0.00205 to 0.00250 cm. in diameter and of galena particles, 0.00052 to 0.000635 cm. in diameter; and the third is entirely quartz, of diameter from 0.00052 to 0.00205 cm.

Advantages of Hindered Settling.—It has been mentioned above that if the particles interfere with each other, the process is called *hindered settling* and Equation (345) does not apply.

If, however, instead of ρ, the density of the separating fluid, there be substituted the apparent density of the solid-liquid mixture, Equation (345) will approximately represent the process. The particles, interfering with each other as they settle, act as a sorting bed of higher density than the fluid itself. It will be seen from Equation (350) that if ρ is increased, and if ρ_a' is smaller than ρ_b', the ratio of the diameters of equal settling particles $(D_a:D_b)$ becomes larger. This effect is produced by hindered settling, and the separation is improved. If there is an intermediate fraction in a particular case of free settling, this intermediate fraction is reduced in size range, or eliminated entirely, if submitted to hindered settling conditions.

Hindered settling has other supplementary advantages, also. In the first place, the capacity of the apparatus is increased, since a larger weight of material can be passed through a given cross-section area. In the second place, the collisions among particles tend to knock off any small or light particles that may adhere to larger or heavier particles.

Example 46.—The classifier of Example 45 is operated on the same feed mixture but under hindered settling conditions, so that the apparent average specific gravity of the sorting bed is 1.6. Calculate the particle size ranges of quartz and galena in the intermediate fraction under these conditions.

Solution.—If there is an intermediate fraction, the largest galena particle in it will have a diameter of

$$D_g = (0.00250)\left(\frac{2.65 - 1.6}{7.5 - 1.6}\right) = 0.000445 \text{ cm.}$$

Since all of the galena particles in the feed are larger than this, there is no intermediate fraction, and a complete separation is obtainable.

PRACTICE OF HYDRAULIC CLASSIFICATION AND SEPARATION

It is apparent from the theory of settling that settling methods can be used for separation on the basis both of size and of specific gravity. It is true that screens also perform the former function, but screening large tonnages on an industrial scale becomes an expensive process when the particle size is much less than 1 mm., since these fine particles tend either to agglomerate together or adhere to the screen wires.

By the use of free settling methods, larger particles may be separated from small ones. An apparatus that uses such free settling methods is commonly known as a *classifier* and effects

a separation on the basis of size alone. By utilizing hindered settling, a fraction of reasonably uniform size range, but containing particles of two different specific gravities, can often be separated on the basis of material, and devices for this purpose are usually called *separators*.

Again, fine particles suspended in a liquid can often be settled to form more concentrated sludge, which can be filtered, or dried. This process is called *sedimentation*.

Hydraulic Classifiers

Classifiers are divided into two main classes: those using only the water in the feed to effect the separation of large and small particles, and those using auxiliary water, which is usually termed *hydraulic water*.

Simple Classifiers.—The first type of classifier, which does not utilize hydraulic water, applies the principles of free settling and merely separates larger particles from smaller, the smaller particles going on to subsequent treatment by means of which particles with different specific gravities are separated. The larger particles after being reground are again given a similar treatment.

Spitzkasten.—One of the best known classifiers of this type is the Spitzkasten. This consists of a series of inverted pyramidal containers. Each succeeding box is larger and deeper than the preceding one. The suspension of finely ground ore in water, often called *pulp*, flows in a horizontal direction from the smallest to the largest container. As a result of this, the velocity decreases in each compartment, and the largest particles settle to the bottom of the first box. Each succeeding box collects particles of smaller and smaller size, and the smallest ones are carried on by the water and flow out at the end of the series. Figure 281 illustrates this classifier. If the discharge is from the points of the boxes, the hydrostatic head on the discharge makes the velocity of discharge too high and causes undue stirring inside the box. By bringing the discharge part way up the side this difficulty is avoided.

The Dorr Classifier.—The capacity of Spitzkasten is rather limited, and a large amount of floor space is occupied for a relatively small production. For a larger capacity some such device as the Dorr classifier is usually used. Figure 282 represents this

classifier. It consists of a rectangular tank with a sloping bottom. A series of rakes *A* is moved a short distance up the sloping bottom *B* and then raised and moved back to the original position. The vertical movements are obtained by the bell

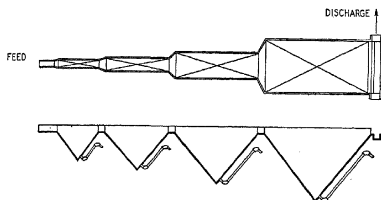



Fig. 281.—Spitzkasten.

cranks *C* and *D*, actuated by the cam *E*. The reciprocating motion parallel to the bottom of the tank is obtained by the pitman *F* and the eccentric *G*. A hand-operated winch *H* is provided to lift the rakes in case the power fails, to prevent

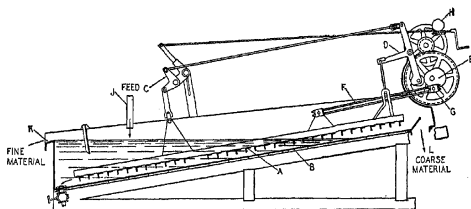



Fig. 282.—Dorr classifier. *A*. Rakes. *B*. Bottom of tank. *C*, *D*. Bell-crank levers. *E*. Operating cam. *F*. Pitman. *G*. Eccentric. *H*. Hand winch. *J*. Inlet for feed. *K*. Overflow for fine material. *L*. Discharge of coarse material.

them from being buried. The feed enters near the lower end at *J* and the fine material is discharged in suspension at *K*. The coarse particles settle to the bottom and are gradually moved up and out of the water by the rakes, to be discharged at *L*.

Through the action of the rakes, the material is turned over and over, thus allowing a more complete separation of the slime or fine particles that have been carried down mechanically by the coarse particles. The Dorr classifier is much used in connection with wet grinding processes, inasmuch as the size of the product may be varied over a wide range. This variation in size is regulated by the rate of feed and the rapidity of motion of the rakes. A spray of wash water may be added near the upper end; and several units such as shown may be placed end to end and operated in series to improve the degree of separation obtained.

Classifiers Using Hydraulic Water.—Classifiers using hydraulic water go a little farther than the mere separation of solids on the basis of size. It is possible by means of these classifiers to accomplish some separation on the basis of specific gravity. For instance, if a mixture of two compounds of different specific gravity has been ground to approximately uniform size, it is possible to separate the two compounds by allowing them to settle through a rising stream of water. If the velocity of the stream of water is a little greater than the free settling velocity of the lighter compound, the heavier compound will settle to the bottom of the tank and the lighter one will be carried out by the rising stream of water. Such a device, of course, requires much more careful previous grinding and screening and is in many cases preceded by classification in the first type of classifier described. By arranging a series of such classifiers with varying velocities in each one, it is possible to effect a gradation of products in a sludge that has not been previously classified or otherwise treated to produce particles of equal size. It is obvious from the theory of settling that these devices do not carry out the separation according to specific gravity so sharply as do the hydraulic separators using hindered settling.

Double-cone Classifier.—The double-cone classifier shown in Fig. 283 is an example of this type of classifier. The feed enters the inner cone *A* and hydraulic water is introduced at *B*. The particles settling from the inner cone meet a rising stream of water at the point *C*. The fine particles pass upward and escape by the peripheral launder *D*, while the coarse particles settle into the chamber *E* and are drawn off at intervals. The degree of separation accomplished is regulated both by regulating

the water supply at B and by regulating the height of the inner cone A by the hand wheel F.

Hydraulic Separators

It has been pointed out that a hydraulic separator may achieve a separation of particles of different specific gravities by utilizing hindered settling

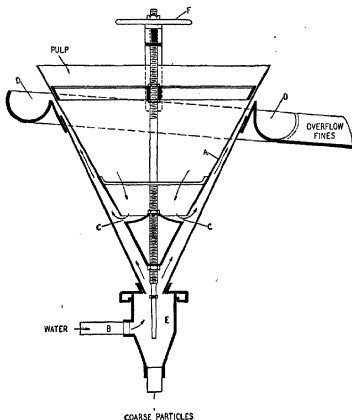



Fig. 283.—Double-cone classifier. *A*. Inner cone. *B*. Supply of hydraulic water. *C*. Mixing port. *D*. Discharge launder. *E*. Collecting box for coarse material.

Hydraulic Jig.—Probably the best example of this operation is the hydraulic jig. The jig (Fig. 284) consists of a compartment with a hopper bottom and a partition in the middle extending only about halfway down. On one side of the compartment is a screen A upon which the material to be separated is placed. The box is filled with water to a point above the surface of the charge. In the other side of the compartment there is a plunger or piston B which is given a small reciprocating motion by

means of a cam *C* on a revolving shaft. This reciprocating motion of the piston causes the water to be moved rapidly up and down through the layer of material on the screen in the other compartment. Hydraulic water is usually added on the side occupied by the plunger and serves to carry the finest material over the upper edge of the separating chamber into the next unit. As a result of the reciprocating motion, the charge of sludge is separated into four layers. The upper layer consists

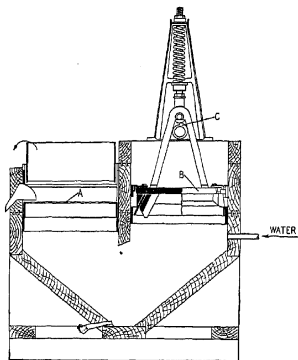



FIG. 284.—Hydraulic jig. *A*. Screen. *B*. Plunger. *C*. Eccentric cam.

of the smallest particles and includes the medium and fine particles of the lighter substance and the finest particles of the heavier substance. As the operation goes on, this layer is carried out by the stream of water to the next unit. The second layer consists of large particles of the lighter substance mixed with medium-sized particles of the heavier substance. This layer is mechanically removed at intervals and reground. The third layer consists of particles of the heavier substance too large to pass through the screen. This layer is also removed intermittently. The fourth layer consists of particles of the heavier substance that are small enough to pass through the screen

and are collected in the hopper below the jig chamber. The reciprocating motion above referred to is called *teetering* and serves mainly to keep the smaller particles in suspension. The net result of this condition is an apparent increase in the specific gravity of the liquid, giving the heavier substance a greater free settling ratio.

Wilfley Table.—The Wilfley table (Fig. 285) is another classifier that operates under the principles of hindered settling. It also belongs to the class of separators known as *jerking tables.* It consists of a flat table with parallel cleats or riffles extending longitudinally and ending along a diagonal line. The whole table is offset from the horizontal about 3 deg., the incline being away from the feed side. The water carrying the fine particles

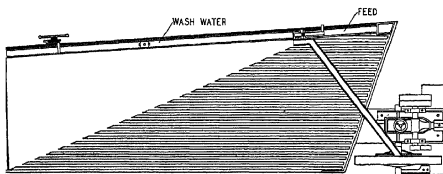



FIG. 285.—Wilfley table.

is allowed to flow diagonally across the table. The whole table is given a longitudinal reciprocating motion by means of a pitman and toggles. The heavier particles settle down behind the cleats and through the jerking motion of the table are gradually forced in a direction parallel to the cleats and discharged along the diagonal line already referred to. The fine particles are carried over the cleats and are discharged on the opposite side of the table from the feed.

Figure 286[1] shows diagrammatically the operation of a Wilfley table. The clean heavy material (concentrates) escapes at *A*. The middlings *B* carry some heavy material and are reworked, and the tailings *C* carry still less and can usually be discarded. The fine lighter mineral, or slime, escapes at *D*.

[1] RICHARDS, "Ore Dressing," McGraw-Hill Book Company, Inc., **3**, 1470 (1909).

AIR SEPARATION AND CLASSIFICATION

A special case of separation by settling methods is the use of an air stream for the separation of materials by size. It is apparent that the fluid used to carry the particles can be air instead of water. In such a case Equation (345) usually applies, since the particles are settling in the eddying resistance stage and under free settling conditions. However, ρ is much less than ρ', and Equation (345) simplifies to

$$u = k_e\sqrt{\frac{D\rho'}{\rho}} \tag{351}$$

An example of an air separator is shown in Fig. 287. The feed enters at *A* and falls on to the rotating plate *B*. Driven by the

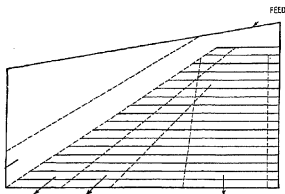



FIG. 286.—Diagram of Wilfley table. *A*. Concentrates. *B*. Middlings. *C*. Coarse tailings. *D*. Fine tailings.

same shaft is a set of fan blades *C*, which produce a current of air as shown by the arrows. Fine particles are picked up by the draft and carried into the space *D*, where the air velocity is sufficiently reduced so that they are dropped and are removed at *E*. Particles too heavy to be picked up by the air stream fall to the bottom and are removed at *F*.

SEDIMENTATION

Sedimentation may really be considered as a type of classification, since the aim of the process is to separate finely divided particles from a fluid in which they are suspended. This is also accomplished by filtration which is discussed in Chap. XIII. Sedimentation is usually applied to suspensions that settle readily

and do not therefore require filtration until after the sludge has been thickened by sedimentation.

Sedimentation Apparatus.—The simplest device for carrying out sedimentation is a vertical tank equipped with a variable-level draw-off for decanting the clear liquor, and a cone bottom fitted with a draw-off for the thickened sludge.

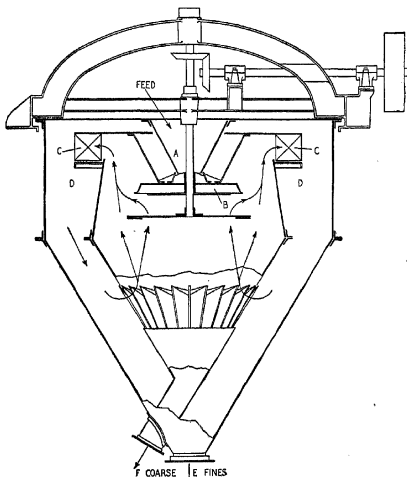



Fig. 287.—Air separator. *A*. Feed hopper. *B*. Rotating plate. *C*. Fan blades. *D*. Settling chamber. *E*. Discharge for fine product. *F*. Discharge for coarse product.

The most important settling device now used, however, is the *Door thickener*, described on page 420.

Methods of Increasing Settling Rate.—From the theory of settling given above it is apparent that the size of the particle is of great importance, since this variable appears as the square in

Equation (344). Sedimentation is therefore considerably facilitated by increasing the size of the particles. This may be accomplished by

a. Causing larger particles to increase at the expense of smaller ones by agglomeration.

b. Aggregating small particles through adding a coagulating electrolyte.

Theory of Sedimentation.—The rate of settling of the upper surface of the suspension is determined by the size of the smallest particle. If the range of sizes is not too great, the line between the sediment and the supernatant liquid is quite sharp. It is of importance to be able to estimate the time necessary for this line to settle through a given distance if the conditions of operation are given.

Since the sedimentation of fine particles, as ordinarily carried out, is a settling process under viscous resistance conditions, it can be expected that Stokes' law, or Equation (344), should apply. It is found by experiment that the rate of settling of a slurry of uniform-size particles is a constant for the first part of the process, where the particles are far apart and do not interfere with each other, but the rate soon begins to decrease and becomes slower and slower as the final level is reached. Robinson[1] has shown that the slowing of the rate is due to the facts that, as the slurry becomes concentrated, both the density and the viscosity of the suspension are increased, because of the packing together of the particles. Both ρ and μ of Equation (344), therefore, change as the operation proceeds, and the rate of settling decreases. Equation (344) then should be written differentially in the form

$$\frac{dX}{d\theta} = kD^2\left(\frac{\rho' - \rho}{\mu}\right) \tag{352}$$

where

X = height of sludge.
D = average particle size.
ρ' = density of particle.
ρ = density of the suspension.
μ = viscosity of the suspension.
k = constant.

If Equation (352) is written as

$$\frac{\mu dX}{\rho' - \rho} \quad kD^2 d\theta \tag{353}$$

it can be integrated graphically, if ρ and μ can be found as functions of X.

The viscosity μ is most conveniently determined as a function of the *relative concentration*, which is defined by Robinson as the ultimate height

[1] *Ind. Eng. Chem.*, **18**, 869 (1926).

of the sludge at infinite time divided by the actual height of the sludge. The relative concentration increases as the process goes on and reaches a maximum value of unity when the process has stopped, or at infinite time.

The relative concentration can be calculated from the initial height and concentration of the sludge and the bulk density and true density of the solid particles. At the end of the process, the particles are, theoretically at least, piled on each other as if the liquid were not present, and therefore the solids in the completely settled sludge exhibit the ordinary bulk density of the particles. Thus, consider an initial height of sludge of X_o cm. containing an initial concentration of C_o cc. of solid per cc. of sludge; let ρ_b' be the bulk density of the solid, and let the cross-section area of the column be 1 sq. cm. Then the ultimate height of the sludge will be (X_oC_o) (ρ'/ρ_b') and the relative concentration R at the time when the sludge has reached a height X will be

$$R = \frac{X_oC_o\rho'}{\rho_b'X} \tag{354}$$

The dependence of μ on X is best found by determining, experimentally, the relation between μ and R. For example, such a relation for silica particles of average size of 0.00895 cm. is shown in Fig. 288.[1]

The average density of the suspension ρ is calculated for any value of X by noting that at this time the total volume of slurry is X cc. while the weight of the slurry is $C_oX_o\rho' + (X - X_oC_o)$ g., and, therefore,

$$\rho = \frac{C_oX_o\rho' + (X - X_oC_o)}{X} = \frac{C_oX_o(\rho' - 1) + X}{X} \tag{355}$$

By assuming values of X, therefore, the corresponding values of R and ρ can be calculated by Equations (354) and (355) and the corresponding value of μ read from a viscosity curve such as Fig. 288.

Example 47.—The following data were taken on the settling rate of silica particles in water:[1]

Average diameter of particles in centimeters......	0.00174
Density of silica in grams per cubic centimeter....	2.54
Bulk density in grams per cubic centimeter......	1.16
Initial concentration in water, per cent by volume of solid..................................	21.0
Initial height................................	100 cm.
Height after 1,000 sec.........................	60 cm.

The viscosity *vs.* relative concentration curve is that of Fig. 288.

a. Calculate the constant k in the equation

$$\frac{dX}{d\theta} = kD^2\left(\frac{\rho' - \rho}{\mu}\right)$$

[1] Robinson, *loc. cit.*

b. It is desired to calculate the settling curve (X *vs.* θ) for silica particles 0.000895 cm. in diameter, starting with an initial concentration of 14.4 per cent by volume. The viscosity *vs.* relative concentration curve is the same as in part *a*. The bulk density of these particles is 0.935 g. per cc. Calculate and plot the desired curve on the basis of 200 cm. original height over the range $X = 200$ cm. to $X = 100$ cm.

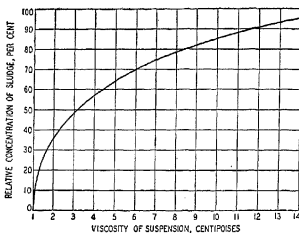



FIG. 288.—Relation between concentration and viscosity of sludge.

Solution. *Part a.* *Calculation of Constant k.*—In this case,

$$X_o = 100 \text{ cm.}$$
$$C_o = 0.21 \text{ cc. solid per cc. total}$$
$$\rho_b' = 1.16 \text{ g. per cc.}$$
$$\rho' = 2.54 \text{ g. per cc.}$$
$$D = 0.00174 \text{ cm.}$$

The relative concentration is, therefore,

$$R = \frac{(100)(0.21)(2.54)}{1.16X} = \frac{46}{X} \tag{356}$$

and the density of the suspension is

$$\rho = \frac{(0.21)(100)(2.54 - 1) + X}{X} = \frac{32.35 + X}{X} \tag{357}$$

Equation (353) becomes

$$\frac{\mu dX}{2.54 - \dfrac{32.35 + X}{X}} = k(0.00174)^2 d\theta$$

or

$$\int_{60}^{100} \frac{X\mu dX}{1.54X - 32.35} = (k)(0.00174)^2 \int_{1,000}^{0} d\theta = -(1,000)(0.00174)^2(k) \tag{358}$$

The limits in Equation (358) follow from the facts that when X is 100, θ is 0; and when X is 60, θ is 1,000.

The integral on the left-hand side of Equation (358) is evaluated graphically by assuming values of X, obtaining corresponding values of R and μ from Equation (356) and Fig. 288, respectively, plotting $\frac{X\mu}{1.54X - 32.35}$ *vs.* X, and measuring the area bounded by this curve, the X-axis, and the ordinates $X = 100$ and $X = 60$. The integral so determined is -160, and therefore

$$k = \frac{160}{(1{,}000)(0.00174)^2} = 52{,}800$$

Part b. Plot of Settling Curve for Smaller Particles.—In this case,

X_o : 200 cm.
C_o : 0.144 cc. solid per cc. total
D : 0.000893 cm.
ρ_b'
ρ' : 2.54 g. per cc.
k : 52,800

The relative concentration is

$$R = \frac{(200)(0.144)(2.54)}{0.935X} = \frac{78.2}{X}$$

and the density of the suspension is

$$\rho = \frac{(0.144)(200)(2.54 - 1) + X}{X} \quad \frac{44.4 + X}{X}$$

Then Equation (353) applied to this case is

$$\int_X^{200} \frac{\mu dX}{2.54 - \frac{44.4 + X}{X}} \quad (52{,}800)(0.000893)^2 \int d\theta = 0.0423\theta.$$

The integral curve of this equation gives the settling curve desired. The integral is evaluated in the usual manner for chosen values of X and the corresponding values of θ determined by dividing the integral values by 0.0423. The results are given in the following table:

Height X, centimeters	Integral	Time θ, seconds
200	0	0
180	35.5	840
160	79.1	1,870
140	134.4	3,180
120	208.2	4,930
100	317.5	7,500

The desired settling curve is plotted in Fig. 289.

FLOTATION

During the last 25 years, the so-called *flotation process* has been used more and more in the separation and classification of solids. This has been particularly true in the metallurgical industries, since the process has not yet been introduced to any great extent into the purely chemical industries. There is no reason why it should not be used generally, and flotation methods probably will become more common after their principles and applications are better understood. The development of the flotation process has been marked by a great deal of litigation and considerable inconclusive experimenting; and it can be truly said that the art of the process has far outdistanced the science. Some of the

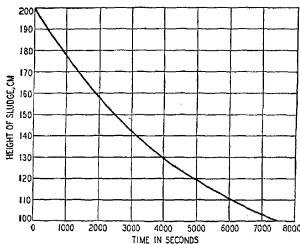



FIG. 289.—Solution of Example 47.

underlying principles are still controversial. There is no doubt of the utility of the process; for though in 1912 only a few thousand tons a year of ores was being treated by flotation methods, at present approximately three-quarters of all ore concentration is done by flotation.

Classification of Flotation Processes.—Flotation may be divided into three classes:

a. Surface-tension methods.

b. Bulk-oil methods.

c. Froth-flotation methods.

Surface-tension Methods.—The surface-tension methods depend upon the fact that some substances are easily wetted by water or other liquids while other substances are not. For

example, some sulfides are not wetted by water and therefore when present as very fine particles tend to float upon the surface, exactly as it is possible to make a steel needle float upon the surface of water, even though the specific gravity of the needle is several times that of water. Silicates and oxides which make up the bulk of the impurities in ores are readily wetted by water and sink. It is, therefore, possible to separate these two classes of substances by this difference in behavior toward water. The first flotation methods applied this principle almost entirely, but since the operation was a very delicate one and recovery rather low, these methods have been discarded. They yield a high grade of concentrate, however.

Bulk-oil Methods.—The bulk-oil methods of flotation depend on the fact that some minerals are wetted preferentially by oil in the presence of water; as a result of this they agglomerate to a certain extent and form groups of particles that are lighter than an equal volume of water. These groups of particles rise to the surface and can be skimmed off and a separation accomplished thereby. These processes were never very successful and find no application in industry today.

Froth-flotation Methods.—The froth-flotation methods are the only ones that have been uniformly successful and, as the name implies, involve during the process the formation of a permanent froth. In carrying out these processes, finely divided mineral, water, and oil are agitated together in the presence of air. In some types of apparatus the air is used to accomplish the agitation. To obtain a persistent froth it is necessary to increase the viscosity of the bubble films. Oil accomplishes this. It adsorbs or concentrates (at the surface of the bubbles) the mineral particles in the pulp so as to form an interface that is more viscous than either the oil or the water or the mixture of the two. It is the presence of the solid matter that contributes to the viscosity of the bubble films in the froth.

The amount of oil required in froth flotation depends upon three factors:

1. Proportion of mineral to be concentrated.
2. Amount of water.
3. Degree of aeration.

The success of the froth method seems to lie in the increase of the viscosity at the bubble interface. Any conditions, there-

fore, that tend to increase this viscosity will materially assist the process. Different types of oil as well as various other substances increase this viscosity, and this has given rise to the large list of so-called *frothers* and *collectors* which one finds in flotation literature.

A great many machines for carrying out separation by flotation have been devised. Some of these have been successful and some not. As a general rule, the successful machines have been those of the froth type and it will suffice to describe one of these.

The Callow Cell.—Figure 290 illustrates the modified Callow cell which has been one of the successful froth machines. It

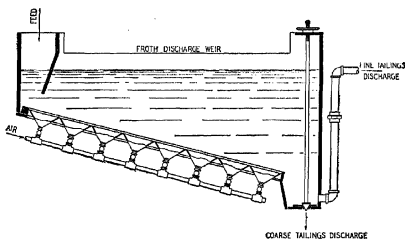



Fig. 290.—Callow flotation cell.

consists of a cell with a sloping bottom and a discharge at the lowest point. The sloping portion of the bottom is porous and covered with a layer of canvas. Through this canvas a steady stream of air is passed. The feed, which consists of a mixture of oil, water, and mineral, is introduced at the shallow end of the tank. The air agitates the mixture violently and causes the formation of a thick layer of froth on the upper surface of the water. This froth carries the mineral that is to be recovered, flows over the side of the cell, and is taken to devices for breaking down the froth. The material that is not carried up by the froth is removed in two fractions. The coarse material settles to the bottom and is discharged at intervals. The fine material stays in suspension and is removed by a separator pipe. These cells may be run either in series or in parallel, and one cell 9 ft. long

and 2 ft. wide will concentrate from 35 to 80 tons of ore per day.

Disposal of Flotation Froth.—The last stage in the flotation process is the disintegration of the froth. Some froths may be readily broken down by passing them over a Wilfley table, but the most common method is by the use of thickeners of the Dorr type followed by filtration. The Oliver filter is frequently used in these processes.

Nomenclature of Chapter XVI

A = area
a, b = individual substances in mixture of solids
B = constant
C = concentration
D = length or diameter
F = force
k = constant
R = relative concentration
u = velocity
X = height of sludge
sub a = one material
sub b = other material
sub e = for turbulent flow
sub g = galena
sub o = initial
sub q = quartz
sub v = for viscous flow
α = constant
μ = viscosity
ρ = density of fluid
ρ' = density of solid
ϕ' = a function

Problems

1. What will be the settling velocity of a spherical steel particle 0.015 in. in diameter through an oil of sp. gr. 0.82 and a viscosity of 10 centipoises, if Stokes' law applies? The specific gravity of steel is 7.87.

2. If a particle of a given material of diameter 0.1 in. settles through water at a constant rate of 8 ft. per sec., under conditions of eddying resistance, how fast will a particle of this same material 0.05 in. in diameter settle through a brine of specific gravity 1.22? Assume that both particles are of the same shape and of specific gravity 2.35.

3. It is desired to separate into two pure fractions a mixture of quartz and galena of a size range from 0.00058 to 0.00250 cm. by using a hindered settling process. What is the minimum apparent density of the sorting bed that will give this separation? If the solid particles in the sorting bed are

30 per cent galena and 70 per cent quartz by weight, what is the minimum consistency of the bed in volume of solid per total volume?

4. A homogeneous silica-water sludge containing 10 per cent solids by volume is to be settled to one-half of its initial height of 200 cm. The average particle size is 0.00095 cm. and the bulk density of the particles is 0.951 g. per cc. Calculate the time necessary for this settling.

CHAPTER XVII

CONVEYING

The transportation of solids is an operation coordinate with the transportation of fluids. The term *conveying* is usually applied to this operation. Although, in general, materials are handled in the fluid form wherever possible, many cases remain where solid materials must be transported. The choice of equipment for this purpose depends on a large number of factors, the most important of which are capacity necessary, shape and size of material, and whether the material is to be transported horizontally, vertically, or on an incline.

In any case the equipment used is ordinarily designed on the basis of empirical experience rather than by any rational methods of calculation. This is due largely to the inherent variation in the properties of the materials transported and also to the very wide range of processes where material handling is involved. A convenient classification of some of the more important conveyors follows:

1. Belt conveyors.
2. Chain conveyors.
   - *a*. Scraper conveyors.
   - *b*. Apron conveyors.
   - *c*. Bucket conveyors.
   - *d*. Bucket elevators.
3. Screw conveyors.
4. Pneumatic conveyors.

BELT CONVEYORS

The belt conveyor is essentially a very simple piece of equipment. It consists of an endless belt on which the solids are transported. Actually, however, a large-scale belt conveyor is rather intricate, its component parts are built in a wide variety of forms, and the design and installation of such an apparatus is a problem for a specialist. A belt conveyor must contain the following elements: first, the belt itself; second, the drive; third,

the supports; and, fourth, the tightener. In addition, unless the belt is to be loaded and unloaded by hand (which is done when package goods are being transported), feeding and discharge devices are necessary.

Ordinarily, the belt is either of canvas or of rubber, although balata is sometimes used. There are some installations using very thin rolled carbon-steel ribbons. The belts may be of one or more plies, depending upon the width and degree of service required.

Belt-conveyor Drives.—Several methods of driving belt conveyors are shown in Fig. 291. The simplest possible drive is a

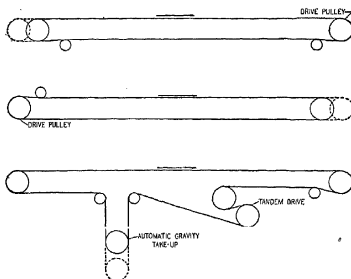



Fig. 291.—Methods of driving belt conveyors.

bare steel pulley actuated by some source of power. This method is satisfactory where the power that must be transmitted is low enough to be carried by the friction of the belt on the pulley. In this type of drive, however, both the area of contact between the belt and the pulley and the coefficient of friction are small. The next step is to utilize pulleys covered with rubber or leather so that the coefficient of friction is increased. Where this does not meet the power requirements, tandem drives, whereby the belt is brought around one pulley, back over a second pulley, and then on, are used. Although the drive of the belt conveyor is ordinarily at the head or discharge end, it can be put at the tail or feed end. This latter

arrangement involves a greater stress in the belt for a given power input and is therefore not used unless necessary.

Belt conveyor Supports.—The supports for the belt are rollers on shaft supports and are usually called *carriers*. They are built in a large variety of forms. The most expensive are carried on roller bearings equipped with pressure grease-gun lubrication. The cheaper ones are carried on ordinary bushings and lubricated with grease cups. In general, the carriers are troughed so as to allow the belt to be depressed in the center and the edges to be raised. This permits a belt of a given width to carry more material per linear foot without spillage. The belt return is ordinarily carried on lighter non-troughing rolls and is sometimes mounted on the same base as the top carriers. An example of a belt-conveyor carrier is shown in Fig. 292.

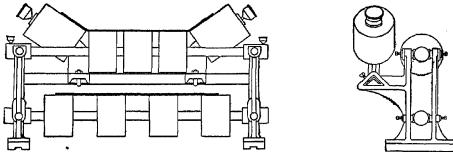



Fig. 292.—Belt-conveyor carrier.

Belt-conveyor Take-ups.—For any but the shortest conveyors, changes in load or in weather, especially in temperature and humidity, result in a variation in belt length of sufficient magnitude to give an uneven tension if there is no provision for keeping the belt taut. Accordingly, a tightener or take-up must be installed to maintain an even tension on the belt under all conditions. Figure 293 shows some common take-ups. The simplest take-up (Fig. 293*a*) consists of a cast-iron bed with a traveling block moving along a screw. The block carries a plain bearing box. Figure 293*b* is a more elaborate type, in which a split journal box rides on a steel angle frame between two fixed support blocks. In both of these and similar types, the shaft of either the head or the tail pulley (preferably the latter) is mounted in a pair of these take-ups and the requisite tension applied by turning up the take-up screws by hand. The

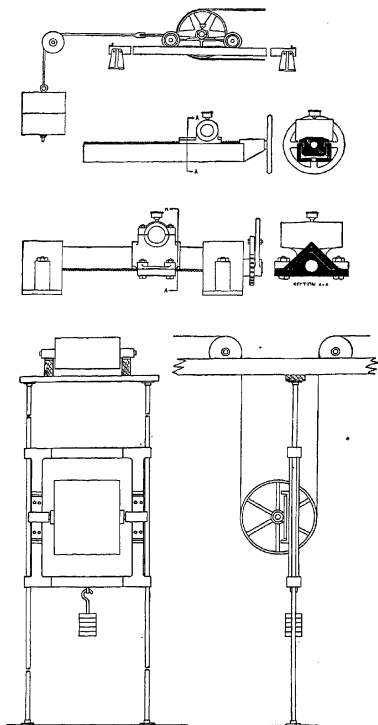



FIG. 293.—Belt-conveyor take-ups. *a*. Cast frame. *b*. Steel angle frame. *c*. Horizontal gravity. *d*. Vertical gravity.

take-ups of Figs. 293*c* and *d* depend on gravity to maintain the tension, and their operation is obvious from the figure.

Feeders.—The simplest method of feeding a belt conveyor is by means of a hopper. When a hopper is used, the slope of the side should be such that the horizontal component of the velocity of the material as it slides on to the belt is nearly the same as that of the belt itself. More elaborate feeding devices include short belt or apron conveyors (page 617) discharging onto the main belt conveyor, shaking screens, rotary-drum feeders, reciprocating-plate feeders, and rotary-vane feeders. Diagrams of typical examples of belt-conveyor feeders are shown in Fig. 294.

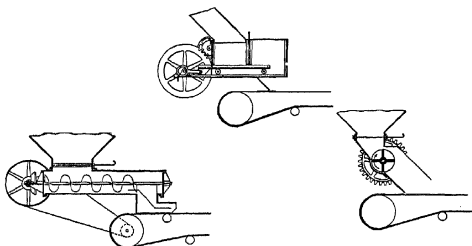



FIG. 294.—Belt-conveyor feeders.

Belt-conveyor Discharge Method.—The method used to discharge a belt conveyor depends on whether or not the discharge is from the end of the conveyor or at some intermediate point, and whether or not the discharge is to be at a single point or to cover the entire length of a bin. For end discharge the belt is self-discharging—the material simply falls over the end. For discharge at intermediate points, however, some special devices are necessary. These methods may be listed as follows: (1) scrapers, (2) tipping idlers, (3) trippers, and (4) shuttle conveyors.

A *scraper*, as its name implies, is a plank or a strip of metal laid diagonally across a belt and diverting the material to one side. The *tipping idler* takes the place of one of the regular

carriers but has its axis on an angle so that the material slides off the belt to one side as the belt passes over the tipping idler. This method is often very unsatisfactory, since a considerable section of belt must be tipped, and the material is discharged over a considerable length of the belt rather than at a definite point.

A *tripper* (Fig. 295) consists essentially of two pulleys A and B in a frame. The pulleys are so mounted that the belt is doubled back for a short distance. The material coming to the tripper on the belt is dropped over the end of the belt as it is turned back, is caught in a chute, and diverted to one side or both sides.

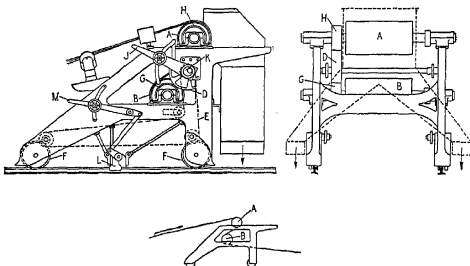



FIG. 295.—Tripper. *A*. Discharge pulley. *B*. Return pulley. *D*. Friction drive pulley. *E*. Drive chains. *F*. Drive wheels. *G*, *H*. Friction pulleys. *J*. Friction-pulley adjusting lever. *K*. Friction-pulley eccentric. *L*. Clamps. *M*. Clamp lever.

Trippers may be stationary, self-propelling, or hand-propelled, and the movable trippers allow the belt to be discharged at any point in its travel. Tracks must be provided along the side of the belt on which the tripper can run. The particular tripper shown in Fig. 295 is driven by power furnished by the belt itself. A friction pulley D is connected by chains E to the drive wheels F. The friction pulley D is so mounted that by means of the handle J and eccentric K, it may be pressed against pulley G on the same shaft as roll B, or against pulley H on the same shaft as roll A. In this way the tripper may be made to travel in either direction. If it is to remain stationary, it may be clamped to the rails by clamps L, actuated by lever M. Trippers may

be set at one point till that part of the receiving bin is filled, and then advanced to another position. Another method is so to arrange the lever J that it is automatically reversed by striking a stop. In such a case the tripper travels continuously back and forth along the length of the receiving bin.

A shuttle conveyor is a short movable conveyor, usually traveling at right angles to the main conveyor. The whole shuttle conveyor travels back and forth over the bin to be filled, automatically reversing the direction of the belt as the movement of the conveyor is reversed.

Belt-conveyor Design

The complete design of large belt-conveyor installations should be undertaken only by one experienced in this field. This is because a belt conveyor is an expensive installation. Consequently, it should be engineered rather carefully. It is possible, however, to give simple rules by which the average engineer can get an approximate idea of his requirements for estimating purposes.

Width and Speed of Belt.—The capacity of a belt conveyor is determined by two factors: first, the cross-section of the load; and, second, the speed of the belt. The cross-section of the load is, in turn, determined by three factors: the width of the belt, the shape of the belt (that is, whether it is flat or troughed), and the size of the material. For relatively fine material the load will assume a fairly uniform cross-section if properly fed. If the material contains large pieces, they are apt to roll off a narrow belt, especially if the feed is irregular.

Every manufacturer of belt-conveyor equipment publishes approximate charts or formulas for the estimation of conveyor sizes and power requirements. Figures 296, 297, and 298, taken from the catalogue of the Robins Conveying Belt Company, are convenient for these calculations. The first step (finding the width and speed of the belt) is carried out with the help of Fig. 296. The chart is entered at the left with the required number of tons per hour or at the top with the required number of cubic feet per hour. Assuming that the load in tons per hour is known, read horizontally across the chart to the intersection with the diagonal that represents the weight of the material in pounds per cubic foot. Then pass vertically downward to the

intersection with the diagonal representing the size of the material and then horizontally to the right-hand edge where the necessary speed in feet per minute is found. For instance, the heavy line of the chart indicates that if 225 tons per hr. of a

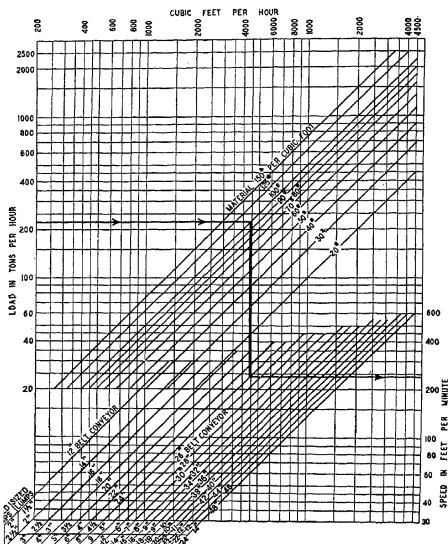



FIG. 296.—Chart for calculating conveyor-belt width.

material weighing 100 lb. per cu. ft. and containing lumps up to 8 in. is to be handled, then a belt 24 in. wide, traveling at a speed of 240 ft. per min., is necessary.

It should be noted that sometimes the width of the belt is determined by the size of the largest pieces rather than the actual

weight of the average cross-section of the load. Thus in the above example if the material, instead of containing 8-in. lumps, were sized material not over 3½ in. in diameter, or if it had no lumps larger than 5 in., then a 20-in. belt running 350 ft. per min. would be satisfactory.

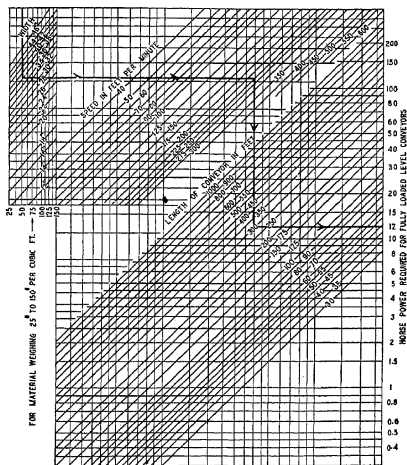



Fig. 297.—Chart for calculating power consumed in belt conveyors.

In general, belt conveyors should not be run at speeds much less than 200 ft. per min. The first cost of the whole conveyor is nearly the same no matter what its operating speed may be. At low speeds the weight of material conveyed per hour is apt to be too small to justify the expense. A narrower belt at higher speed will handle the same load at a lower first cost. On the other hand, speeds much over 500 ft. per min. should be avoided, since they cause undue wear on the belt. Also at these higher speeds. fine material is apt to be blown off the belt.

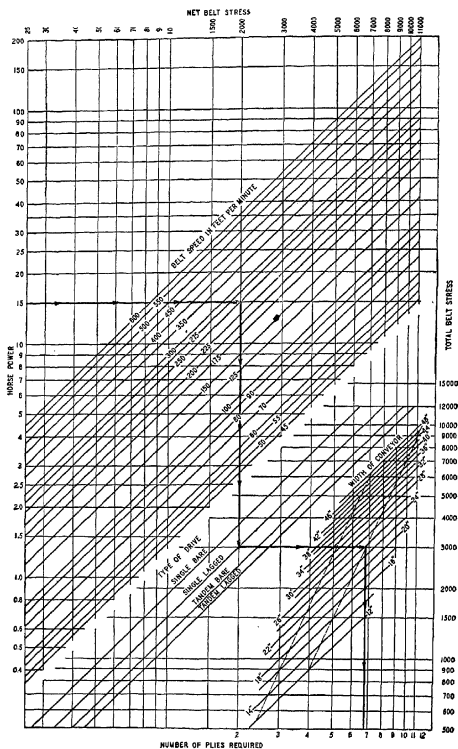



FIG. 298.—Chart for calculating weight of conveyor belt

Power Requirements.—The power consumed by a belt conveyor may be divided into several items: (1) power necessary to give velocity to the load; (2) power necessary to give velocity to the belt itself; (3) power necessary to overcome friction in the idlers; (4) power necessary to operate trippers; (5) and power necessary to elevate material (in the case of inclined conveyors). The complete formulation of all these factors is somewhat complicated, especially since the various friction factors are not well known and the weight of the belt itself has not been determined at this stage of the design. The formulas given in the various manufacturers' catalogues involve considerable simplification. A graphic solution of such formulas is shown in Fig. 297. The chart is entered at the top with the weight of the material in pounds per cubic foot. Pass down to the width of the belt and then horizontally to belt speed in feet per minute. This part of the calculation is no more than a determination of the load in tons per hour. From the intersection with the lines representing belt speed, pass down to the line representing the length of conveyor, and then horizontally to the right-hand margin, where the horsepower requirement of the fully loaded conveyor is read. The heavy line in Fig. 297 shows the solution of this problem: What is the power required to drive a conveyor 36 in. wide, 300 ft. long, running 250 ft. per min., carrying material weighing 50 lb. per cu. ft.? The answer is seen to be 12 hp.

This calculation is based on plain bearing idlers. Idlers containing roller bearings will require 40 per cent less power. For inclined conveyors add 0.001 hp. for each foot-ton of lift per hour. The maximum incline permissible is 15 to 20 deg. The power required for trippers is given in the table on page 608.

Weight of Belt.—It is not sufficient to know the width of the belt, but its thickness must also be determined. This thickness depends on the maximum safe working stress that may be assumed for each ply of fabric per inch of width. Different concerns use figures ranging from 18 to 30 lb. per in. of width per ply for similar grades of duck. The difference merely represents variations in the factor of safety. The tendency in recent designs is toward the higher figure.

In order to drive the conveyor at all, a certain initial tension must be placed on the belt in order to prevent its slipping on the pulleys. When the conveyor is loaded, the top or conveying

run is under this tension plus the tension equivalent to the power consumed. The bottom run is subject to the initial tension only. Therefore, in order to calculate the stress in the conveying run of the belt, it is necessary to know not only the working horsepower pull but the initial tension of the belt as well. This, in turn, is a function of the type of drive. For a load requiring a given horsepower, if the driving pulley is a bare steel pulley, a greater initial tension must be impressed on the belt to prevent slipping than if the same horsepower were transmitted by a lagged pulley or a tandem drive. Figure 298 is a chart that

Width of belt	Horsepower for one tripper	
	Plain bearings	Roller bearings
12	0.75	0.50
14	1.00	0.75
16	1.00	0.75
18	1.50	1.25
20	1.50	1.25
24	1.75	1.25
30	2.50	1.75
36	3.00	2.50
42	4.00	3.00
48	5.00	3.25
54	6.00	5.00
60	7.00	6.00

gives an approximate solution for the problem of determining the number of plies and also gives a check on the type of drive selected. It is based on a rather low stress per inch per ply and therefore gives conservative belt thicknesses. The chart is entered at the left with the known horsepower. Pass across to the right to the intersection with the diagonal line representing belt speed. From this intersection pass downward to an intersection with the diagonals representing the type of drive, horizontally to the right to the intersection with the diagonal representing the belt width, and then down to the lower margin where the number of plies is read. The intersection of the line for the horsepower with the line for belt speed may be read along the top of the chart in terms of "net belt stress." This represents the pull in the top run necessary to transmit the power for moving

the load only. The intersection of a vertical from this point with the diagonals representing the type of drive may be read to the right as total belt stress in the top run, or the total pull in pounds. From the width of the belt, the number of plies, and this total belt stress the stress per ply per inch of width may be calculated. The heavy line in Fig. 298 shows the solution of this problem: What weight of belt is needed for a 24-in. conveyor running 250 ft. per min. using 15 hp., and having a single-lagged drive pulley? The answer is seen to be 6.6 plies, hence a 7-ply belt would be used. The total stress in the conveying run is 3,000 lb., and hence the stress per ply per inch of width is $3{,}000/(7 \times 24)$ or 17.8 lb.

It will be noted that the lines representing width of conveyor are crossed by two dotted diagonal lines. Only those widths of belt that are obtained from intersections between these two lines should be used. Figures obtained from an intersection to the left of the strip represent belts so thin that they will sag between the idlers. Intersections to the right of this strip will call for belts so heavy that they will be too thick to trough well. If the intersection falls to the left of the strip, a sufficiently heavy belt to fall within the strip should be used even though it is thicker than necessary to transmit the power. If the intersection falls to the right of this strip, a better type of drive should be chosen, thereby reducing the initial belt tension and bringing the intersection in the desired area.

It should be remembered that these charts give approximate solutions and are for estimating purposes only. It should not be assumed that when these calculations have been completed, the belt conveyor has been designed. There are many other factors that must be taken into consideration before a complete installation can be built. Errors or mistakes in the design of belt conveyors are not equally serious. Some cause excessive wear on the belt, some can be remedied after the conveyor is in service. The one error that cannot be remedied is to make the belt too narrow for the size of the material or the total capacity handled.

CHAINS

A large and very important group of conveyors is built around chains and chain attachments. In contrast to the belt conveyor, which is an expensive installation and is usually purchased

complete, the chain conveyor is simple and cheap, is adapted to a wide variety of problems, and is usually constructed on the job. With a good knowledge of chains and chain attachments, and with the use of a little ingenuity, one can solve a wide variety of conveying problems for himself.

The types of chains available are so varied that it is difficult to bring them all into one simple classification. The major types, however, may be classified as follows:

1. Malleable detachable chain.
2. Malleable pintle chain.
   a. Plain.
   b. Interlocked.
   c. Plain bushed.
   d. Ley bushed.
   e. Roller bushed.
   f. Special forms such as sawdust, transfer, etc.
3. Combination chain.
4. Steel chain.
   a. Ice chain.
   b. Flat and round.
   c. Roller bushed.
   d. Straight-side long-pitch.

Malleable Detachable Chain.—Malleable detachable chain is by far the commonest type of chain used in conveying equipment. Typical links and their attachments are shown in Fig. 299. As indicated in the illustration, the links are so cast that they can be assembled and detached without the use of tools. The sizes are exactly standardized and are known by arbitrary numbers. For instance, a No. 88 link made by one manufacturer should fit a No. 88 link made by any other manufacturer. For each size, not only the dimensions but also the safe working strength are tabulated. They range from No. 25, which has 133 links in 10 ft. and an ultimate strength of 700 lb., up to No. 124 with 30 links in 10 ft. and an ultimate strength of 15,000 lb. Sizes and safe working strengths of malleable detachable chain are given in Appendix IX.

By far the most important features of these chains are the attachments. These are special links which, however, have the same dimensions and strength as the straight links of the same number. The attachments are known by letter and number and a very wide variety is listed. Certain attachments are

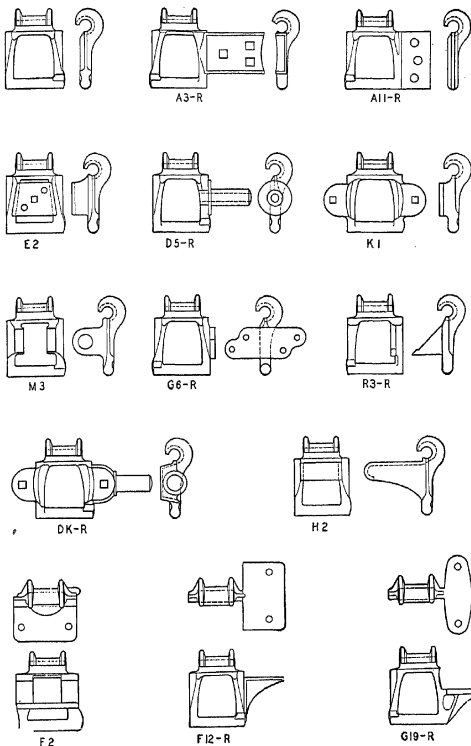



FIG. 299.—Attachments for malleable detachable chain.

standard in all the common sizes of chain, certain others are standard for a few sizes only, while others are special and can be obtained only on order. The key letters of the most important groups of attachments are as follows:

A. Side lugs.
C. Pushers.
D. Pins for rollers.
F. Vertical lugs.
G. Side lugs.
K. Flat lugs on both sides of the chain.
L. Pushers.

A group of these attachments is shown in Fig. 299, and many uses will at once suggest themselves. These will be discussed in more detail under the subject of chain conveyors.

Pintle Chain.—A link of malleable chain is complete in itself. The pintle chains, on the other hand, are all characterized by having a separate pin that forms the joint between two succeeding links. Pintle chains are used where the service is too severe or the load too great for the ordinary malleable detachable chain. In general, the pintle is prevented from turning by a T-head or a flat side. In this way, as the chain goes over the sprocket, the friction is between the head of the link and the pin, thus distributing the wear over a larger surface than would be the case if the pin turned in the shank of the link. A number of types of pintle chain are shown in Fig. 300. In the plain chain (Fig. 300*a*) the tension on the chain is carried entirely by the resistance of the pin to shearing. In the *interlocking* types (Fig. 300*b*) a recess is cast in the inside of the shank, and a projection on the head of the next link fits into this recess, thus relieving the pin of strain. *Bushed chains* (Fig. 300*c*) have a hardened bushing inserted into the head of the link so that if there is any wear, it comes between this bushing and the pin. Since the pin is softer, it is the part that ultimately fails, but it is easily renewed. *Roller chains* (Fig. 300*d*) are also obtainable. In these chains, where the chain goes over the sprocket, there is rolling friction against the side of the sprocket tooth instead of rubbing friction between the tooth and the link. The *Ley bushed chain* (Fig. 300*e*) is a special form of link in which the underside of the link casting is cut away so that only the cast-hardened bushing comes in contact with the sprocket tooth.

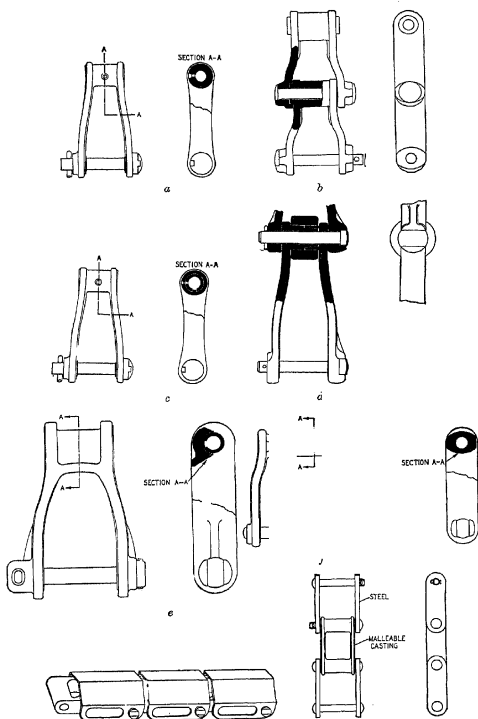



FIG. 300.—Pintle chains. *a*. Plain. *b*. Interlocking. *c*. Plain bushed. *d*. Roller. *e*. Ley bushed. *f*. Sawdust chain. *g*. Transfer chain. *h*. Combination chain.

All of these chains have attachments that follow the same general lines as the corresponding attachments on malleable detachable chain. The attachments for the pintle chains are given the same designation as the corresponding attachments on malleable detachable chain.

There are many higher specialized forms of pintle chain for special purposes. The so-called *sawdust link* (Fig. 300*f*) is used as a scraper conveyor. When a chain made up of these links is run in a shallow trough, it acts as a conveyor in itself without any

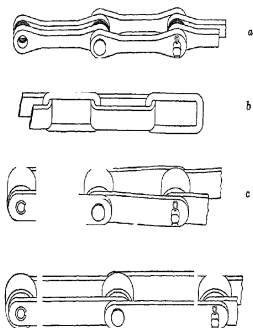



Fig. 301.—Steel chains. *a*. Ice chain. *b*. Flat and round. *c*. Roller bushed. *d*. Straight-side long-pitch.

further attachments. *Transfer chains* (Fig. 300*g*) are widely used in handling long pieces such as lumber, which may be loaded on the conveyor at right angles to the direction of the conveyor travel. The *combination chain* (Fig. 300*h*) is a succession of links of malleable iron connected by steel side bars. This reduces the wear of the malleable iron, and the steel side bars are easily renewed.

Steel Chain.—Steel chain is similar to malleable pintle chain except that the side bars are made of steel stampings, and therefore such chain can be made stronger and heavier than any chain containing cast links. A number of forms are shown in Fig. 301.

Ice chain (Fig. 301*a*) is merely a series of flat bars held apart by washers. Its construction is obvious from the figure. *Flat and round chain* (Fig. 301*b*) contains the flat links to carry attachments, and the sprockets are so designed that the round links fit over the sprocket teeth. The construction of *roller-bushed chain* (Fig. 301*c*) is also obvious, and, finally, the heaviest and strongest chains are the *straight-side long-pitch chains* (Fig. 301*d*) used in the very heaviest conveyors.

In the case of steel chains, attachments are easily made by riveting the proper stampings or castings to the steel side bars. The variety of standard attachments obtainable in this way is smaller than in the case of malleable chain but is ample for all ordinary purposes.

Chain Conveyors

Scraper Conveyor.—The scraper conveyor is the simplest and cheapest type of conveyor. Its advantages are its low first cost, its adaptability to a wide variety of conditions, its suitability for steeper inclines than the belt conveyor, and its ability to handle large pieces. The disadvantages of the scraper conveyor are its relatively heavy power requirement and heavy repair charges if the service is continuous. These conditions contrast sharply with the belt conveyor, which in each particular may be characterized in just the opposite terms. Consequently, where the load is heavy, the distance long, and first cost unimportant in comparison to power consumption and repairs, the belt conveyor is used. When the distance is short, the load light or intermittent, the first cost important, and power consumption relatively unimportant, the scraper conveyor is used.

The scraper conveyor may, to a large extent, be improvised to suit particular conditions. The simplest possible conveyor is one such as shown in Fig. 302*a*. A malleable detachable chain with F-2 attachments has blocks of wood fastened to these attachments to act as scrapers. The conveyor runs in a wooden trough. In Fig. 302*a* the upper run is the conveyor and the lower run is the return. Such a conveyor might be used for sawdust, chips, or any other light material that would not be injured by having the chain running in the material conveyed. The lower run might have a solid bottom and be used for conveying if it were desirable to keep the chain out of the material handled. In this case the top run would be the return.

More elaborate constructions employ steel frames and wearing bars for the flights, as shown in Fig. 302*b*. Here the same chain and the same attachments are used, but the flights are of sheet steel, the trough is of sheet steel, and the frame is built of angle iron. Wearing bars are riveted to the corners of the flights so that the flight itself does not bear on the trough. A still more elaborate form, using roller pintle chain, is shown in Fig. 302*c*. Since this is a double-chain conveyor, it will handle heavier loads for the same weight of chain.

Many similar constructions will suggest themselves from an inspection of chain attachments and a consideration of the type

a *b* *c*

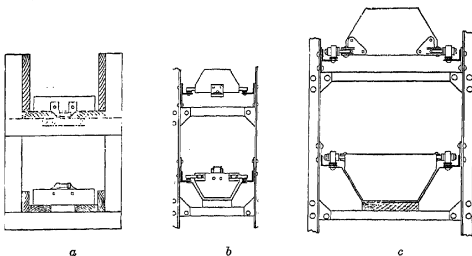



Fig. 302.—Scraper conveyors. *a*. Malleable chain, wood flights. *b*. Steel flights with wearing bars. *c*. Double roller chains, steel flights.

and size of material to be handled. As mentioned on page 614, sawdust chain run in a shallow trough makes its own conveyor.

Design of Scraper Conveyors.—The capacity of scraper conveyors on material weighing about 50 lb. per cu. ft. and operating on a level is as follows:

Size of Flights, Inches	Pounds per Flight
4 × 10	15
4 × 12	19
5 × 12	23
5 × 15	31
6 × 18	40
8 × 18	60
8 × 20	70
8 × 24	90
10 × 24	115

On other materials the capacity will be proportional to the weight per cubic foot. The speed of scraper conveyors is usually about 100 ft. per min. A conveyor running up an incline has the following percentage capacity of that of a corresponding horizontal conveyor:

Incline, Degrees	Capacity, Per Cent of Horizontal
20	77
30	55
40	33

The following formula for computing the horsepower requirements of flight conveyors is given by the Stephens-Adamson Manufacturing Company:

$$Hp. = \frac{0.06SLFW_c + T(L'F' + H)}{600} \tag{359}$$

where

Hp = horsepower at motor.
S = speed of conveyor in feet per minute.
L = overall horizontal projected length of conveyor in feet.
F = friction factor for chains and flights.

Flights on sliding shoes,	$F = 0.6$
Malleable chain (dragging),	$F = 0.6$
Malleable roller chains,	$F = 0.35$
Roller-bushed chains,	$F = 0.2$

W_c = weight of chains and flights (both run and return) in pounds per foot of conveyor length.
T = tons of material handled per hour.
L' = horizontal projected length of loaded section of conveyor in feet.
F' = friction factor for load (0.6 for coal).
H = vertical lift in feet.

Apron Conveyors.—Apron conveyors are used for the widest variety of purposes but usually for heavy loads and short runs. They range from forms that can be improvised for simple cases up to elaborate and expensive conveyors that would be purchased from manufacturers who specialize in this field.

The simplest apron conveyor (Fig. 303a) consists of two chains made up entirely of malleable detachable links carrying A-attachments. Wooden bars are fastened to these attachments between

the chains, and the whole conveyor drags on the support. This forms a practically continuous moving platform. For heavier

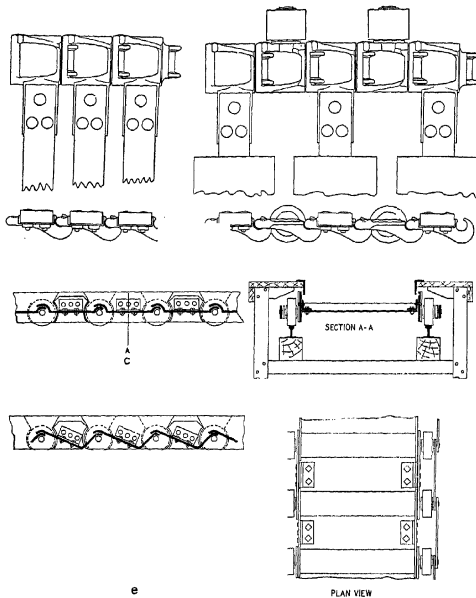



FIG. 303.—Apron conveyors. *a*. Malleable detachable chain with A-attachments, chain dragging. *b*. Malleable detachable chain with alternate A- and D-attachments, chain on rollers. *c*. Overlapping flat steel plates riveted to long-pitch straight-side roller chain. *d*. Overlapping stamped plates. *e*. Recessed plates (this form approximates the bucket conveyor).

loads or longer runs, malleable chains consisting alternately of A-links and D-links might be used with rollers on the D-links

(Fig. 303*b*). The next step would be roller pintle chain with A-attachments. For still heavier work or rougher usage, steel plates might be used instead of the wooden bars. In the simplest cases, these plates may be flat, but where the conveyor is to be used for loose solids rather than packages, the plates may overlap and be stamped to the proper radius so that they will cover each other as the chain goes over the sprockets (Fig. 303*c*). More elaborate types involve the use of long-pitch straight-side chains that carry steel plates with a depression stamped in each (Fig. 303*d*). This style of conveyor, when carried to the extreme case, becomes practically a series of horizontal buckets (Fig. 303*e*). In these latter styles, the inner side bar of the chain is developed to a considerable height to form the side of the conveyor, and this makes possible larger capacities.

Bucket Conveyors.—The deep apron conveyors, as has been said, develop gradually into a type known as bucket conveyors. The simpler bucket conveyors consist merely of deep steel stampings with overlapping edges carried on long-pitch straight-side steel chain. If the buckets are sufficiently deep, there is no distinction in construction between the horizontal conveyor and a sharply inclined bucket elevator. Very elaborate forms of bucket conveyors are used for handling coal in power houses and other places where the most expensive type of conveyor is justified. In this case, cast-iron or stamped-steel buckets are pivoted between two long-pitch straight-side steel chains. They are so constructed that on the horizontal runs the buckets overlap each other, and the feed to such a conveyor may therefore be a continuous stream of material. The buckets are so pivoted that on the vertical runs they hang freely between chains, and the conveyor acts as an elevator. A tripper may be located at any point in the horizontal run to discharge the buckets by inverting them over any desired portion of the bin. Such conveyors are often arranged so that they receive coal near the end of the bottom run, elevate it to the coal bunkers, discharge it at any point in the run over the bunkers, and handle ashes on the bottom run. They are elaborate and expensive installations, and their design does not come within the scope of this book.

Elevators.—A belt, scraper, or apron conveyor may be used to lift material as well as convey it, provided that the lift is

short in comparison with the horizontal run, so that the angle of the conveyor to the horizontal is not great. Belt conveyors are seldom run at angles greater than 15 to 20 deg., and scraper conveyors seldom over 30 deg. If the lift must be more abrupt than this, or if a straight vertical lift is necessary, some form of elevator is used. The usual type of elevator consists of a series of buckets carried either on chains or on a belt.

Buckets may be of many forms and some types are shown in Fig. 304. The Minneapolis-type bucket (Fig. 304*b*) is almost universally used for grain and any other dry pulverized material.

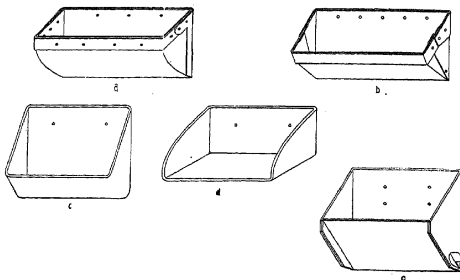



Fig. 304.—Elevator buckets. *a*. General-service bucket. *b*. Minneapolis type. *c*, *d*. Buckets for wet or sticky materials. *e*. Stamped-steel bucket for crushed rock.

For materials that tend to be sticky, flatter buckets (Fig. 304*c* and *d*) are used; and for large lumps and heavy material, such as coal or crushed stone, the heavier stamped-steel buckets (Fig. 304*e*) are employed.

Where the elevator must be kept as clean as possible, as for handling grain and food stuffs, the buckets may be attached to a belt. A much commoner construction is to fasten the buckets to a chain. By the use of K-1 or K-2 links, the buckets can be suspended from a single chain. The attachments are slightly above the level of the chain so that the end of the sprocket tooth does not strike the back of the bucket. For heavier loads, two chains with G-1 attachments may be used. The attachments

are riveted to the ends of the buckets. For heavier chains and more severe service, G-attachments may be obtained that are hinged to the chain to prevent binding if the chains stretch unequally under load. The heaviest conveyors carry the stamped-steel buckets of Fig. 304*e* between long-pitch straight-side chain. In such cases the buckets usually overlap so that the elevator may be fed with a continuous stream from a spout.

Belt or chain bucket elevators handling light materials may be operated at a speed of from 150 to 250 ft. per min. At this speed the material is usually thrown from the buckets at the top of the elevator so that a spout placed to clear the head sprocket will receive all the discharge. For heavier loads or lower speeds, the so-called *perfect discharge* may be used. In this design an idler sprocket bends the chain back under the head sprocket so that the buckets turn completely upside down over a spout placed just under the head sprocket. In the case of elevators for heavy materials using overlapping buckets, the buckets are so shaped that the back of one bucket acts as a discharge spout for the next bucket, and such elevators will discharge clear of the head sprocket at very low speeds.

Elevators are invariably driven from the head sprocket, but because of the weight of the conveyor and the corresponding stretch of either chain or belt under load, some device must be provided for altering the position of the sprocket or pulley at the foot of the elevator. On ordinary granular materials, the buckets fill by digging into the loose material. These two functions, namely, feeding the bucket and tightening the chain, are usually combined in the structure called the *boot*. A typical elevator boot construction is shown in Fig. 305.

Elevators may be run without any casing around them. It is more common, however, to enclose the elevator completely; and the casing may be of wood or sheet steel, as conditions may dictate. Occasionally, a separate casing is made around each leg of the elevator, but it is far more common to enclose the whole structure in a single casing.

SCREW CONVEYORS

An important type of conveyor for transporting material in the form of finely divided solids or pasty solids is the screw conveyor. This apparatus consists essentially of a spiral

blade revolving around an axis in the bottom of a U-shaped trough.

Screw-conveyor Flights.—The screw element is called a flight (Fig. 306) and may be sectional, helicoid, or special. The sectional conveyor (Fig. 306*a*) is made up of short sections, each of which is stamped as a circular disc, cut along one radius, and then given the proper twist to develop the spiral. Each disc provides for one full turn of the conveyor, and the various turns are riveted together. The helicoid flight, however (Fig. 306*b*), is made from a single long ribbon that is twisted and warped into a spiral shape and then welded to the central shaft.

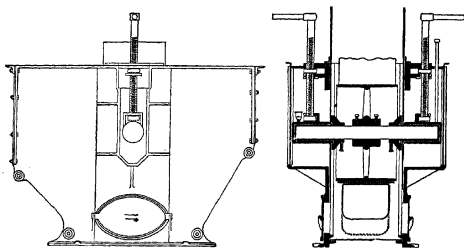



Fig. 305.—Elevator boot.

The shaft is standardized and is extra-heavy iron pipe. For service where temperature or abrasion necessitates cast iron, cast-iron flights are assembled on a standard shaft (Fig. 306*c*). For sticky materials, ribbon flights (Fig. 306*d*) are used. For mixing, the flights of Fig. 306*e* are used. These are made by cutting into a standard flight and bending back the part of the helix between the cuts.

For the simplest and least expensive type of screw conveyor, when a length of conveyor is purchased, there is supplied with it one hanger, one coupling for joining sections of spiral, four coupling bolts, and a half-round liner. The standard coupling consists of a short section of solid shaft fitting into the hollow conveyor shaft (Fig. 307). The conveyor pipe is reinforced at the ends, and a part of the coupling shaft acts as the journal. For

more expensive and more elaborate conveyors there are available better-grade hangers, various types of spiral, different constructions of trough, more elaborate and more expensive bearings, and many other accessories.

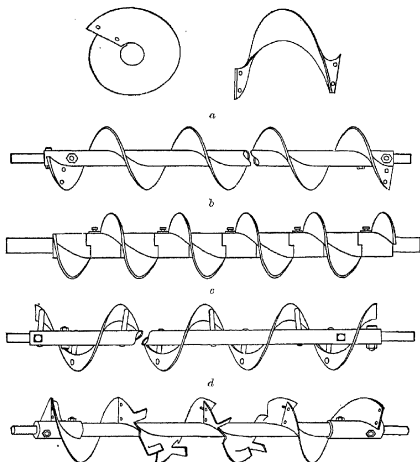



FIG. 306.—Screw-conveyor flights. *a*. Sectional. *b*. Helicoid. *c*. Cast iron. *d*. Ribbon. *e*. Cut flights.

Screw-conveyor Troughs.—The trough (Fig. 308) is ordinarily made of sheet steel. Standard sections come in lengths of 8, 10, and 12 ft. In the simplest type (Fig. 308*a*), only the half-round section at the bottom of the trough is made of steel and the straight sides are formed by the wooden trough in which the conveyor is installed. The steel liner for this type of trough

is regularly supplied with each length of conveyor flight. Figure 309*b* shows a more elaborate trough in all-steel construction.

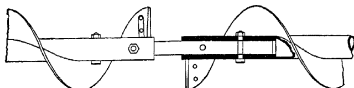



Fig. 307.—Screw-conveyor coupling.

It is necessary that the shaft be suspended in suitable bearings in order that it be kept in alignment. Two of the bearings are carried in the end plates of the conveyor, but hangers must also be provided along the length of the trough. Ordinarily there is a

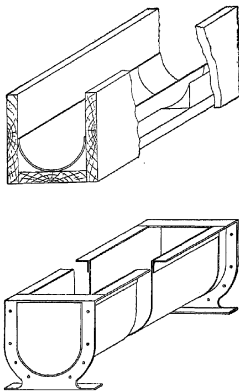



Fig. 308.—Screw-conveyor troughs. *a*. Wood trough with steel liner. *b*. All-steel trough.

hanger for each section. Figure 309 shows a few types of hangers. Figure 309*a* is one of the simplest and cheapest forms. Figure 309*b* has a split bearing box instead of a solid one. These

two hangers fit over wood box sides. Figure 309*c* fits inside a steel trough. More elaborate forms have adjustable bearings and better lubrication. Since the material that is being transferred through the conveyor is in contact with the bearings of the hanger, oftentimes oil or grease is objectionable because of contamination, and wood bushings soaked with oil or bushings made of white cast iron (Fig. 309*d*) run without lubricant are used.

The types of construction used in the ends of the conveyors or *box ends* also exhibit a great variety. Some of these are shown in

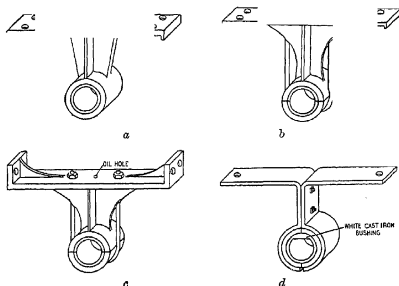



Fig. 309.—Screw-conveyor hangers. *a*. Simple hanger for wood trough. *b*. Hanger with split bearing for wood trough. *c*. Hanger with split bearing for wood or steel trough. *d*. Hanger for bushing to run without lubrication.

Fig. 310. In general, the drive end (Fig. 310*d*) is different in construction from the discharge end, in order to bring the shaft through and connect it to the drive, which is usually by bevel gears. As in all other types of conveyors, the selection of suitable parts is determined by the expense that is justified by the scale of operation, severity of service, value of the material, and other peculiarities of the problem at hand.

Design of Screw Conveyors.—The following table of capacities is based on data given by H. W. Caldwell & Son Co. The capacities as given are conservative, but in order to obtain them the conveyor must be fed regularly, and the hangers used must not

be so large as to obstruct the flow of material more than does the ordinary hanger. The capacities are based on the recommended

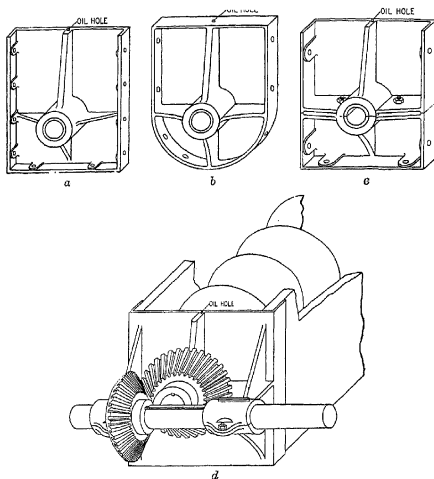



FIG. 310.—Screw-conveyor box ends. *a*. Plain end for wood trough. *b*. Plain end for steel trough. *c*. Split bearing for wood trough. *d*. Drive end for wood trough.

maximum speed of conveyor for each material. Slower speeds will result in proportionate decreases in capacity.

GRAIN

Size of screw conveyor, inches	3	4	5	6	7	8	9	10	12	14	16
Speed, r.p.m.	200	200	190	180	175	175	170	165	165	160	160
Cubic feet per hour	34	72	175	243	352	734	910	1,205	2,180	2,935	5,110
Bushels per hour	27	58	140	195	282	586	728	965	1,745	2,350	4,100

Sand, Gravel, or Ashes
Dry, and with no large lumps or stones

Size of screw conveyor, inches	3	4	5	6	7	8	9	10	12	14	16
Speed, r.p.m.	125	125	120	115	110	105	100	95	90	85	80
Cubic feet per hour	20	43	95	126	178	359	421	540	933	1,200	2,000

Coal
Screenings, or small-sized coal, with no lumps larger than 1 in.

Size of screw conveyor,[1] inches	7	8	9	10	12	14	16
Speed, r.p.m.	110	105	100	95	90	85	80
Cubic feet per hour	269	544	650	838	1,460	1,905	3,260
Tons (2,000 lb.) per hour	6.7	13.6	16.3	20.9	36.5	47.5	79.0

[1] Small sizes of screw conveyor not recommended for handling coal.

Cement

Size of screw conveyor, inches	6	7	8	9	10	12	14	16
Speed, r.p.m.	125	115	110	100	100	90	90	85
Cubic feet per hour	167	233	468	541	725	1,210	1,625	2,730
Barrels per hour	42	58	117	135	181	303	408	683

Power Requirements of Screw Conveyors.—The same company proposes the following formula for power consumption. The formula is conservative and power actually consumed will probably be lower than that calculated.

$$Hp. = \frac{KC\rho L}{2,000,000} \qquad (360)$$

where

K = 1.2 for grains.
2.5 for fine coal, cement, etc.
4.0 for sand, ashes, etc.
C = capacity in cubic feet per hour.
ρ = weight of material in pounds per cubic foot.
L = length of conveyor in feet.

Pneumatic Conveyors.—A method extensively used for the conveying of light and bulky materials is the pneumatic conveyor. In this system the material is transferred in suspension in a stream of air. The household vacuum cleaner is a familiar illustration of this method. There are a variety of systems, but they all involve a pump or fan for producing the stream of air, a receiver for separating the larger particles, and usually, but not

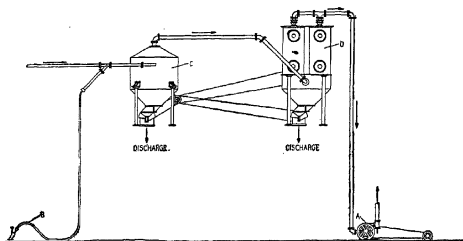



Fig. 311.—Pneumatic conveyor. *A*. Suction pump. *B*. Nozzle. *C*. Cyclone separator. *D*. Dust filter.

necessarily, a bag filter for removing the dust. In the simplest form (Fig. 311), a pump *A* of the cycloidal type produces a moderate vacuum and its suction is connected to the conveying system. The material is sucked up through a nozzle *B* which may be fixed or movable (usually the latter). The stream of air with the solid in suspension goes to a cyclone separator *C*, of the type described on page 566, and then to the pump. Where the material carries dust that would injure the pump or that would be harmful if discharged into the air, a bag filter *D* of the type described on page 567 is placed between the separator and the pump.

This system of conveying is primarily indicated for materials that would be unpleasant or injurious, such as pulverized soda ash or pulverized materials containing such poisonous constituents as lead or arsenic. It is also suitable for bulky materials such as wood chips, dried beet pulp, and similar materials.

The velocity of the air may be from a few hundred up to 5,000 ft. per min. There will be used from 50 to 100 cu. ft. of air

for every pound of material handled, depending on the nature and weight of material, the distance conveyed, vertical lift, etc. The disadvantage of this system is the relatively large power requirement.

Nomenclature of Chapter XVII

c = capacity of screw
F = friction factor for chains and flights
F' = friction factor for load (0.6 for coal)
H = vertical lift in feet
Hp = horsepower at motor
K = constant for screw conveyors
L = horizontal length
L' = horizontal projected length of loaded section of conveyor in feet
S = speed of conveyor in feet per minute
T = tons of material handled per hour
W_c = weight of chains and flights (both run and return) in pounds per foot of conveyor length
ρ = density

Problems

1. A belt conveyor is to handle 50 tons per hr. of dry salt weighing 60 lb. per cu. ft. The conveyor is 750 ft. long, and the carriers have plain bearings. What are the power consumed, and the width and thickness of belt, if the drive is a single lagged pulley? What would be the result if a single bare drive pulley were used?

2. Recalculate the above conveyor on the basis of roller-bearing carriers. What are now the power consumption, the width and weight of belt, and the type of drive? If the conveyor is in service 24 hr. per day, 300 days per year, if power costs 1.2 cts. per kw.-hr., and if fixed charges on the equipment are 15 per cent per year, what is the maximum sum available to convert the conveyor of Problem 1 to a roller-bearing conveyor?

3. A scraper conveyor consists of two strands of No. 88 malleable detachable chain with 4 by 10-in. scraper flights of ¼-in. plate steel spaced 2 ft. apart. It runs 100 ft. per min., is level, and carries coal. The total overall length is 125 ft., and the actual distance the coal is carried is 100 ft. What is the power required? What is the factor of safety involved in the use of two strands of No. 88 chain?

4. Could the above conveyor be built with a single strand of No. 488 plain pintle chain, weight 29 lb. per 10 ft., ultimate breaking strength 11,000 lb., without decreasing the factor of safety?

5. Number 88 malleable detachable chain costs $0.35 per ft., while a roller malleable pintle chain weighing 33 lb. per 10 ft. and having an ultimate strength of 11,000 lb. costs $1.10 per ft. The conveyor is in service 10 hr. per day, 300 days per year, power costs 1.25 cts. per kw.-hr., and repairs,

depreciation, and fixed charges are 20 per cent per year. Is the conveyor of Problem 3 more economical if made with two strands of No. 88 malleable detachable chain or one strand of this roller pintle chain?

6. An 8-in. screw conveyor running 80 r.p.m. is conveying salt (50 lb. per cu. ft., friction factor the same as for coal). The conveyor is 50 ft. long. What is the power used?

APPENDIX I

RESISTANCE OF SCREW FITTINGS EXPRESSED IN EQUIVALENT LENGTHS OF STRAIGHT STANDARD PIPE[1]

(In feet)

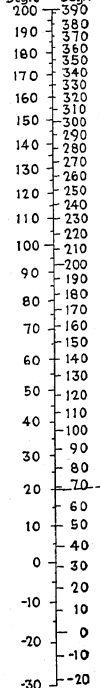
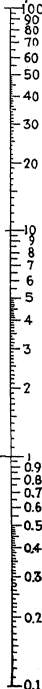
Nominal pipe size, inches	Form of resistance							
	Gate valve	Long-sweep elbow or on run of standard tee	Medium-sweep elbow or on run of tee reduced in size ¼	Standard elbow or on run of tee reduced in size ½	Angle valve	Close return bend	Tee through side outlet	Globe valve
½	0.31	0.41	0.52	0.84	1.12	1.25	1.66	2.50
¾	0.44	0.57	0.73	1.17	1.57	1.75	2.33	3.50
1	0.57	0.77	0.98	1.57	2.11	2.34	3.11	4.68
1¼	0.82	1.07	1.37	2.19	2.94	3.27	4.35	6.54
1½	0.98	1.29	1.64	2.63	3.52	3.92	5.21	7.84
2	1.32	1.74	2.23	3.55	4.77	5.30	7.05	10.60
2½	1.64	2.16	2.75	4.39	5.91	6.56	8.71	13.12
3	2.13	2.81	3.59	5.72	7.69	8.54	11.40	17.08
3½	2.53	3.34	4.26	6.80	9.10	10.13	13.50	20.26
4	2.96	3.90	4.97	7.94	10.65	11.84	15.75	23.68
4½	3.27	4.45	5.66	9.05	12.14	13.50	17.95	27.00
5	3.88	5.11	6.52	10.40	13.95	15.51	20.60	31.02
6	4.81	6.35	8.09	12.90	17.35	19.27	25.60	38.54
7	5.75	7.59	9.66	15.40	20.70	23.02	30.60	46.08
8	6.70	8.85	11.20	17.90	24.10	26.80	35.60	53.60
10	8.75	11.54	14.70	23.40	31.50	35.00	46.60	70.00
12	10.90	14.40	18.35	29.30	39.30	43.70	58.10	87.40

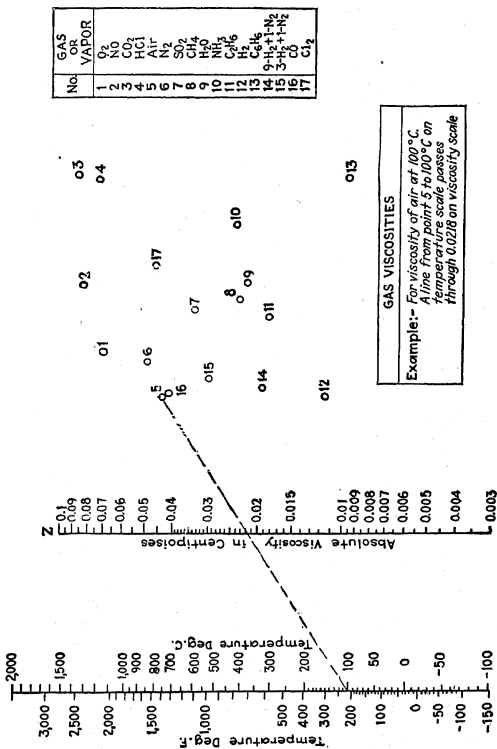
[1] FOSTER, *Trans. A. S. M. E.*, **42**, 647 (1920), recalculated in PERRY, p. 741.

APPENDIX II[1]

LIQUID VISCOSITIES

TABLE			
Point	Liquid	Point	Liquid
18	Acetic Acid 100%	7	Glycerol 50%
12	Acetic Acid 70%	31	Heptane
34	Acetone	36	Hexane (N)
17	Allyl Alcohol	15	Mercury
39	Ammonia	26	Methyl Alcohol 100%
8	Aniline	24	Methyl Alcohol 90%
25	Benzene	13	Methyl Alcohol 30%
11	Butyl Alcohol	32	Methyl Acetate
40	Carbon Dioxide	9	Naphthalene
33	Carbon Disulfide	14	Nitrobenzene
21	Carbon Tetrachloride	28	Octane
22	Chlorbenzene	38	Pentane
29	Chloroform	5	Phenol
37	Ether	35	Sulfur Dioxide
19	Ethyl Alcohol 100%	2	Sulfuric Acid 111%
10	Ethyl Alcohol 40%	3	Sulfuric Acid 98%
30	Ethyl Acetate	6	Sulfuric Acid 60%
23	Ethylene Chloride	27	Toluene
4	Ethylene Glycol	16	Turpentine
1	Glycerol 100%	20	Water

Temperature
Deg.C Deg.F.

Viscosity in
Centipoises

[1] McAdams, p. 340.

APPENDIX III[1]

[1] McAdams, p. 341.

APPENDIX IV

BRIGG'S STANDARD GAGE FOR IRON PIPE[1]

Size, in.	Size, mm.	Diameters		Nominal thickness, in.	Circumference		Transverse areas			Length of pipe per sq. ft. of		Length of pipe containing one cu. ft., feet	Nominal weight per foot, plain ends	Number of threads per inch of screw
		External, in.	Approximate internal, in.		External, in.	Internal, in.	External, sq. in.	Internal, sq. in.	Metal, sq. in.	External surface, ft.	Internal surface, ft.			
1/8	3	0.405	0.269	0.068	1.272	0.845	0.129	0.057	0.072	9.431	14.199	2,533.775	0.244	27
1/4	6	0.540	0.364	0.088	1.696	1.144	0.229	0.104	0.125	7.073	10.493	1,383.789	0.424	18
3/8	10	0.675	0.493	0.091	2.121	1.549	0.358	0.191	0.167	5.658	7.747	754.360	0.567	18
1/2	13	0.840	0.622	0.109	2.639	1.954	0.554	0.304	0.250	4.547	6.141	473.906	0.850	14
3/4	19	1.050	0.824	0.113	3.299	2.589	0.866	0.533	0.333	3.637	4.635	270.034	1.130	14
1	25	1.315	1.049	0.133	4.131	3.296	1.358	0.864	0.494	2.904	3.641	166.618	1.678	11½
1¼	32	1.660	1.380	0.140	5.215	4.335	2.164	1.495	0.669	2.301	2.767	96.275	2.272	11½
1½	38	1.900	1.610	0.145	5.969	5.058	2.835	2.036	0.799	2.010	2.372	70.733	2.717	11½
2	50	2.375	2.067	0.154	7.461	6.494	4.430	3.355	1.075	1.608	1.847	42.913	3.652	11½
2½	64	2.875	2.469	0.203	9.032	7.757	6.492	4.788	1.704	1.328	1.547	30.077	5.793	8
3	76	3.500	3.068	0.216	10.996	9.638	9.621	7.393	2.228	1.091	1.245	19.479	7.575	8
3½	90	4.000	3.548	0.226	12.566	11.146	12.566	9.886	2.680	0.954	1.076	14.565	9.109	8
4	100	4.500	4.026	0.237	14.137	12.648	15.904	12.730	3.174	0.848	0.948	11.312	10.790	8
4½	113	5.000	4.506	0.247	15.708	14.156	19.635	15.947	3.688	0.763	0.847	9.030	12.538	8
5	125	5.563	5.047	0.258	17.477	15.856	24.306	20.006	4.300	0.686	0.756	7.198	14.617	8
6	150	6.625	6.065	0.280	20.813	19.054	34.472	28.891	5.581	0.576	0.629	4.984	18.974	8
8	200	8.625	7.981	0.322	27.096	25.073	58.426	50.027	8.399	0.442	0.478	2.878	28.554	8
10	250	10.750	10.020	0.365	33.772	31.479	90.763	78.855	11.908	0.355	0.381	1.826	40.483	8
12	300	12.750	12.000	0.375	40.055	37.699	127.676	113.097	14.579	0.299	0.318	1.273	49.562	8

[1] Crane Company.

APPENDIX V

Standard Condenser-tube Data[1]

Outside diameter of tube, inches	No. gage, B.W.G.	Weight per linear foot, pounds[2]	Weight per 100 sq. ft. outside surface, pounds[2]	Thickness, inches	Inside diameter, inches	Inside circumference, inches	Surface outside, square feet: Per linear foot	Surface outside, square feet: Per linear inch	Surface inside per linear foot, square feet	Inside sectional area per tube, square inches	Velocity per 1 g.p.m., feet per second	G.p.m. per 1 ft. per sec. velocity
5/8	16	0.421	256.7	0.065	0.495	1.559	0.16362	0.01364	0.1299	0.193	1.633	0.602
	17	0.380	232.2	0.058	0.509	1.598	0.16362	0.01364	0.1331	0.204	1.579	0.635
	18	0.326	199.5	0.049	0.527	1.660	0.16362	0.01364	0.1382	0.218	1.472	0.680
	20	0.238	145.5	0.035	0.555	1.745	0.16362	0.01364	0.1455	0.242	1.328	0.754
	22	0.193	118.2	0.028	0.569	1.790	0.16362	0.01364	0.1491	0.254	1.265	0.790
3/4	16	0.514	262.0	0.065	0.620	1.950	0.19635	0.01637	0.1627	0.302	1.069	0.938
	17	0.464	236.1	0.058	0.634	1.990	0.19635	0.01637	0.1660	0.3155	1.019	0.983
	18	0.396	220.0	0.049	0.652	2.047	0.19635	0.01637	0.1706	0.334	0.9617	1.037
	20	0.289	147.0	0.035	0.680	2.138	0.19635	0.01637	0.1780	0.364	0.883	1.133
1	16	0.700	268.0	0.065	0.870	2.737	0.26180	0.02179	0.2279	0.595	0.540	1.850
	17	0.632	241.7	0.058	0.884	2.775	0.26180	0.02179	0.2315	0.6135	0.524	1.910
	18	0.540	206.0	0.049	0.902	2.832	0.26180	0.02179	0.2360	0.638	0.503	2.000
	20	0.389	149.0	0.035	0.930	2.920	0.26180	0.02179	0.2433	0.679	0.474	2.120
1 1/4	16	0.890	272.0	0.065	1.120	3.520	0.32708	0.02725	0.2935	0.985	0.3262	3.060
	17	0.800	244.6	0.058	1.134	3.561	0.32708	0.02725	0.2970	1.011	0.3172	3.150
	18	0.680	208.0	0.049	1.152	3.620	0.32708	0.02725	0.3020	1.045	0.3075	3.255
	20	0.490	150.0	0.035	1.180	3.710	0.32708	0.02725	0.3092	1.096	0.2930	3.415
1 1/2	10	2.110	537.5	0.134	1.232	3.880	0.3925	0.03275	0.3237	1.195	0.2688	3.720
	12	1.75	445.5	0.109	1.282	4.025	0.3925	0.03275	0.3382	1.292	0.2488	4.020
	14	1.36	346.0	0.083	1.334	4.189	0.3925	0.03275	0.3500	1.400	0.2298	4.365
	16	1.080	265.0	0.065	1.370	4.320	0.3925	0.03275	0.3600	1.473	0.2180	4.585
2	10	2.890	553.0	0.134	1.730	5.440	0.5233	0.04360	0.4534	2.350	0.1365	7.330

[1] Braun, *Am. Soc. Nav. Eng.*, **29**, 345 (1917).

[2] Weights given are for 60-40 brass. For pure copper multiply by 1.05.

APPENDIX VIa[1]

THERMAL CONDUCTIVITY OF METALS AT 212° FAHRENHEIT (100° CENTIGRADE)

Metal	k
Aluminum	119
Brass (90–10)	68
Brass (70–30)	63
Brass (67–33)	62
Brass (60–40)	69
Cast iron	28
Copper	222
Lead	20
Nickel	34
Silver	240
Steel, mild	26
Tin	34
Wrought iron	35
Zinc	62

[1] McADAMS, p. 308.

APPENDIX VIb[1]

THERMAL CONDUCTIVITY OF WATER
(English units)

Temperature, degrees Fahrenheit	k	Temperature, degrees Fahrenheit	k
32	0.320	155	0.382
35	0.322	160	0.384
40	0.326	165	0.385
45	0.329	170	0.386
50	0.333	175	0.387
55	0.336	180	0.388
60	0.340	185	0.389
65	0.343	190	0.390
70	0.346	195	0.391
75	0.349	200	0.392
80	0.352	205	0.392
85	0.355	210	0.393
90	0.358	215	0.393
95	0.360	220	0.394
100	0.362	230	0.395
105	0.364	240	0.396
110	0.367	250	0.396
115	0.369	260	0.396
120	0.371	270	0.396
125	0.373	280	0.396
130	0.375	290	0.396
135	0.376	300	0.396
140	0.378	325	0.394
145	0.379	350	0.390
150	0.381	400	0.382

[1] Calculated from SCHMIDT and SELLSCHOPP, *Forsch. Geb. Ingenieurw.*, **3**, 277–286 (1932).

APPENDIX VIc[1]

THERMAL CONDUCTIVITY OF GASES

Gas	k at 32 degrees Fahrenheit	Increase per degree Fahrenheit
Air	0.0129	0.000020
Ammonia	0.0116	0.000034
Carbon dioxide	0.0079	0.000020
Hydrogen	0.0917	0.000139
Methane	0.0170	0.000040
Nitrogen	0.0131	0.000020
Oxygen	0.0134	0.000020
Water vapor	0.0085	0.000023

[1] McADAMS, *loc. cit.*, pp. 323–325; BADGER, "Heat Transfer and Evaporation," Chemical Catalog Co., p. 12, 1926.

APPENDIX VId[1]

Thermal Conductivity of Liquids Other than Water

Material	Temperature, degrees Fahrenheit	k
Acetic acid 100 %......	32	0.123
	122	0.085
Acetic acid 50 %.......	32	0.182
	176	0.257
Acetone..............	32	0.106
	212	0.092
Aniline...............	32	0.105
	212	0.106
Benzene..............	32	0.085
	122	0.064
25 % $CaCl_2$ brine......	−4	0.274
	176	0.368
25 % NaCl brine.......	−4	0.226
	176	0.322
Butyl alcohol (n)......	32	0.098
	212	0.093
Carbon tetrachloride...	50	0.070
	158	0.058
Ethylene glycol.......	32	0.154
Ethyl acetate.........	54	0.084
	114	0.071
Ethyl alcohol 100 %...	32	0.107
	176	0.101
Ethyl alcohol 90 %.....	32	0.092
	176	0.167
Glycerol 100 %........	32	0.163
	122	0.167
Heptane..............	39	0.082
Hexane (n)...........	86	0.080
	140	0.079
Hydrochloric acid 30 %	68	0.244
	140	0.282
Kerosene.............	68	0.087
	176	0.080
Methyl alcohol........	32	0.130
	176	0.117
Octane...............	86	0.087
	140	0.085
Petroleum oils........	86–212	0.078–0.083
Sulfuric acid 98 %.....	68	0.155
	140	0.163
Sulfuric acid 60 %.....	68	0.094
	140	0.108
Toluene..............	32	0.095
	212	0.082

[1] McAdams, *op. cit.*, pp. 319–320.

APPENDIX VIe[1]

THERMAL CONDUCTIVITY OF NON-METALLIC SOLIDS

Material	Temperature, degrees Fahrenheit	k
Asbestos board	68	0.43
Boiler scale	200–300	1.3–2.1
Brick:		
Building	68	0.4
Carborundum	900	8.9
Carborundum	1,500	6.9
Fire	900	0.6–0.7
Fire	1,500	0.65–0.8
Silica	900	0.8
Silica	1,500	0.9
Sil-o-cel	400	0.042
Sil-o-cel	1,500	0.044
Calcium carbonate	86	1.3
Calcium sulfate	100	0.22
Celotex	70	0.028
Concrete	....	0.5
Cork board	70	0.025
Hard rubber	32	0.09
Glass	....	0.2–0.7
Glass, Pyrex	90–170	0.65
Ice	32	1.27
Magnesia (85%) pipe covering	70	0.04
Porcelain	....	0.6
Wood (across grain)	....	0.2–0.09

[1] McADAMS, *op. cit.*, pp. 311–318.

APPENDIX VII[1]

Specific Heats of Gases in B.T.U. per Pound

SPECIFIC HEATS-GASES
1 ATM.

Deg. C.: -200, -100, 0, 100, 200, 300, 400, 500, 600, 700, 800, 900, 1,000, 1,100, 1,200, 1,300, 1,400, 1,500, 1,600, 1,700, 1,800, 1,900, 2,000

Deg. F.: 0, 500, 1,000, 1,500, 2,000, 2,500, 3,000, 3,500

p: 0.80, 0.70, 0.60, 0.50, 0.40, 0.30, 0.20, 0.10, 0

NO.	SUBSTANCE
9	Air
1	Ammonia
21	Bromine
6	Carbon dioxide below 1341° F.
7	" " above 1341° F.
8	" monoxide
18	Chlorine
15	Fluorine
4	Hydrogen (Multiply by 10)
19	" Bromide
14	" Chloride
4	" Fluoride (HF)
20	" Iodide
5	" Sulfide
22	Iodine
16	Methane (Multiply by 10)
10	Nitric oxide (NO)
8	Nitrogen
11	Oxygen
17	Sulfur (S_2)
12	" Dioxide below 1341° F.
13	" " above 1341° F.
3	Water below 1341° F.
2	" above 1341° F.

[1] McAdams, *op. cit.*, p. 337.

APPENDIX VIII

THE TYLER STANDARD SCREEN SCALE

(This screen scale has as its base an opening of 0.0029 in. which is the opening in 200-mesh 0.0021-in. wire, the standard sieve, as adopted by the Bureau of Standards of the U. S. Government.)

Mesh	Clear opening, inch	Clear opening, millimeters	Opening, fraction of inch (approx.)	Diameter of wire, decimal of an inch
...	1.050	26.67	1	0.148
[1]	0.883	22.43	7/8	0.135
...	0.742	18.85	3/4	0.135
[1]	0.624	15.85	5/8	0.120
...	0.525	13.33	1/2	0.105
[1]	0.441	11.20	7/16	0.105
...	0.371	9.423	3/8	0.092
2½[1]	0.312	7.925	5/16	0.088
3	0.263	6.680	1/4	0.070
3½[1]	0.221	5.613	7/32	0.065
4	0.185	4.699	3/16	0.065
5[1]	0.156	3.962	5/32	0.044
6	0.131	3.327	1/8	0.036
7[1]	0.110	2.794	7/64	0.0328
8	0.093	2.362	3/32	0.032
9[1]	0.078	1.981	5/64	0.033
10	0.065	1.651	1/16	0.035
12[1]	0.055	1.397	....	0.028
14	0.046	1.168	3/64	0.025
16[1]	0.0390	0.991	....	0.0235
20	0.0328	0.833	1/32	0.0172
24[1]	0.0276	0.701	....	0.0141
28	0.0232	0.589	....	0.0125
32[1]	0.0195	0.495	....	0.0118
35	0.0164	0.417	1/64	0.0122
42[1]	0.0138	0.351	....	0.0100
48	0.0116	0.295	....	0.0092
60[1]	0.0097	0.246	....	0.0070
65	0.0082	0.208	....	0.0072
80[1]	0.0069	0.175	....	0.0056
100	0.0058	0.147	....	0.0042
115[1]	0.0049	0.124	....	0.0038
150	0.0041	0.104	....	0.0026
170[1]	0.0035	0.088	....	0.0024
200	0.0029	0.074	....	0.0021
For coarser sizing—3 to 1½-in. opening				
			3	0.207
			2	0.192
			1½	0.148

[1] These screens, for closer sizing, are inserted between the sizes usually considered as the standard series. With the inclusion of these screens the ratio of diameters of openings in two successive screens is as $1:\sqrt[4]{2}$ instead of $1:\sqrt{2}$.

APPENDIX IX

MALLEABLE DETACHABLE CHAINS
(LINK BELT Co., Catalogue 500)

Number	Average pitch, inches	Approximate number links per 10 ft.	Average ultimate strength	Weight per 10 ft., pounds
25	0.902	133	700	2.3
32	1.154	104	1,100	3.5
33	1.394	86	1,190	3.3
34	1.398	86	1,300	4.0
42	1.375	88	1,500	5.5
45	1.630	74	1,600	5.2
52	1.506	80	2,300	8.0
55	1.631	74	2,200	7.2
57	2.308	52	2,800	8.6
62	1.654	73	3,100	10.6
67	2.308	52	3,300	11.5
75	2.609	46	4,000	13.4
77	2.297	52	3,600	14.5
78	2.609	46	4,900	18.8
88	2.609	46	5,750	24.0
103	3.075	39	9,600	40.0
114	3.250	37	11,000	53.0
124	4.063	30	15,000	66.0

APPENDIX X

Properties of Water and Saturated Steam

Temperature, degrees Fahrenheit	Temperature, degrees Centigrade	Absolute pressure, pounds per square inch[1]	Latent heat of evaporation, B.t.u. per pound[1]	Specific volume, cubic feet per pound[1]	Density of liquid water, pounds per cubic foot	Viscosity of liquid water, centipoises[2]
32	0.00	0.0887	1,073.4	3,301	62.42	1.794
35	1.67	0.1000	1,071.8	2,946	62.43	1.692
40	4.44	0.1217	1,069.1	2,445	62.43	1.546
45	7.22	0.1475	1,066.3	2,037.2	62.42	1.420
50	10.00	0.1780	1,063.6	1,704.8	62.42	1.310
55	12.78	0.2140	1,060.9	1,431.8	62.40	1.213
60	15.56	0.2561	1,058.2	1,208.0	62.37	1.129
65	18.33	0.3054	1,055.4	1,022.7	62.34	1.052
70	21.11	0.3628	1,052.7	869.0	62.30	0.982
75	23.89	0.4295	1,050.0	740.9	62.26	0.919
80	26.67	0.5067	1,047.3	633.8	62.22	0.862
85	29.44	0.5960	1,044.6	543.8	62.17	0.810
90	32.22	0.6980	1,041.8	468.5	62.11	0.764
95	35.00	0.8149	1,039.1	404.9	62.06	0.721
100	37.78	0.9487	1,036.3	350.8	62.00	0.682
105	40.56	1.1009	1,033.5	305.0	61.93	0.647
110	43.44	1.274	1,030.8	265.8	61.86	0.616
115	46.11	1.470	1,027.9	232.3	61.79	0.586
120	48.89	1.692	1,025.1	203.6	61.71	0.559
125	51.67	1.941	1,022.2	178.9	61.63	0.535
130	54.44	2.221	1,019.4	157.64	61.55	0.511
135	57.22	2.536	1,016.5	139.17	61.46	0.490
140	60.00	2.887	1,013.6	123.22	61.38	0.470
145	62.78	3.280	1,010.6	109.31	61.29	0.451
150	65.56	3.716	1,007.7	97.23	61.20	0.433
155	68.33	4.201	1,004.7	86.66	61.10	0.417
160	71.11	4.739	1,001.8	77.40	61.00	0.401
165	73.89	5.334	998.8	69.28	60.90	0.386
170	76.67	5.990	995.8	62.14	60.80	0.372
175	79.44	6.716	992.8	55.82	60.69	0.359
180	82.22	7.510	989.8	50.28	60.58	0.347
185	85.00	8.382	986.8	45.36	60.47	0.336
190	87.78	9.336	983.8	41.01	60.36	0.325
195	90.56	10.385	980.8	37.12	60.24	0.315
200	93.33	11.525	977.7	33.67	60.12	0.306
205	96.11	12.771	974.6	30.59	60.00	0.295
210	98.89	14.123	971.5	27.83	59.88	0.287
212	100.00	14.696	970.2	26.82	59.83	0.284
215	101.67	15.594	968.2	25.37	59.76	
220	104.44	17.188	965.1	23.16		
225	107.22	18.91	961.8	21.17		
230	110.00	20.78	958.6	19.388		
235	112.22	22.80	955.2	17.779		
240	115.56	24.97	952.0	16.324		
245	118.33	27.31	948.6	15.011		

[1] Condensed from Keenan, "Steam Tables," *A. S. M. E.*, 1930.
[2] Calculated by the authors from data in "International Critical Tables.'

PROPERTIES OF WATER AND SATURATED STEAM.—(*Continued*)

Temperature, degrees Fahrenheit	Temperature, degrees Centigrade	Absolute pressure, pounds per square inch[1]	Latent heat of evaporation, B.t.u. per pound[1]	Specific volume, cubic feet per pound[1]	Density of liquid water, pounds per cubic foot	Viscosity of liquid water, centipoises[2]
250	121.11	29.82	945.2	13.824		
260	126.67	35.43	938.4	11.762		
270	132.22	41.85	931.4	10.061		
280	137.78	49.20	924.2	8.644		
290	143.33	57.55	917.0	7.459		
300	148.89	67.01	909.6	6.464		
310	154.44	77.68	902.1	5.623		
320	160.00	89.65	894.4	4.910		
330	165.56	103.03	886.5	4.303		
340	171.11	117.99	878.5	3.784		
350	176.67	134.62	870.2	3.338		
360	182.22	153.01	861.7	2.954		
370	187.78	173.33	853.0	2.622		
380	193.33	195.70	844.1	2.333		
390	198.89	220.29	834.9	2.0816		
400	204.44	247.25	825.5	1.8608		

[1] Condensed from KEENAN, "Steam Tables," *A. S. M. E.*, 1930.

[2] Calculated by the authors from data in "International Critical Tables."

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